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A TEXT BOOK OF PHYSICS

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FOR THE USE OF STUDENTS OF
SCIENCE & ENGINEERING

BY

J. DUNCAN, WH. EX., M.I.MECH.E.

AND

S. G. STARLING, B.SC., A.R.C.SC.

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A TEXT BOOK OF PHYSICS

FOR THE USE OF STUDENTS OF SCIENCE AND ENGINEERING

BY

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HEAD OF THE DEPARTMENT OF CIVIL AND MECHANICAL ENGINEERING AT THE
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PART V

MAGNETISM AND ELECTRICITY

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PREFACE

THE preparation of this volume was undertaken to meet a demand that has been growing for some years past for a text-book of Physical Science which should connect more intimately than has hitherto been usual the scientific aspects of Physics with its modern practical applications. The reader must be left to judge how far the authors have succeeded in thus combining the outlooks of the man of science and the engineer.

The contents have been selected to meet the requirements of various classes of students: those preparing for Intermediate and other examinations of London and other Universities; and those entering for appointments in the Army, Navy, and Civil Service, or offering themselves for examination in Electrical Engineering (Grade I.) by the City and Guilds of London Institute.

The book has been arranged in parts, in accordance with the divisions of the subject found convenient in most schools and colleges. Part I., Dynamics, comprises the sections of Mechanics and Applied Mathematics usually studied, and includes sections on motion, statics, and the properties of fluids. Part II., Heat; Part III., Light; Part IV., Sound; and Part V., Magnetism and Electricity; deal respectively with the principles of these subjects and their applications.

Complete courses of laboratory work have been provided in each Part. Many physical laboratories are equipped with apparatus differing in some respects from the instruments here described, nevertheless the guidance given will enable intelligent use to be made of other forms of apparatus designed for the same or similar purposes.

Attention is directed to the experimental treatment of dynamical principles, because its neglect, which is unfortunately common, makes it difficult for a student to secure a thorough and systematic knowledge of physical science. The complete course of experimental work has been devised to meet both the requirements of the physicist and of the engineer; in cases where the methods of treatment adopted by these differ radically, the teacher or student may choose the experiment which best suits his special needs.

In Part V., the treatment of the Dynamo, Telegraph, and so on, is that which follows naturally and logically from the earlier theoretical principles explained; technical considerations of design and construction have been omitted as unsuitable in a text-book of Physics.

A large number of worked-out examples have been included to assist the student to understand the text and to solve the exercises at the ends of the chapters. Many of these exercises have been taken, with the permission of the authorities to whom grateful acknowledgments are made, from examination papers, the source being given in each case. Questions marked L.U. are from examination papers of the London University and those with C.G. from papers of the City and Guilds of London Institute.

Answers have been supplied in the case of numerical exercises, but it is too much to hope that these are entirely free from errors. The authors will welcome any corrections which readers may send to them.

The authors are glad of this opportunity to express their indebtedness to Prof. Sir Richard Gregory and Mr. A. T. Simmons for constant assistance and invaluable hints while the book was in preparation and passing through the Press.

J. DUNCAN.
S. G. STARLING.

1918.

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PART V

MAGNETISM AND ELECTRICITY

CHAPTER LXI

MAGNETISATION

Lodestone.—The peculiar properties of a mineral called **lodestone**, found in the neighbourhood of Magnesia in Asia Minor, were known in very early times, and consist in its power of attracting fragments of the same material, and of setting in one particular direction when suspended. This mineral is now known as **magnetite**, and is an oxide of iron. If a piece of magnetite be dipped into iron filings it will be found that they adhere to it, more particularly at certain places. There are in general two such places on any piece of magnetite where the filings adhere in greatest quantity. On suspending a piece of magnetite in a stirrup by means of a piece of silk, it will be found that it will not rest in just any position, but only in such a position that the line joining the two places where the filings adhere in greatest quantity, points north and south.

Magnets.—One of the most important properties of these natural magnets is that they can communicate their properties to pieces of steel. Thus, if a steel knitting needle be stroked from one end to the other with a piece of magnetite, using for the place of contact that at which the filings adhere most freely, the needle will now possess the property of attracting iron filings, and of setting north and south when suspended. The effect may be increased by stroking the needle several times, but always with the same part of the magnetite and in the same direction.

Such a needle is said to be a **magnet**. Magnets are now made of bars of steel, which are many times more powerful than the needle described, and their method of manufacture will be considered later. Their essential properties, however, differ in no way from the magnetised needle; they are simply more powerful, and for this reason are generally used in experimental work.

EXPT. 161.—**Magnetisation of needle.** Dip a new knitting needle into iron filings and notice that they do not adhere to it. Place it in a thin wire stirrup S (Fig. 708) supported by a single silk fibre. Note that the needle will remain in any direction in which it is placed. Now remove the needle and stroke it from one end to the other with the end of a bar magnet. Dip the needle into iron filings; it will be found that the filings adhere at the ends, but not to the middle portion. Replace it in the stirrup, and notice that it will no longer remain in just any direction, but always sets north and south.

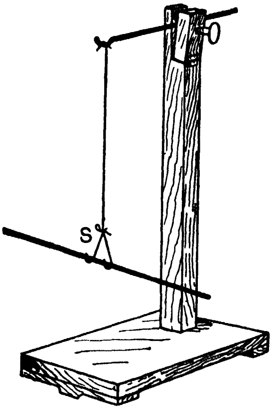


FIG. 708.—Suspended magnetised needle.

Magnetic poles.—On dipping a bar magnet into iron filings it is found that they adhere most freely at and near the ends. These places are called **poles**. It was seen in Expt. 161 that the magnetised needle has poles, one at each end. If the end which points to the north be now marked by placing a piece of paper on it, it will be observed that this end always points north; the other end cannot be made to rest pointing north. Thus, one pole always points north, and is called the **north-seeking** or N pole. The other is the **south-seeking** or S pole.

Force between poles.—Magnetic poles always exert forces upon each other. The force between any pair of poles depends upon their distance apart, becoming greater as they approach each other, but it is universally true that **N poles repel each other and S poles repel each other, while a N and a S pole attract each other. Thus there is a repulsion between poles of a like kind and an attraction between unlike poles.**

EXPT. 162.—**Forces between poles.** Magnetise two knitting needles and suspend them in turn in the stirrup as in Fig. 708; in each case mark the N pole. Now, with one of them suspended, bring a pole of the other near a pole of the suspended needle, trying all the four pairs of poles in the same way in turn. It will be found that like poles repel each other while unlike poles attract each other.

Molecular theory of magnetisation.—In early times many theories were suggested to account for the facts described, but one theory was suggested which has been modified and improved until at the present time it is held universally. This is, that every magnetic substance, or substance capable of being rendered a magnet, consists of smaller parts which are themselves magnets. These minute magnets, which are sometimes called *molecular magnets*, are not arranged in any one particular direction when the substance is unmagnetised, but are orientated indiscriminately in all directions. The act of magnetisation consists in arranging them in one direction. Fig. 709 (*a*) represents diagrammatically the arrangement in an unmagnetised bar of iron, the strokes representing diagrammatically the molecular magnets, the arrow-heads being the N poles; (*b*) represents the same bar when magnetised. It will be noticed that in (*b*) the little *n* and *s* poles of the molecular magnets are close to each other within the material, but at the N end are the free *n* poles without any corresponding *s* poles, and at the S end we have the free *s* poles. This explains very well the reason for the poles of a bar magnet being situated at the ends and not in the middle of the bar.

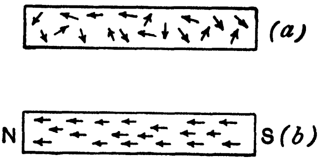


FIG. 709.—Magnetisation.

The term *magnetic molecule* requires some consideration. We do not mean the chemical molecule or necessarily the atom; in fact, at this stage we must not endeavour to state precisely their nature, but merely to consider that they are extremely small bodies, each having a N and S pole, and free to turn in any direction into which external magnets tend to set them.

EXPT. 163.—**Poles produced on magnetising a needle.** Mark one end of a knitting needle and stroke it several times from the unmarked to the marked end, with the N pole of a bar magnet. Then suspend it and note that the unmarked end points north. Repeat with the S pole of the bar magnet, and note that the marked end now points north. Repeat, stroking from the marked to the unmarked end. From these results it will be seen that the end at which the applied pole of the bar magnet leaves the needle has always the opposite polarity to that by which it is stroked.

Evidence for molecular theory.—The last experiment is easily explained on the molecular theory of magnetisation. For on reference to Fig. 709 it will be seen that on stroking the bar from, say, left to right with the N pole of a bar magnet, the S poles of the molecular magnets will turn towards the N pole of the bar magnet, and, since the latter leaves at the right-hand end, this end will be a S pole.

Again, **if a magnet be broken in the middle**, new poles will appear at the freshly produced ends, as in Fig. 710 (*b*). This is easily explained; for the act of breaking leaves a set of molecular S poles to the left of the gap, and of N poles at the right. A further breaking will produce additional poles, as in Fig. 710 (*c*).

The production of a magnet may be imitated by placing a **test-tube of steel filings** with one end near the pole of a strong bar magnet. On removing the tube **without shaking**, and testing it by hanging it near a suspended magnet (Fig. 708), it will be found that the ends of the tube of filings are poles, that which was placed near the pole of the bar magnet being opposite in kind to the pole of the bar magnet. 'The fragments of steel become permanent magnets and act like the molecular magnets in the bar. On shaking the tube all traces of poles at the ends disappear.

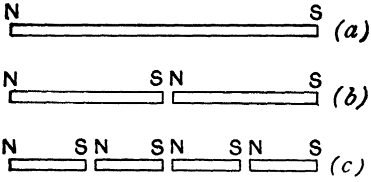


FIG. 710.—Effect of breaking a magnet.

If an unmagnetised bar of steel be placed with one end on the pole of a bar magnet and **tapped with a hammer**, it will be found to become a fairly strong permanent magnet. If the bar be afterwards tapped, its magnetism is gradually lost. The tapping assists the arrangement of the molecular magnets in the first part of the experiment when the bar magnet is near, and destroys their arrangement in the second part when the bar magnet is absent.

Magnetisation disappears at high temperatures. This fact may be shown by heating a magnetised knitting needle to red heat by a long burner which will heat the whole of it at the same time, and then allowing it to cool in an east and west direction (the reason for this will be seen later). On testing the needle with iron filings, and by bringing it near a suspended needle, it will be found to be unmagnetised.

The amounts of N and of S pole on a magnet are equal.—Perhaps the strongest evidence in favour of the molecular theory of magnetisation lies in the fact that on any piece of iron or steel **the total amount of N pole is always equal to the total amount of S pole**, because the act of magnetisation does not involve the putting of pole on the bar or needle, but merely the arrangement of the molecular magnets as in Fig. 709. This may be proved by floating a bar magnet upon a piece of wood on water. The magnet is then free to move upon the surface in any direction. It will be found that the magnet rotates until it points north and south, but it does

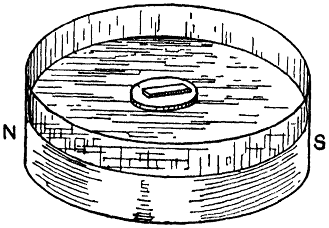


FIG. 711.—Magnet floated on water.

not move bodily either north or south. Thus, the forces on the N and S poles are equal and opposite, giving rise to a couple but not to a resultant force of translation. This shows that the total amounts of N and of S pole are equal.

The **phenomenon of saturation** also points to the truth of the molecular theory. It will be seen later that a piece of iron or steel cannot be magnetised to more than a certain amount; when all the molecular magnets have been turned into the same direction it is obviously impossible to increase the magnetisation any further.

Soft iron and steel.—The chief magnetic difference between iron and steel is that iron is easily magnetised and readily demagnetised, while steel is not so easily magnetised, but retains its magnetisation much more than does iron.

Thus, if the pole of a strong bar magnet be applied to one end of a bar of soft iron, the bar of iron itself becomes a strong magnet and will pick up a considerable amount of iron filings. On removing the bar magnet the filings immediately drop off the iron rod. Also, if its polarity be tested by bringing its poles near those of a suspended needle, they will be found to be arranged as in Fig. 712.

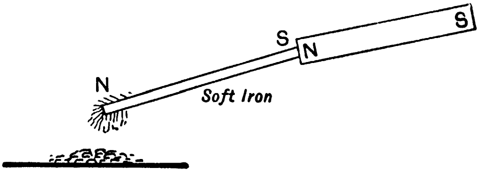


FIG. 712.—Magnetisation of soft iron.

On repeating this experiment with a steel rod it will be found that it is not nearly so strongly magnetised by contact with the magnet as was the iron rod, but on removing the magnet the steel rod is found to retain its magnetisation.

It is thus seen that the presence of a magnet always magnetises a piece of soft iron, and may magnetise a piece of steel if the previous magnetisation of the steel is sufficiently feeble. Hence, if a piece of soft iron be brought near to the N pole of a suspended magnet, the nearer parts of the iron become a S pole, as would be expected from the molecular theory, and there will now be an attraction between the iron and the N pole of the needle. This is characteristic of soft iron. The presence of a magnet produces such poles in the iron that **there is always an attraction between the iron and the magnet.** In the case of steel which is already magnetised, and has therefore so-called **permanent magnetisation**, there may be an attraction between it and a magnet, or there may be repulsion, as was seen on p. 770.

Inverse-square law.—In all cases where an effect is radially and uniformly distributed with respect to a point, the effect falls off inversely as the square of the distance from the point. We have already seen (p. 544) such a distribution in the case of light radiated from a point. The effect on a point magnetic pole due to another follows the same law. It is usual to attempt to prove this inverse-square law in the case of each phenomenon to which it applies, and on p. 786 we shall do so for the law of force between magnetic poles. For the present, however, it will be taken for granted. Thus, **the force between two point poles varies inversely as the square of their distance apart,** or, expressed mathematically,

$$F \propto \frac{m_1 m_2}{d^2}.$$

That the force between the poles varies directly as the product of their strengths m_1 and m_2, follows from the fact that it is independent of the presence of other poles. Thus, there is a certain force between two given poles A and B; if a third pole, C, be added to A, we have a force between (A+C) and B which is equal to the sum of the separate forces between A and B and between C and B. If the process be continued, A, B, C, etc., being all unit poles, we see that the force between any two compound poles is proportional to the product of the number of units of pole in each.

Unit pole.—In the relation $F \propto m_1 m_2/d^2$, where F is the force in dynes between the two poles m_1 and m_2 situated d centimetres apart, let the poles be chosen of such a strength that, when they are equal, and d is one centimetre, F is one dyne. Each of these is then taken as the **unit of magnetic pole**, and, measuring m_1 and m_2 in terms of these units, we may then write our equation,

$$\text{Force} = \frac{m_1 \times m_2}{d^2} \text{ dynes.}$$

Thus, **the unit magnetic pole may be defined as such a one, that when placed one centimetre from an equal pole, the force between them is one dyne, the poles being situated in air.**

EXAMPLE.—Magnetic N poles of strength 50 and 90 units are placed at the corners B and C of an equilateral triangle ABC of side 10 cm. If a S pole of strength 80 be placed at A, find the resultant force on A.

$$F = \frac{m_1 m_2}{d^2}.$$

$$\text{For poles A and B,}\quad F=\frac{50\times 80}{10^2}=40 \text{ dynes.}$$

$$\text{,, ,, A and C,}\quad F=\frac{80\times 90}{10^2}=72 \text{ dynes.}$$

Since both these forces are attractions, draw AD to scale equal to 40 and AE equal to 72. Complete the parallelogram ADFE, and the diagonal AF then represents the resultant force on A. Its magnitude may be calculated from the relation on p. 82.

Thus,

$$AF=\sqrt{(72)^2+(40)^2+(2\times72\times40\times\cos 60^\circ)}$$

$$=\sqrt{5184+1600+2880}$$

$$=\sqrt{9664}=98 \text{ dynes.}$$

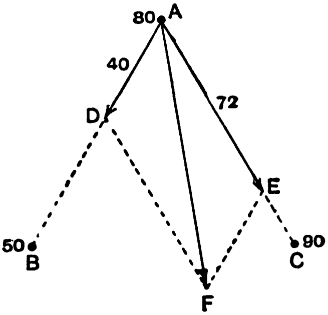


FIG. 713.—Forces between magnetic poles.

EXERCISES ON CHAPTER LXI.

1. Describe how you would magnetise a knitting needle so that one particular end should be a N pole. How would you test this when made ?

2. How would you magnetise a needle so that it should have a N pole at each end, and a S pole in the middle ?

3. What kinds of forces are exerted between magnetic poles ? Describe how you would demonstrate the proof of your statements.

4. A soft iron rod is held with an end near the south pole of a bar magnet. Give a diagram illustrating the magnetisation that the bar will acquire, and explain why it should become magnetised in this way.

5. A magnetic pole of strength 180 units is situated in line with a magnetised knitting needle, and at a distance of 30 centimetres from its middle point. If the length of the needle is 20 cm. and its strength of pole 40 units, find the force on the given pole.

6. Find the force on a magnetic pole of strength 40 units, situated at a distance of 30 cm. from each end of a magnetised needle whose length is 20 cm. and strength of pole 30 units.

7. Find the force between two magnets placed in the same straight line with a distance of 18 cm. between their middle points, if one has length 12 cm. and pole strength 60, and the other, length 6 cm. and pole strength 45 units.

8. Two magnetised knitting needles, each of mass 4 grams, are suspended with their two S poles together and the N pole ends hanging downwards. The magnets are free to turn about the S poles, and the N poles move apart until the distance between them is 4 cm. Find the pole strength of each needle, assuming that the N poles are situated 20 cm. below the S poles, and the centres of gravity are 10 cm. from the S poles.

9. A knitting needle is magnetised and then broken into four pieces equal in length. What will be the magnetic condition of the different pieces ? What view of the nature of a magnet does such an experiment suggest ?

10. State the law of action between magnet poles.

Two north poles repel one another with a force of 2·4 dynes when their distance apart is 2 cm. What will be their distance apart when the force is 3·6 dynes ? Find also their repulsive force when their distance apart is 3 cm. Calcutta University.

CHAPTER LXII

MAGNETIC FIELD

Magnetic field near a magnet.—In Chapter LXI. certain effects of one magnet upon another were studied. Thus, there is a region around any magnet throughout which its influence may be detected. This is widely extended in the case of powerful magnets, and restricted for feeble magnets. The term **magnetic field** is sometimes applied in a general manner to this region, but the term has further a definite and more restricted meaning.

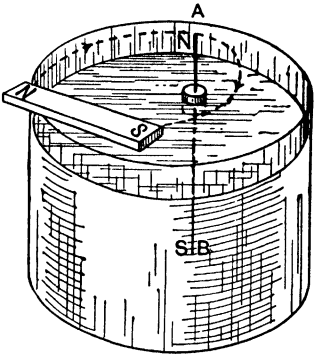


FIG. 714.—Experiment to illustrate magnetic field.

At every point near a magnet, or system of magnets, a magnetic pole would experience a force in some definite direction, and if the pole is free to move it would travel in this direction. This direction is called the **direction of the magnetic field** at that point. Although free poles cannot be obtained, their effect may be illustrated roughly by passing a long magnetised needle through a cork and floating it on water (Fig. 714). On bringing a bar magnet NS near, and on a level with the upper pole A, the force on this will be much greater than that acting upon the lower pole B. If A be a N pole, it will, on starting near N of the bar magnet, be driven along a curved path, and eventually reach S. Thus, at each point of its path it experiences a force driving it in the direction of the magnetic field at that point.

Again, a suspended needle, or compass needle. experiences two forces, one on its N pole in the direction of the magnetic field, and the other on its S pole in the opposite direction. These in general form a couple (Fig. 723) tending to twist it into the direction of the magnetic field, but when it attains this direction, the couple becomes zero and it is then in equilibrium. Thus, a suspended needle or compass needle, if small enough, comes to rest in the direction of the magnetic field. Of course, if the needle is large, its two poles may be in places where the direction of the field is not the same, and the finding of its position of equilibrium is not then so easy.

Lines of force.—A line whose direction at each point is that of the magnetic field at that point is called a **magnetic line of force.** Thus, the magnetic pole A (Fig. 714) traces in its motion, a line of force. A line of force may also be defined as the path along which a N pole, perfectly free to move, would travel. As a perfectly free N pole cannot be obtained in practice, magnetic lines of force are usually obtained by means of a small compass needle, which, as we have seen, always comes to rest in the direction of the magnetic field.

EXPT. 164.—**Lines of force of a bar magnet.** Place a bar magnet on a sheet of drawing paper, and draw in its outline. Divide up this outline by a number of approximately equidistant marks. Place a small compass needle with one pole as nearly as possible upon one of these marks, and make a mark A (Fig. 715) on the paper at the other pole of the compass. Now place the first end of the compass on A and make another mark B, and so on, until the line traced out reaches the edge of the paper or returns to the magnet. Draw an even curve through the points obtained, and mark it with an arrow to show the direction in which the compass pointed. Repeat the process until every mark upon the outline of the magnet is either the beginning or end of a line of force. The lines of force in Fig. 716 were obtained in this way.

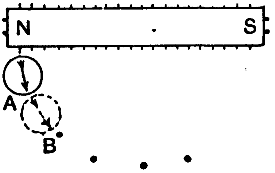


FIG. 715.—Plotting lines of force.

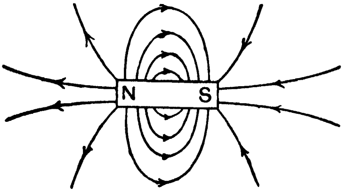


FIG. 716.—Lines of force of a bar magnet.

Note.—Lines of force arise on a N pole and end on a S pole; also no two lines can meet or cross each other, for if they did it would mean that the compass would have two directions at the same time, where the lines cross, which is absurd.

EXPT. 165.—**Lines of force of two bar magnets, unlike poles together.** Place two bar magnets on a sheet of drawing paper as in Fig. 717, and map out the lines of force as in Experiment 164.

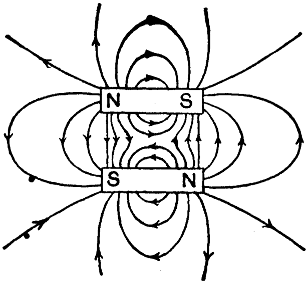


FIG. 717.—Lines of force of two bar magnets, unlike poles together.

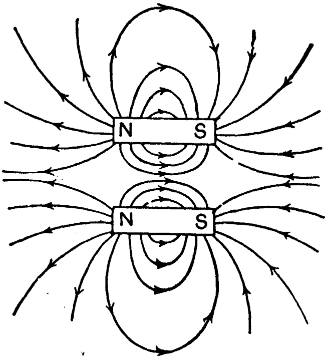


FIG. 718.– Lines of force of two bar magnets, like poles together.

EXPT. 166.—**Lines of force of two bar magnets, like poles together.** Repeat Experiment 165, but with the N poles together as in Fig. 718.

EXPT. 167.—**Lines of force by means of iron filings.** Place a piece of drawing paper over the magnets in Expts. 164, 165 and 166, and sprinkle iron filings on the paper. Tap the paper gently until the filings arrange themselves in lines, as in Fig. 719. Each filing becomes a magnet in the field of the bar magnet, and the filings then hang together in chains. By coating the paper with molten paraffin wax before the experiment and allowing it to cool, a smooth surface is presented to the filings. After they have taken their proper arrangement the lines can be rendered

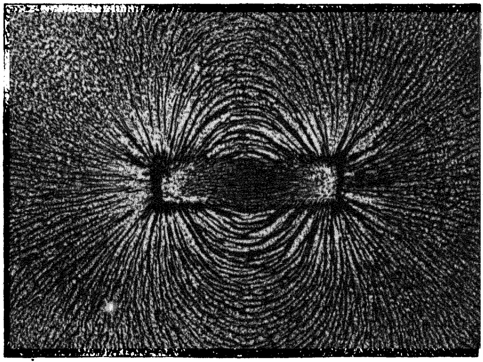


FIG. 719.—Lines of force by means of iron filings.

permanent by passing a bunsen flame over the sheet, which melts the wax and imbeds the filings. Figs. 719, 720 and 721 were obtained in this way.

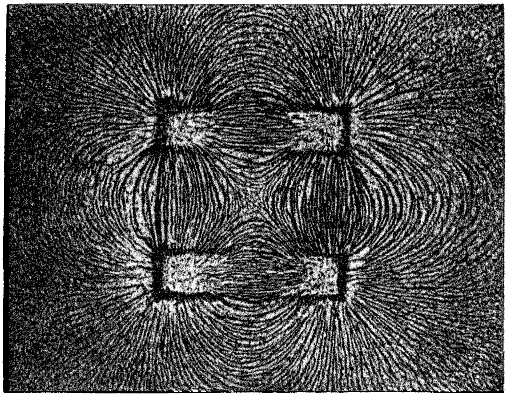


FIG. 720.—Lines of force by means of iron filings.

Strength of magnetic field, or magnetic intensity.—Having seen that the magnetic field has a definite direction at every point, its strength must now be considered. It is fairly obvious from Figs. 716-721 that the field is stronger where the lines of force are packed closely together than where they are far apart. In order, however, to define exactly the strength of field at any point, a unit pole must be imagined to be placed at the point. This pole experiences a

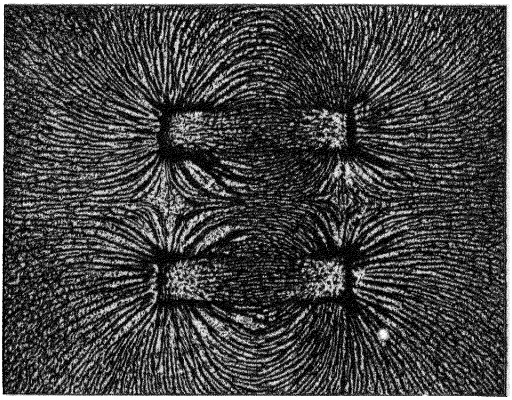


FIG. 721.—Lines of force by means of iron filings.

force, which force is taken as the measure of the strength of field. Thus, the **strength of field, or magnetic intensity at any point is the force that would be exerted on a unit N pole situated at this point.** This quantity is usually denoted by the letter H. Thus, the force on a pole *m*, situated in a magnetic field of intensity H, is *m*H dynes.

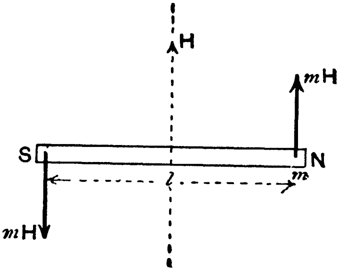


FIG. 722.—Couple acting on a magnet.

Magnetic moment of a magnet.—Consider a magnet NS (Fig. 722) having a pole of strength *m* at either end, to be situated at right angles to a magnetic field of intensity H. Then each pole experiences a force *m*H dynes, these forces act in opposite directions, and therefore constitute a couple (p. 125) whose turning moment is H*ml*, where *l* is the length of the magnet or distance between the poles. This couple consists of two parts, the field H and the part *ml* belonging to the magnet. The quantity *ml* is called the **magnetic moment** M of the magnet.

Hence, Couple = HM.

As a rule magnets do not have the two poles situated exactly at the ends, and hence both *m* and *l* are indefinite quantities. Still, the magnetic moment M is not indefinite, for it may be measured by mechanical means, by determining the couple necessary to hold the magnet at right angles to a given field. Thus, the magnetic moment may also be defined as **the couple required to hold a magnet at right angles to a field of unit intensity.**

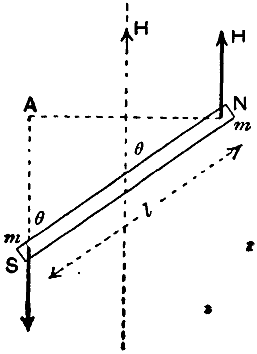


FIG. 723.—Couple acting on a magnet inclined to a magnetic field.

Couple acting on a magnet in any position.—If the magnet be inclined at an angle θ to the field H (Fig. 723) the force on each pole is still H*m*, but the perpendicular distance between the forces is

$$AN = l \sin\theta\,;$$

$$\therefore\ \text{couple} = Hml \sin\theta$$

$$= HM \sin\theta.$$

From this it is seen that when $\theta = 90°$, the couple is HM, as found above, and when $\theta = 0°$ the couple is zero, since $\sin 0° = 0$. Hence, the only position in which a freely suspended magnet is in equilibrium in a given field is when its direction is coincident with that of the field.

Field due to a bar magnet.—The general form of the field due to a bar magnet is shown in Fig. 716. The strength of field at certain points may be calculated without difficulty. Thus, take a point P (Fig. 724) on the line passing through the poles of the magnet of pole strength m. Let l be half the length of the magnet and d the distance of P from its middle point. Then the distance of P from N is $(d-l)$, and from S the distance is $(d+l)$.

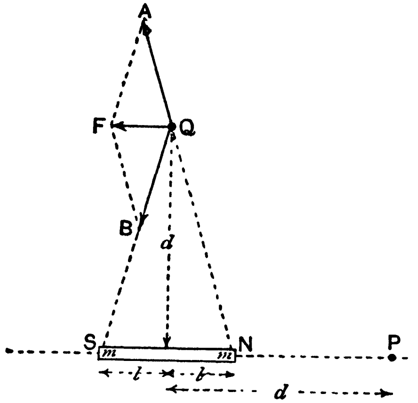


FIG. 724.—Strength of field due to a bar magnet, by calculation.

Imagine a unit N pole placed at P.

Force on unit pole due to N $= \dfrac{m}{(d-l)^2}$.

Force on unit pole due to S $= \dfrac{m}{(d+l)^2}$.

Since these two forces are in the same line but have opposite senses, we have

$$\text{Resultant force} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

$$= m\,\frac{(d+l)^2-(d-l)^2}{(d^2-l^2)^2}$$

$$= \frac{4ml\,.\,d}{(d^2-l^2)^2}.$$

This is the force on unit pole, and is therefore the strength of the field at P. Also, since $2ml$ is the magnetic moment M of the magnet,

$$\text{Strength of field at P} = \frac{2\text{M}d}{(d^2-l^2)^2}.$$

If P is at a considerable distance from the magnet, l^2 may be neglected in comparison with d^2, and the strength of field is then $2\text{M}/d^3$.

At a point Q on the line bisecting the magnet at right angles (Fig. 724), the strength of field may be determined in a somewhat similar manner, by placing a unit N pole at Q and finding the force acting on it. Thus,

$$\text{Force due to N} = \frac{m}{\text{QN}^2},$$

$$\text{,, \quad ,, \quad S} = \frac{m}{\text{QS}^2}.$$

Representing these forces by QA and QB, the resultant force will be represented by QF.

From the similarity of the triangles FBQ and NQS,

$$\frac{\text{FQ}}{\text{BQ}} = \frac{\text{NS}}{\text{QS}}.$$

Now, $\text{BQ} = \frac{m}{\text{QS}^2}$, and $\text{NS} = 2l$;

$$\therefore \text{FQ} = \frac{2ml}{\text{QS}^3}.$$

Also, since $\text{QS}^2 = d^2 + l^2$, $\text{QS}^3 = (d^2 + l^2)^{\frac{3}{2}}$;

$$\therefore \text{FQ} = \frac{2ml}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{\text{M}}{(d^2 + l^2)^{\frac{3}{2}}}$$

which is therefore the strength of field at Q due to the magnet.

If d is so great compared with l, that l^2 may be neglected in comparison with d^2, the strength of field is M/d^3.

Neutral point.—In plotting the lines of a force of a magnet (Fig. 716), it is the resultant of the fields due to the magnet and the earth that has been found. Near the magnet, the earth's field has little effect, as it is feeble in comparison with that of the magnet. At greater distances, however, the earth's field is more important, and at very great distances from the magnet the earth's field only is of consequence. It will therefore happen that, at some points, the fields of the earth and the magnet are equal, and if they are also opposite in direction, the resultant is zero. The compass needle would set in any direction at such points. They are called **neutral points.**

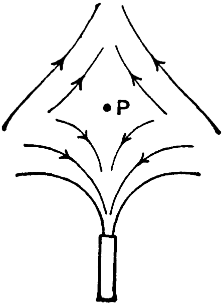


FIG. 725.—Neutral point.

EXPT. 168.—**Neutral point.** Place the bar magnet on a sheet of drawing paper with its S pole pointing north. Plot the lines of force diverging from the S pole. It will be found that they are of the form shown in Fig. 725 in the neighbourhood of some such point as P. Draw the lines nearer and nearer to P until its position is determined. Measure the distance d from P to the middle of the magnet, and, taking the earth's field strength as 0·18, calculate the magnetic moment of the magnet from the relation $2M/d^3 = 0{\cdot}18$; or use the more exact relation

$$\frac{2Md}{(d^2 - l^2)^2} = 0{\cdot}18.$$

The magnetometer.—It has already been seen that a suspended needle points north and south, that is, it sets in the direction of the earth's magnetic field. If, however, a magnet be brought near the needle, it may be deflected from the true north and south position; it will set along the resultant magnetic field due to the earth and the magnet.

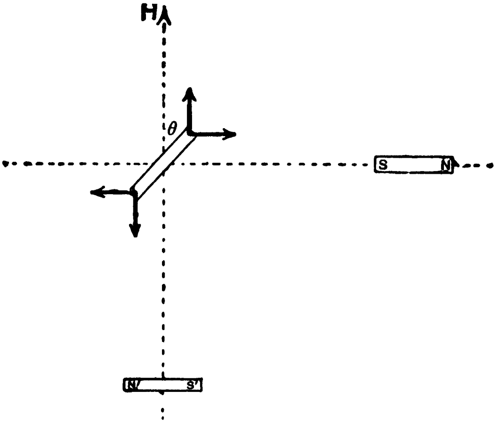


FIG. 726.—Diagram of the magnetometer.

Let the field due to the magnet NS be at right angles to that due to the earth, as in Fig. 726. The suspended magnet will take up some such position as shown, making an angle θ with the earth's field.

Hence, the couple due to the earth's field is $Hm \sin\theta$ (where m is the magnetic moment of the suspended magnet), and tends to rotate it into the direction of the earth's field (p. 781). In a similar manner the deflecting couple due to the magnet's field F, is $Fm \cos\theta$, and the suspended magnet is in equilibrium when these two couples, which tend to rotate it in opposite directions, are equal,

$$\therefore\ Fm \cos\theta = Hm \sin\theta;$$

$$\therefore\ \frac{F}{H} = \tan\theta.$$

For the magnet NS in the position shown, which is known as the **end on**" position, $F=2M/d^3$ (p. 782), so that

$$\frac{2M}{d^3}=H\tan\theta, \quad \text{or} \quad \frac{M}{H}=\frac{d^3}{2}\tan\theta.$$

If the magnet is placed at N′S′, then $F=M/d^3$, and

$$\frac{M}{d^3}=H\tan\theta, \quad \text{or} \quad \frac{M}{H}=d^3\tan\theta.$$

This is known as the "**broadside**" position.

The **magnetometer** has a variety of forms, a common one being shown in Fig. 727. The magnetic needle may be supported on a needle point, or suspended by a fibre. It is provided with a long pointer pp', so that its position with respect to a horizontal circular scale may be read to half a degree or less.

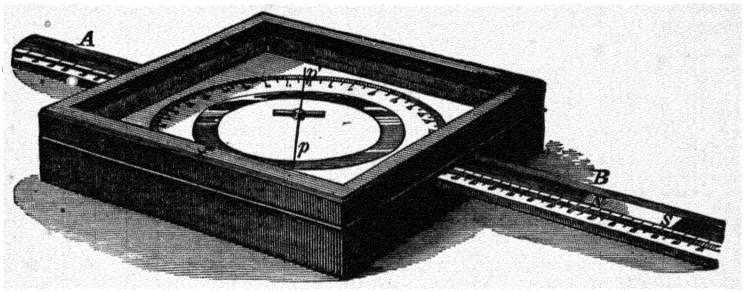


FIG. 727.—The magnetometer.

In Fig. 727 the magnet producing the deflection is shown in the **end on** position at B, and is either east or west of the suspended needle. For the **broadside** position the magnetometer must be turned round so that the magnet is either north or south of the suspended needle. Its distance may be measured upon the linear scale.

Use of the magnetometer.—The magnetometer may be used for comparing magnetic fields, magnetic moments, or finding the ratio of the values of a magnetic moment and a field. In all cases the procedure is as follows:

(i) The instrument is turned round, and levelled if necessary, until both ends of the pointer are at 0° on the scale when there is no deflecting magnet near.

(ii) The magnet B is placed with its middle point at a certain distance d from the needle, and the deflection indicated by **both**

ends of the pointer observed. This is to correct for want of symmetry of the scale and pointer.

(iii) If the magnet was east of the needle in the first position, it is now removed to an equal distance west of it, or *vice versa*, and the deflection again observed.

This is to correct for the fact that the zero of the linear scale may not be exactly at the middle of the suspended needle.

(iv) The magnet is now turned over so that the end which pointed east now points west, and all the observations repeated. The magnet may not be symmetrical in its polarity, but this is corrected for by this reversal of its direction.

There are now eight readings of the deflection, and the mean of the eight is the deflection, free from the above errors. The readings may with advantage be recorded as follows :

	Deflection.	
Magnet E of needle, N pole pointing E - -		
,, ,, ,, ,, W - -		
Magnet W of needle, N pole pointing W - -		
,, ,, ,, ,, E - -		
Distance $d=$ cm.	Mean deflection $\theta=$.	

Expt. 169.—**To prove the relation $\frac{M}{H}=\frac{(d^2-l^2)^2}{2d}\tan\theta$ for the end on position.**

Follow the above instructions, with the magnet at 50 cm. from the needle in the end on position, and observe the mean deflection. Repeat at distances of 45 cm., 40 cm., and 35 cm. In each case calculate the value of $(d^2-l^2)\tan\theta/2d$, and show that this is approximately constant; for each distance calculate the percentage error introduced in using $(d^3\tan\theta)/2$ instead of the more exact relation.

Expt. 170.—**To prove the relation $\frac{M}{H}=(d^2+l^2)^{\frac{3}{2}}\tan\theta$ for the broadside position.**

Repeat the last experiment, using the magnet in the broadside instead of the end on position. Calculate $(d^2+l^2)^{\frac{3}{2}}\tan\theta$ for each distance, and find the percentage error at each distance if $d^3\tan\theta$ were used.

Proof of inverse square law.—It will have been noticed by the student that the formula for the strength of field due to a magnet was calculated on p. 782, by assuming the inverse square law to be true ; hence, the validity of the results depends upon the truth of the law, and the constancy of the values for M/H in the magnetometer experiments is a good indication of the truth of the law.

Comparison of magnetic moments.—By employing different magnets in the magnetometer experiment, the ratio of their magnetic moments may be found. Using the first magnet of magnetic moment M_1 and obtaining the corresponding mean deflection θ_1, as on p. 786, for distance d_1.

$$\frac{M_1}{H} = \frac{d_1^3}{2} \tan \theta_1,$$

and for the second magnet,

$$\frac{M_2}{H} = \frac{d_2^3}{2} \tan \theta_2;$$

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3 \tan \theta_1}{d_2^3 \tan \theta_2}.$$

If more exact results are required, the longer expressions for M/H must be used.

Expt. 171.—**To compare magnetic moments.**—Find $d^3 \tan \theta$ for each of two magnets in turn, using three distances d, in each case, and obtain the ratio M_1/M_2 as above.

Comparison of magnetic fields.—The strengths of magnetic field at two places may be compared by performing the magnetometer experiment with the same magnet at both places. Thus, at the first place let H_1 be the strength of field,

then,
$$\frac{M}{H_1} = \frac{d_1^3}{2} \tan \theta_1.$$

At the second place, where H_2 is the magnetic field,

$$\frac{M}{H_2} = \frac{d_2^3}{2} \tan \theta_2;$$

$$\therefore \frac{H_1}{H_2} = \frac{d_2^3 \tan \theta_2}{d_1^3 \tan \theta_1}.$$

Equivalent length of magnet.—Since the poles of a magnet are not merely at its ends, but are spread over an appreciable part of the sides, it is not strictly correct to take l as half the actual length of the magnet in the above cases. For every magnet, however, there must be some equivalent length l, such that $2ml$ is the moment of the magnet, and it is this value of l that should be used in the above experiments.

This equivalent length l may be found for any magnet by making observations and finding $(d^2 - l^2)^2 (\tan\theta)/2d$ for two distances d_1 and d_2. These results should be equal, hence

$$\frac{(d_1{}^2 - l^2)^2}{2d_1}\tan\theta_1 = \frac{(d_2{}^2 - l^2)^2}{2d_2}\tan\theta_2,$$

and, since d_1, d_2, $\tan\theta_1$ and $\tan\theta_2$ are now known, l may be calculated.

The following instructive method, due to Mr. E. Edser, may be employed. Using the broadside position, we have,

$$\frac{\mathsf{M}}{\mathsf{H}} = (d^2 + l^2)^{\frac{3}{2}} \tan\theta,$$

or

$$d^2 + l^2 = \left(\frac{\mathsf{M}}{\mathsf{H}}\right)^{\frac{2}{3}} \frac{1}{\tan^{\frac{2}{3}}\theta} = \left(\frac{\mathsf{M}}{\mathsf{H}}\right)^{\frac{2}{3}} \cot^{\frac{2}{3}}\theta.$$

If a series of values of d and θ be observed, d^2 and $\cot^{\frac{2}{3}}\theta$ may be calculated, and if plotted in the form of a graph, will give a straight line, such as AB (Fig. 728). This line may be produced backwards until it cuts the d^2 axis in the point C, for which point $\cot^{\frac{2}{3}}\theta = 0$, and therefore $d^2 + l^2 = 0$, so that OC is numerically equal to l^2. By taking the square root l is found.

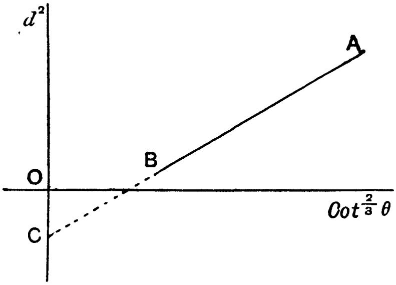


FIG. 728.—Graph for finding the equivalent length of a magnet.

EXPT. 172.—**To find the equivalent length of a magnet.** Perform a set of readings as in Expt. 170, but plot d^2 and $\cot^{\frac{2}{3}}\theta$ as in Fig. 728. Produce the line AB to cut the axis in C. Note the length of OC; find its square root l, and double it to obtain the equivalent length of the magnet. Measure its actual length and evaluate the ratio

$$\frac{\text{equivalent length of magnet}}{\text{real length of magnet}}.$$

Vibration of a suspended magnet.—It will have been noticed that comparisons of magnetic moments and strengths of field may be made by means of the magnetometer, but that neither can be determined in absolute measure. To do this, some other relation between these quantities must be found. This relation is given by the equation for the time of oscillation of a suspended magnet, vibrating with small amplitude in a magnetic field. This relation is

$$\mathsf{T} = 2\pi\sqrt{\frac{\mathsf{I}}{\mathsf{MH}}}, \quad (cf.\ \text{p. } 229\text{A})$$

where T is the time of one complete oscillation and I is the **moment of inertia** (p. 200) of the magnet; M and H have the same meanings as before. The moment of inertia is related to rotational motion, exactly as **mass** is to linear motion, and its value for a rectangular bar magnet about an axis passing through the centre of mass is found to be Mass $\times (l^2+b^2)/12$, where l is the length and b the breadth of the bar. For a circular bar magnet $I = \text{Mass} \times \left(\frac{l^2}{12}+\frac{r^2}{4}\right)$, where r is the radius of the circle.

Comparison of magnetic fields.—Since the moment of inertia of any body with respect to a given axis is constant, a knowledge of the time of vibration of a magnet affords a good method for the comparison of field strengths. For if T_1 is the time of vibration at a place where the field strength is H_1, and T_2 is the time where the field strength is H_2,

$$T_1 = 2\pi\sqrt{\frac{I}{MH_1}}, \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{I}{MH_2}};$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{H_2}{H_1}}, \quad \text{or} \quad \frac{H_1}{H_2} = \frac{T_2^2}{T_1^2}.$$

As a rule, the numbers of vibrations n_1 and n_2 in a given time t are observed; then

$$t = n_1T_1 = n_2T_2;$$

$$\therefore \frac{H_1}{H_2} = \frac{n_1^2}{n_2^2},$$

where n_1 and n_2 are the numbers of vibrations made in the same time at two places where the field strengths are H_1 and H_2 respectively.

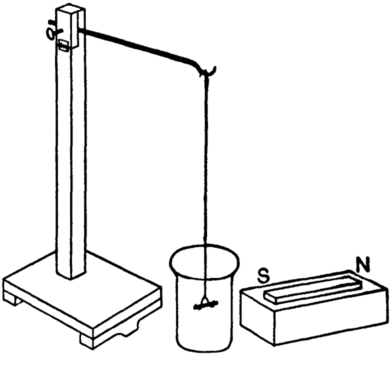


FIG. 729.

EXPT. 173.—**Plotting fields by means of vibrations.** Suspend a piece of magnetised knitting needle or clock-spring about 2 cm. long, by means of a silk fibre, protecting it from draughts by means of a beaker, as in Fig. 729. Either north or south of the needle place the bar magnet NS, pointing north and south, with its nearest pole 10 cm. from the needle. Count the number of vibrations n made by the needle in one minute. Repeat at distances of

15, 20, 25, and 30 cm., and again when the magnet is taken away, so that the needle vibrates in the earth's field alone. Tabulate the result as follows:

Distance.	n	n^2	$(F+H) \propto n^2$.	F
15 20 ... ∞			0·18	0

In the fourth column is the resultant field due to magnet and earth $(F+H)$; the values for this field are found from the fact that n^2 is proportional to the resultant field, and with the magnet absent (or at infinity) the earth's field alone is assumed to be 0·18. The last column, F, is the field due to the magnet alone, and is obtained by taking 0·18 from each of the values of $(F+H)$. Plot a curve with distances from the magnet as abscissae and F as ordinates.

Repeat the experiment with the magnet east or west of the needle and still pointing north and south, measuring the distance from the middle of the magnet.

Determination of earth's magnetic field.—From the magnetometer experiment (Expt. 169) we obtain the relation $\frac{M}{H} = \frac{d^3}{2} \tan \theta$, and from the vibration experiment (p. 789) $T = 2\pi\sqrt{\frac{I}{MH}}$, or $MH = \frac{4\pi^2 I}{T^2}$. Combining these equations we obtain $MH \times \frac{M}{H} = M^2$, or $MH \div \frac{M}{H} = H^2$, and thus both M and H may be found in absolute measure.

Expt. 174.—**Determination of H.** By means of the magnetometer find the value of $\frac{1}{2}d^3 \tan \theta (=M/H)$, as in Expt. 169. Now suspend the bar magnet at the place previously occupied by the magnetometer needle, employing a paper stirrup hung by a silk thread, or the finest cotton that will carry the weight of the magnet. Take the time for fifty oscillations of the magnet by means of a stop-watch, and so obtain T, the time of one oscillation. Care should be taken that the magnet oscillates only a few degrees on either side of its equilibrium position, and complete oscillations must be taken, that is, the time from passing any given position to next passing the same position in the same direction. Now weigh the magnet, measure its length and breadth, and calculate I, the moment of inertia (p. 789). $4\pi^2 I/T^2 (=MH)$ should now be found, and combining this with the result of the magnetometer experiment, M and H should be calculated.

EXERCISES ON CHAPTER LXII.

1. Describe how the strengths of two magnetic fields may be compared by means of a vibrating magnet.

2. Define the magnetic moment of a magnet. Describe how the magnetic moments of two magnets may be compared by a deflection method.

3. Show that the magnetic intensity due to a short bar magnet of moment M at a distance d from the mid-point along a line at right angles to the magnetic axis is approximately M/d^3.

4. A small magnet oscillating in the earth's magnetic field makes 20 vibrations in 150 seconds. It is then placed due north of a bar magnet, which is lying in the magnetic meridian with its north pole pointing north, and is then found to make 20 vibrations in 80 seconds. Find the strength of magnetic field, at the position of the oscillating magnet, due to the bar magnet. ($H = 0{\cdot}18$.)

5.* A small suspended magnet makes 10 oscillations in 35 seconds in the earth's field alone. On bringing a bar magnet with its S. end pointing north to a point due east of the suspended needle, the needle makes 20 oscillations in 55 seconds. Find the time of oscillation of the suspended needle if the bar magnet is now reversed pole for pole.

6. Define the magnetic moment and the magnetic axis of a magnet, and describe how you would determine one of them experimentally.

7. Define the terms *unit magnetic pole* and *strength of a magnetic field at a given point.*
Calculate the field due to a bar magnet, 10 cm. long, and having a pole strength of 100 units, at a point 20 cm. from each pole. L.U.

8. What factors determine the time of oscillation of a magnet when free to swing in a horizontal plane ?
Two bar magnets are bound together side by side and suspended so as to oscillate in a horizontal plane. The time of swing is 12 seconds when like poles are together and 16 seconds when the direction of one magnet is reversed. Compare the moments of the magnets. L.U.

9. How would you define (*a*) unit magnetic pole, and (*b*) the moment of a magnet ?
A thin magnet 20 cm. long, having its north-seeking end pointing south, just balances the earth's field ($H = 0{\cdot}2$ C.G.S.) at distances of 10 cm. from its poles. Find its magnetic moment. L.U.

10. Explain the method of comparing the intensities of magnetic fields by observations of the times of oscillation of a magnetic needle.
A small magnet vibrating horizontally in the earth's field has a period of 4 seconds. When another magnet is brought near to it, 50 swings take 160 seconds. Compare the strength of the field due to the magnet with the earth's horizontal field, assuming that the two fields are in the same direction or in opposite directions. L.U.

11. What is the law of force between two magnetic poles ? The moment of a magnet is 200 units, and the poles are 10 cm. apart. Find the force

which this magnet exerts upon a pole of strength 10 units placed upon its axis at a distance of 25 cm. from the mid point of the magnet.
L.U.

12. Describe some method of measuring the horizontal component of the earth's magnetic field.

13. A bar magnet is 8 cm. long, and has its magnetic poles concentrated at its ends. Determine graphically the direction of the magnetic field at points distant from the poles as follows: (*a*) 4 cm. from the N. pole and 9 cm. from the S. pole; (*b*) 6 cm. from the N. pole and 8 cm. from the S. pole; (*c*) 7 cm. from the N. pole and 5 cm. from the S. pole.
Adelaide University.

14. What is the *magnetic moment* of a magnet ?

A short magnet is placed in a horizontal plane with its axis parallel to the meridian and its N-seeking pole pointing to the south. It is found that at a point on the axis of the magnet 50 cm. to the south of its midpoint the resultant horizontal field is zero. If the intensity of the earth's horizontal field is 0 20 dyne per unit pole, calculate the magnetic moment of the magnet.
L.U.

15. Define Magnetic Field, Strength of Field, Lines of Force, Magnetic Moment, Unit Pole.

What is the couple acting on a magnet 12 cm. long, of pole strength 7 units, placed at an angle of 60° with the direction of a field of strength 0·017 unit ?
Punjab University.

CHAPTER LXIII

TERRESTRIAL MAGNETISM

Earth's resultant magnetic field.—In Chapter LXII. the magnetic field has been treated as though it were horizontal, because it is only the horizontal component of the field that is active in making a compass needle point north and south. That the needle itself remains horizontal is not surprising, for it is always adjusted in its stirrup, or on its needle-point, until it assumes a horizontal position.

In order to determine whether the earth's magnetic field is really horizontal, it is necessary to balance the **needle** carefully **before it is magnetised.** It is then magnetised and mounted so that it is free to rotate in a vertical plane (p. 797). It will then be found that it does not set horizontally; in the northern hemisphere the N end of the needle dips downwards, and in the southern hemisphere the S end dips downwards. The resultant magnetic field is therefore inclined to the horizontal.

Again, if a soft iron rod be placed north and south, it will be magnetised by the earth's field, the end pointing north becoming a N pole. Also, if it be placed vertically, in the northern hemisphere, the lower end will become a N pole ; further, if the rod be placed parallel to the direction in which the dip needle sets, it will be still more strongly magnetised. A slight tapping assists these effects. If an unmagnetised bar of steel be used, a vigorous tapping with a hammer will be necessary for it to become magnetised.

EXPT. 175.—**Magnetisation of a bar in the earth's field.** Place a soft iron bar, about 18 in. long and $\frac{1}{4}$ in. diameter, horizontally north and south and tap it gently. Bring a compass needle near each end in turn, and test its polarity. Repeat with the bar arranged vertically, and test the polarity.

Place the bar in a vertical plane situated N and S, the length of the bar being inclined at about 60° to the horizontal. Tap it gently and test the polarity of its ends with a compass needle.

Magnetic declination and dip.—It is well known that a compass needle does not point true geographical north and south; thus the direction of the earth's magnetic field is not in general horizontal, neither is it in the geographical meridian. If the vertical plane AB (Fig. 730) be taken to represent the geographical meridian, or plane passing through the point of observation and containing the axis of rotation of the earth, the **magnetic meridian,** or vertical plane containing the axis of a freely suspended magnet, is inclined to AB, and may be represented by the plane CDEF. The angle GCD between the geographical and magnetic meridians is called the **magnetic declination,** or in nautical language, the **magnetic variation,** or the **variation of the compass.**

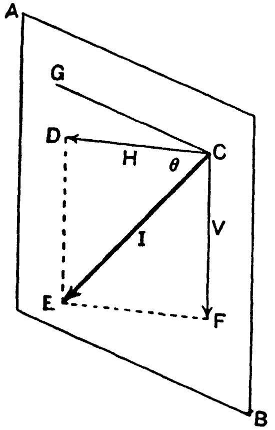


FIG. 730.—Magnetic declination and dip.

The angle DCE between the resultant direction of the earth's magnetic field and the horizontal is called the **magnetic dip**; it is the angle of dip of a magnetised needle which is free to rotate in the plane of the magnetic meridian.

The resultant magnetic intensity CE, represented by I, may be resolved into two components, H and V, one of which is horizontal and the other vertical. Since the triangles CDE and EFC have right angles at D and F, we have

$$\tan\theta = \frac{V}{H}, \quad \text{and} \quad I^2 = H^2 + V^2.$$

These quantities, the declination, dip, H, V and I are known as the **magnetic elements** at any point on the earth's surface, and if the declination and any two of the other elements are known, the whole of them may be calculated.

The three elements usually measured at any place are the **declination, dip,** and the **horizontal component of the earth's field, H.** The measurement of H has already been described.

Measurement of declination.—To determine the declination at any place, the position of the geographical meridian must be found, and also that of the magnetic meridian. The geographical meridian is found by astronomical means, by observing the direction of the sun at a known time, its direction at this particular time being known in terms of the longitude of the place, from astronomical tables. To determine the direction of the magnetic meridian, a suspended needle or compass is employed, but it must be so arranged that when the direction of its axis has been observed, it may be turned over and suspended from the other side and a new observation made. The true direction of the magnetic meridian is obtained by bisecting the angle between the two observed positions of the geometrical axis of the magnet.

The reason for this may best be understood by considering what we mean by the **magnetic axis of a magnet.** In the case of a thin magnetised needle there is little doubt as to the meaning of the term, it is a line joining the poles. But magnets are not as a rule of this simple form. A common type is shown in Fig. 731. The poles are spread over the ends, and if the position of the resultant point poles be defined in a manner somewhat similar to that employed in finding the centre of gravity (p. 107), the line joining these resultant poles is the magnetic axis. The magnet, when suspended, will then set with the magnetic axis in the direction of the magnetic field. The positions of these resultant point poles cannot conveniently be found, but it may be noticed that a suspended magnet always sets with its magnetic axis in the direction of the magnetic field. Hence, we may define the **magnetic axis of a magnet as that line in it which always takes the direction of the magnetic field when the magnet is freely suspended in the field and is allowed to come to rest.**

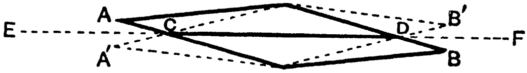


FIG. 731.—Magnetic axis of a magnet.

Thus, if CD is the magnetic axis of the magnet AB in Fig. 731, when the magnet is suspended and comes to rest, CD has the direction of EF, the magnetic meridian. On turning the magnet over and suspending it from the other side, the magnetic axis CD will still be in the meridian, but A′B′ is the position of the magnet. The magnetic meridian EF then bisects the angle between AB and A′B′.

EXPT. 176.—**Determination of the magnetic meridian and axis of a magnet.** Take two equal discs of cardboard and paste them together with several parallel magnetic needles between them. Select two points on the circumference diametrically opposite each other and join them by a thick ruled diameter on each side of the disc. Suspend the disc above, but near to, a sheet of drawing paper; it comes to rest with the ruled diameter in some such position as AB (Fig. 732). Mark the position of AB on the paper. Now suspend the disc from the other side. The ruled diameter comes to rest in the position A′B′. Bisect the angle between AB and A′B′ by the line EF, which is then the direction of the magnetic meridian, and may be checked by suspending a magnetised knitting needle in place of the disc. The position of the magnetic axis CD of the disc coincides with EF.

It should be noticed that, even in the extreme case of a disc whose direction of magnetisation is unsuspected, the magnetic meridian can be found by taking the two positions of rest of some arbitrary fixed mark on the disc, the disc being suspended from its two sides in turn.

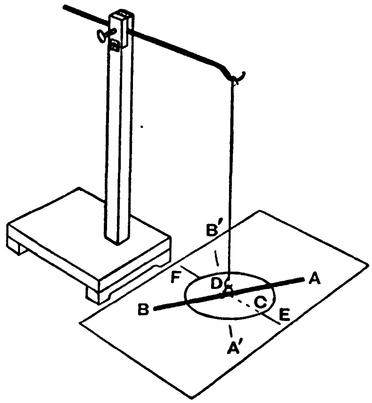


FIG. 732.—Determination of the magnetic meridian.

Measurement of magnetic dip.—The dip is usually measured by an instrument known as the **dip circle.** This consists of a vertical circle (Fig. 733) at the centre of which the dip needle is suspended by a fine steel axle resting upon two agate knife-edges. The circle and supports carrying the knife-edges can rotate about a vertical axis, the azimuth (*i.e.* the angle between the plane of the circle and some fixed plane of reference) being observed by the horizontal circular scale and verniers. The magnet can be raised from the knife-edges, or lowered on to them, by means of two V supports, not shown in Fig. 733, the ends of the magnet's axle resting in the V's. In this way the axis of the magnet may always be brought back to the centre of the circle, and any sticking of the axle on the knife-edges may be prevented.

To use the dip circle, it is first levelled by means of the spirit-level and the levelling screws. It is then turned about its vertical axis until the magnet sets vertically, both ends pointing to 90° upon the scale. The plane of the circle is then at right angles to the magnetic meridian, and by rotating the instrument through 90° by the horizontal scale, the plane of the vertical circle and of rotation of the needle is brought into the magnetic meridian. The reason for this adjustment is, that when the plane of rotation of the magnet is at right angles to the magnetic meridian, the horizontal component of the earth's field H, being parallel to the axis of rotation, does not produce any couple tending to rotate the magnet about this axis (Fig. 734). The vertical component V then sets the magnet vertically.

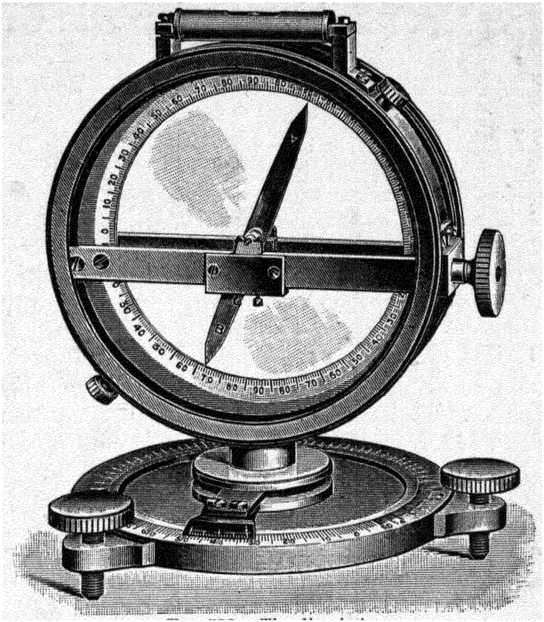


FIG. 733.—The dip circle.

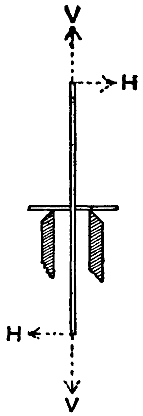


FIG. 734.—Supports of the dipping needle.

In the instrument shown in Fig. 733, the points of the needle are very close to the divisions of the vertical circular scale, and very small error is made in reading their position. By estimation, the readings can be made to about one-tenth of a degree. If greater accuracy is required, the points of the needle are observed by means of low-power microscopes, carried on a framework which rotates about an axis having the same position in space as the dipping needle. The microscopes are provided with cross-wires, and their positions are observed by means of verniers moving over the vertical circular scale.

Having placed the dip circle with its plane in the magnetic meridian, there are four sources of error to be corrected for, involving the making of sixteen observations.

(i) **The axis of rotation of the magnet may not be at the centre of the scale,** as shown to an exaggerated extent in Fig. 735. Both ends of the pointer are therefore observed, and the mean value of the dip is then free from this error.

(ii) **The zero line of the circle may not be truly horizontal.** This would make the dip appear to be too great or too small, as is seen for the positions O—O and O′—O′ respectively in Fig. 736. On rotating the dip circle through 180° about its vertical axis, as measured by the horizontal scale (Fig. 733), this error is reversed, and the readings of (i) are repeated.

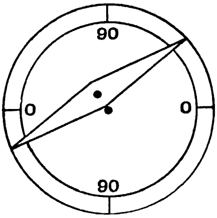


FIG. 735.—Eccentric error.

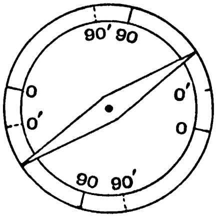


FIG. 736.—Levelling error.

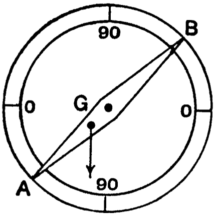


FIG. 737.—Gravitational error.

(iii) **The magnetic axis of the magnet may not coincide with its geometric axis.** This source of error has been discussed on p. 795. The magnet is reversed on its bearings, and the previous readings (i) and (ii) repeated.

(iv) **The centre of gravity of the needle may not lie in the axis of rotation of the magnet.** In this case there would be a couple due to gravity, which would either increase or decrease the dip. In Fig. 737 the end A of the magnet is the heavier, the centre of gravity being at G. The magnet must now be remagnetised, so that the end B dips downwards, and as the previous readings all indicate too great a value for the dip, on now repeating them they will indicate too small a value. Thus, in all, 16 readings are taken, and may be tabulated as follows, the mean being taken as the true dip :

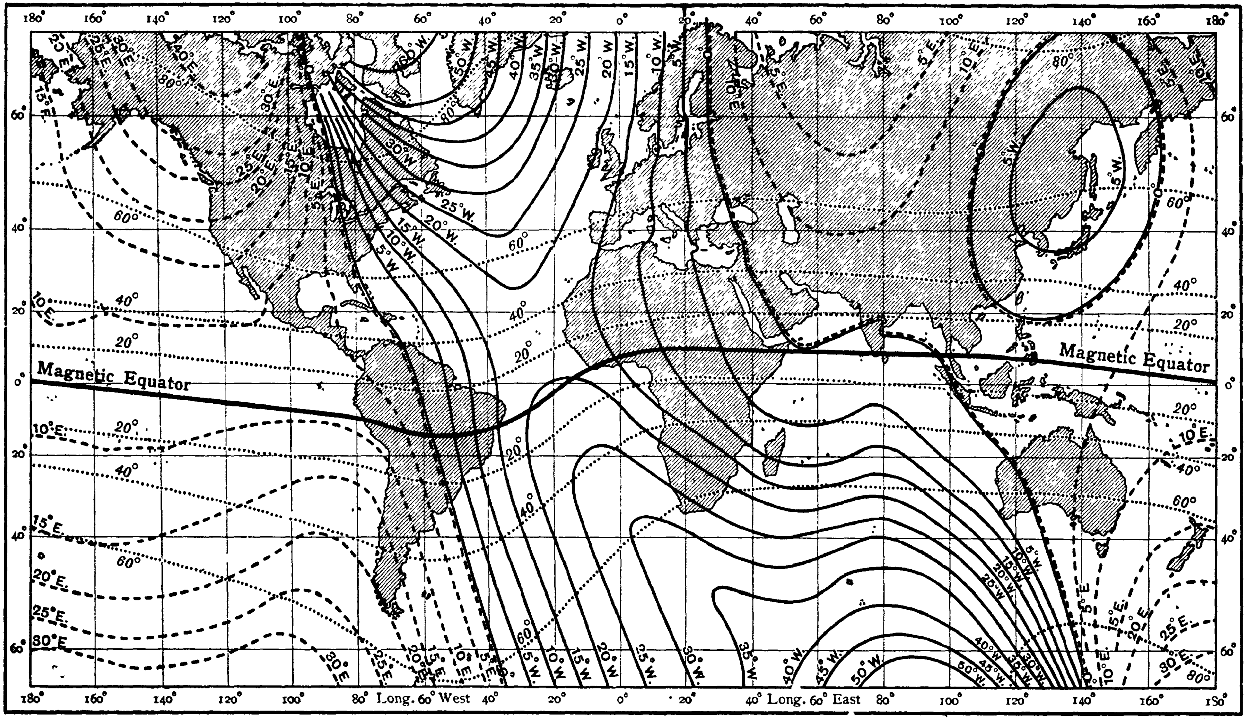
		Reading of Upper end.	Reading of Lower end.
End A dipping	Circle facing E - - - -		
	” ” W - - -		
	Magnet reversed on bearings		
	Circle facing W - -		
	” ” E - - - -		
End B dipping	Circle facing E - - - -		
	” ” W - - -		
	Magnet reversed on bearings		
	Circle facing W - - -		
	” ” E - - - -		
	Total - -		
	Mean value of dip =		

EXPT. 177.—**To measure the magnetic dip.** By means of the dip circle, follow the instructions given above, and determine the angle of dip.

Magnetic maps.—Observations of the magnetic declination, dip, and horizontal intensity have been made at a great number of places on the earth's surface, and the results represented for convenience upon maps. This may be done in a variety of ways, but the commonest is to draw lines through points upon the map for which the value of any particular element is the same.

Isogonals.—**Isogonals** are lines passing through points for which the magnetic declination has the same value. In Fig. 738 the isogonals (shown in full and heavy dotted lines) all pass through the geographical north and south poles. There are also two other points through which they pass. One is situated in latitude about 73° 31′ N., longitude 96° 43′ W., and is called the **magnetic north pole,** and the other in latitude about 72° 21′ S., longitude 155° 16′ E., and is called the **magnetic south pole.**

One **agonic line,** or line at every point of which the declination is zero, or the compass points due north, runs through the American Continent, and the other through Europe, Arabia, the Indian Ocean and Australia. At all points in the "Atlantic" space between these two agonic lines the declination is westerly (full line), that is, the compass points west of geographical north. In the "Pacific" space the declination is easterly (heavy dotted line), except within



Emery Walker Ltd sc.

FIG. 738.—Map of the world showing the isogonals and isoclinals.

an oval agonic line passing through China and Siberia, called the **Siberian Oval**, within which the declination is westerly.

It will thus be seen that the isogonal lines have somewhat the character of lines of longitude, in that their general tendency is to run from north to south. The irregularities, however, are very great.

Isoclinals.—Lines for which the dip has the same value at every point are called **isoclinals.** These are shown in light dotted lines in Fig. 738. The line of no dip, or the **magnetic equator**, follows the general course of the geographical equator. It is, however, south of the geographical equator in America, and north in Africa. At all points upon this line, the dip-needle remains horizontal. The isoclinals resemble the parallels of latitude, and form closed curves about the magnetic poles. At the magnetic poles the dip-needle sets vertically, with the N pole downwards at the north pole and the S pole downwards at the south pole.

Isodynamic lines.—Lines passing through points for which the horizontal intensity of the earth's magnetic field is the same are called **isodynamic lines.** The horizontal intensity is zero at the magnetic poles, and increases to a maximum at the magnetic equator. Thus the isodynamic lines have a similar shape to the isoclinals, but must not be confused with them.

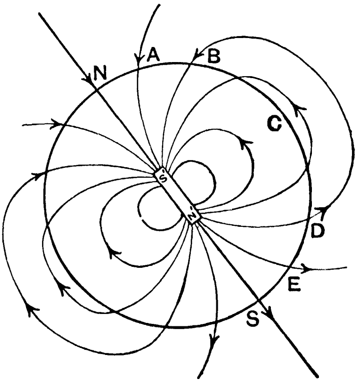


FIG. 739.—Magnetic condition of the earth.

The earth as a magnet.—The cause of the earth's magnetisation has been the subject of a great deal of speculation. We shall here content ourselves with saying that the magnetisation is not entirely due either to internal, or to external causes, but to both. However, the general character of the earth's magnetic field may be represented roughly by imagining a large internal magnet. In order to represent the earth with correct magnetic polarity, the S pole S′ of this internal magnet N′S′ (Fig. 739) must be situated under the magnetic N pole of the earth. The lines of force due to such an internal magnet are shown in the figure. This state of magnetisation may be imitated by taking a disc of cardboard to represent the earth and laying upon it a bar magnet in the position N′S′. A compass needle carried round the disc will set perpendicularly to the circle on reaching N. At A the dip will be less, and will be still smaller at B, until at C which represents the magnetic equator, the dip will be zero. Similarly, at D and E the S pole of the needle will dip and at S the dip is 90°.

Such a magnetic distribution is far from being a true one, but it gives a rough idea of the actual state of affairs. Attempts have been made, by adding a smaller auxiliary magnet, to imitate more closely the earth's magnetic condition, but no representation by a small number of magnets can produce a field presenting all the irregularities of the earth's field.

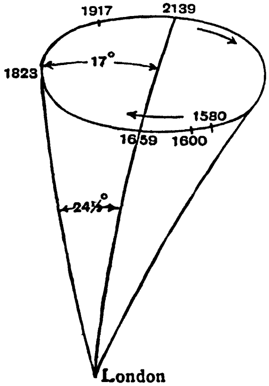


FIG. 740.—Secular variation in the earth's magnetic field.

Variations in the earth's magnetic field.—The form of the earth's magnetic field is never twice the same; it is continually varying. Not only is it undergoing minute irregular variations at every place, but there is a continual periodic change which repeats itself daily, known as the **daily variation.** There are also annual changes, and upon all these there is imposed a **secular variation,** which is large in amount and goes through a cycle of change in about a thousand years.

Secular variation.—The earliest record of the magnetic declination is that of the year 1580, when its value was 11° 15′ E. at London. Successive changes are recorded in later years, the easterly declination diminishing and becoming zero in 1659. At this time the compass needle pointed true north at London. The declination then became westerly, increasing to 24½° W. in 1823, since which time it has been decreasing. This change can be very well represented by considering the magnetic poles to rotate round the geographical poles, the magnetic north pole describing a circle of 17° radius about the geographical north pole, the direction of rotation being clockwise, as shown in Fig. 740. A computation from the changes already recorded shows that the declination may be expected again to be zero at London in the year 2139, which is 480 years from the last occasion. The magnetic north pole will then have described half its circular path, so that the time required for its complete circle is 960 years. The magnetic conditions all over the world go through an approximate cycle in this time.

Annual variation.—The declination also goes through a very small cycle once a year, and this annual cycle is in opposite directions in the northern and southern hemispheres. The declination is about 2¼′ east of its mean position in August at London, and the same amount west of its mean position in February.

Daily variation.—All the magnetic elements undergo fairly regular daily changes, but as these are small, special instruments are necessary to record them. They can be recorded only at fixed observatories, the portable magnetometers generally used in magnetic surveys not being sufficiently sensitive for their observation. A mirror is attached to the magnetic needle, and a beam of light reflected from it is brought to a focus upon a sheet of photographically sensitive paper wrapped round a drum. This drum is rotated by clockwork at uniform speed, so that the motion of the paper is at right angles to the variation to be recorded. Thus, for recording variations in declination, the axis of the drum must be horizontal, so that the motion of the sensitive paper takes place vertically, the movement of the spot of light corresponding to changes in declination taking place horizontally.

The curve in Fig. 741 is of the type usually obtained for the daily variation in declination. The line O—O indicates the mean position of the spot of light. It will be seen that there is a maximum variation of 3′ E. from the mean position just before 8.0 a.m., and 5′ W. at about 1.0 p.m. These variations, however, are not the same from day to day; on specially 'quiet' days they are less than the above, and on days of considerable magnetic disturbance they are greater.

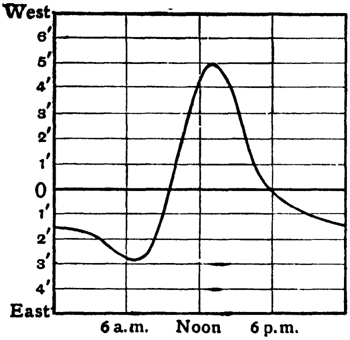


FIG. 741.—Daily variation in declination.

It has been shown that the daily variations are due probably to electric currents in the upper regions of the atmosphere, but their explanation is by no means complete.

Eleven-year period.—Apart from the irregularities which occur, the daily variation undergoes a cyclic variation in magnitude, closely allied to the frequency of occurrence of sun-spots. Thus at a sun-spot maximum the daily variation is greatest, and *vice versa*. Sun-spots go through a periodic change in occurrence once in approximately eleven years. The daily variation in the magnetic elements have been compared with the intensity of sun-spot occurrence since 1855, and the coincidence in the variation of one with the other is remarkable.

Magnetic storms.—It often happens that the suspended needles at the magnetic observatories over considerable parts of the earth are suddenly and violently affected at the same time. These sudden disturbances, called **magnetic storms,** cannot be foretold; they apparently occur quite irregularly. It may, however, be noticed that the sudden appearance of a large sun-spot is usually accompanied by a magnetic storm. Also a prominent display of the Aurora Borealis generally occurs with magnetic storms, although there may be magnetic storms when there is no aurora.

The connection between magnetic storms and the aurora and sun-spots makes it appear likely that the sun emits rays similar to the kathode rays of the vacuum tube (Chap. LXXX.), which, on reaching the earth's atmosphere, render it a conductor of electricity, the electric currents then produced being accompanied by magnetic fields (Chap. LXV). The form of the aurora is just what would be expected from analogy with the vacuum tube, when these kathode rays enter the earth's magnetic field.

The magnetic compass.—Probably the magnetic compass finds its most important application in connection with the navigation of ships. The true geographical bearing, as well as the latitude and longitude, may be determined by astronomical observations, but these are tedious and are generally performed only once a day. The actual steering of the ship is generally performed according to the compass, or magnetic, bearing. The Kelvin compass card is the one most commonly used. It consists of a light disc of aluminium or mica on which the points of the compass are drawn, the magnetic part being composed of several light steel magnets. The whole is supported upon an agate cup resting on a needle point, and in the best compasses is immersed in methylated spirit, to buoy up the card and so take part of its weight off the needle point. The liquid also serves to reduce any oscillations of the compass that would otherwise be troublesome. Passing through the axis of support is the mid line of the ship, indicated by two lines or marks, one fore and the other aft of the card, thus enabling the bearing of the ship according to the compass to be observed directly.

It follows, of course, that in order to know the geographical bearing of the ship, the declination (or magnetic variation) must be added to, or subtracted from, its magnetic bearing. The value of this at all parts of the world is given on charts supplied by the Admiralty.

The magnetic compass has also great importance in the steering of air-craft. An aeroplane compass of the Creagh-Osborne type is shown in Fig. 742. The bowl is of spherical form and is mounted in such a manner as to minimise vibration. The compass card has several steel magnets, and is provided with a vertical mica ring, on which the points of the compass are marked in luminous radium paint. A window at the back of the bowl enables the pilot to read the course by means of the scale on the compass card. The bowl contains liquid to buoy up the card, part of which consists of a hollow float. The liquid thus takes part of the weight off the agate point suspension, and also helps to damp the vibrations of the card.

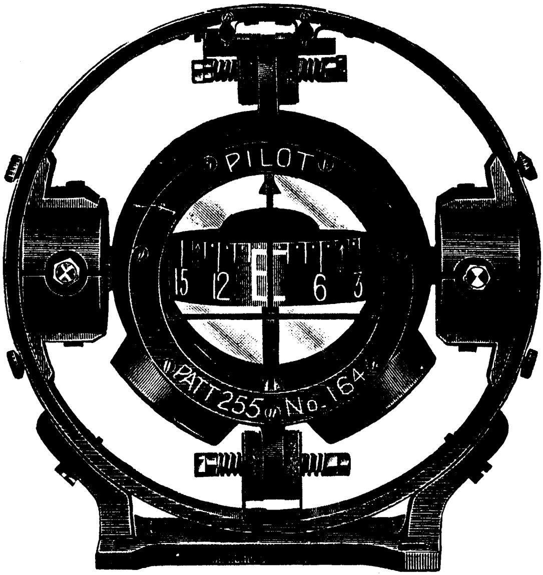


FIG. 742.—The Creagh-Osborne aeroplane compass.

Ship's magnetisation.—Masses of iron or steel situated near the compass will, of course, affect its reading; and since modern ships are built almost entirely of these materials, the errors introduced and the corrections necessitated are considerable.

These errors are of many kinds and cannot all be considered here in detail, but the three principal errors will now be dealt with.

Semicircular deviation.—Most iron ships are permanently magnetised, thus the ship itself is a large magnet, and as it takes different courses it will have different effects upon its compass. This permanent magnetisation is acquired at the time of building the ship, the magnetic axis being in the magnetic meridian, that part of the ship which lies northwards being a N pole (p. 793). Consider the line NS to be the magnetic axis of the ship (Fig. 743). In the position shown, that is, with the N pole west of the magnetic meridian AB, the compass *ns* is evidently deflected so that *n* is east of the meridian. This applies for any position of the ship when N is west of the meridian. When N is east of the magnetic meridian the deviation of the compass is westerly. Hence, as the ship rotates through a complete circle, the deviation due to the permanent magnetisation is east for half the circle and west for the other half. For this reason this is called **semicircular deviation.**

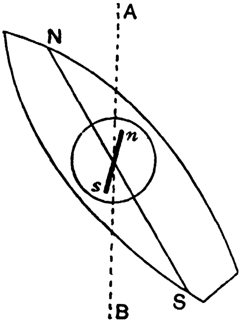


FIG. 743.—Permanent magnetisation of a ship.

There is another cause of semicircular deviation, namely, masses of soft iron situated with their greatest length vertically. A vertical soft iron column will be magnetised by the vertical component of the earth's field (p. 793) and in the northern hemisphere, the N pole will be at the bottom and the S pole at the top. If situated as in Fig. 744 with the N pole near the compass, the deviation is east when the magnet is west of the compass and west when the magnet is east of the compass. But as the ship rotates, the deviation is east for half the rotation and west for the other half. The direction of the deviation is reversed if the upper end of the bar is level with the compass, as in Fig. 745, and, of course, the directions are all the opposite to those in the earth's southern hemisphere, where the N pole of the iron bar would be at the top.

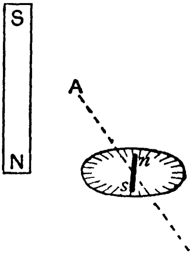


FIG. 744.—Vertically disposed soft iron.

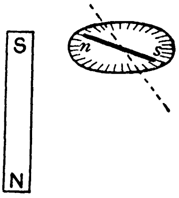


FIG. 745.—Vertically disposed soft iron.

Although there may be many such masses of iron in a ship, the resultant effect will always give rise to a semicircular deviation.

Quadrantal deviation.—When the soft iron is disposed horizontally its effect is more complicated, as its direction of magnetisation changes as the ship rotates. Thus, suppose the bars to be disposed as in Fig. 746 (*a*). The position of the N and S poles are as shown, and the deviation produced is west. On rotation of the ship till the bars are in position (*b*) the deviation will be east. A further rotation through 90° will reproduce case (*a*), for it must be remembered that we are dealing with soft iron, and its magnetisation depends on its position in the earth's magnetic field. The bars have changed places with respect to the compass, but their polarity has changed too, so that the deviation is again west. A further rotation through 90° produces again case (*b*). Thus the deviation changes in direction four times during one rotation of the ship, and is therefore only constant in sign for a rotation of 90°. It is therefore called **quadrantal deviation.**

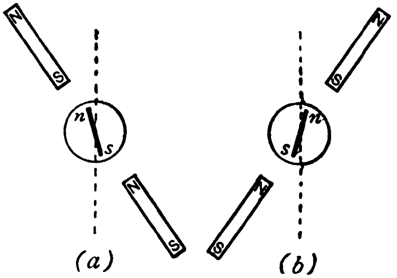


FIG. 746.—Horizontally disposed soft iron.

Swinging the ship.—Owing to the complexity of the errors due to the ship's magnetisation they cannot be foretold, and their resultant must be found by direct observation. This is called **swinging the ship.** The ship is swung round into a number of positions, and in each position the true magnetic bearing and the observed compass bearing are observed. The difference is the error of the compass, due to the ship's magnetisation. A table or diagram is then constructed for future use, so that for any observed compass bearing the correction to be applied in order to obtain the real magnetic bearing is known. To this must then be applied the declination, in order to obtain the geographical bearing.

Methods of correcting for ship's magnetisation.—Although there is no method of compensating completely the disturbances of the compass due to the ship's magnetisation, yet they may be partially compensated in several ways. To compensate for the **quadrantal deviation** two hollow soft iron spheres are placed one on each side of the compass, and on a level with it. If FF (Fig. 747) be the fore and aft line of the ship, and AB the direction of the magnetic meridian, the spheres NS, NS are placed as shown. Since they become magnetised as indicated, they will, by themselves, produce a deviation in an easterly direction. But reference to Fig. 746 shows that the quadrantal deviation is usually westerly for this position of the ship. Hence the spheres placed at a suitable distance will compensate for the quadrantal deviation. From p. 807 it will be seen that the deviation produced by the spheres alone is quadrantal. With spheres 5 inches in diameter a quadrantal deviation of about 2° is corrected when their centres are 9 inches from the compass.

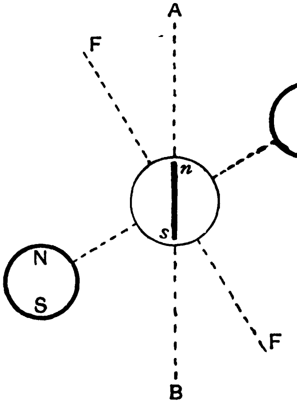


FIG. 747.—Soft iron compensators.

The **semicircular deviation** due to permanent magnetisation is corrected by fixing a number of small permanent magnets near the compass, their correct number and position being found by trial. The part of the semicircular deviation due to vertical soft iron is got rid of by fixing a vertical soft iron bar in front of, or behind, the compass. This is called a **Flinder's bar.**

MEAN VALUES OF THE MAGNETIC ELEMENTS AT GREENWICH.

Year.	Declination W.		H.	Dip.		
	°	′		°	′	″
1914	15	6·3	0·18518	66	51	13
1915	14	56·5	0·18508	66	51	50
1916	14	46·9	0·18494	66	52	45

EXERCISES ON CHAPTER LXIII

1. What are the magnetic elements which are usually measured in order to find the state of the earth's field at any point? How are they related to each other?

2. Describe the precautions necessary in finding the direction of the magnetic meridian.

3. A horizontal bar magnet is brought near a dip-needle in the northern hemisphere, the bar being in the plane of rotation of the needle, and the N pole pointing south. Describe the effect on the observed dip (*a*) when the magnet is immediately north of the needle, and (*b*) when it is vertically over the needle.

4. Describe a dip circle and explain its action.

A dip circle is slowly rotated about a vertical axis. Describe and explain the behaviour of the needle during one complete turn.
Sen. Camb. Loc.

5. Define the angle of dip and explain how it may be measured. Describe roughly how the angle of dip varies from place to place on the earth's surface. L.U.

6. Explain how to determine (*a*) the vertical component of the earth's magnetic force, (*b*) small changes in the magnitude of the horizontal component. L.U.

7. Assuming that the magnetism of the earth is due to a small magnet placed at its centre, find expressions for the horizontal component and the dip of the field at any point of the earth's surface in terms of the magnetic latitude. L.U.

8. A dip-needle placed so that it can oscillate in the meridian is found to make 35 oscillations per minute in a locality where the dip is 60°. At another locality, where the dip is 45°, it is found that the needle makes 40 oscillations per minute. Assuming that the needle is constant in magnetic condition, find (*a*) the ratio of the earth's total intensities, and (*b*) the ratio of the horizontal components of the earth's magnetic field at the two places. L.U.

9. Define declination and dip (in reference to the earth's magnetism). How are they determined ?

Find the resultant force of the earth's magnetism at a place at which the dip is 30° and the horizontal force is 0·18. Calcutta University.

10. Define the angle of magnetic dip and describe a method of measuring it.

A dip circle is placed so that the needle sets vertical. The circle is then rotated through an angle A about a vertical axis and the dip measured in this position. Show how the dip as observed is related to the true dip and to the angle A. L.U.

11. How is the horizontal component of the earth's magnetic force determined in absolute measure ?

At a place A the total magnetic intensity is 0·5 and the dip is 64° ; at a place B the total intensity is 0·6 and the dip is 72°. If a magnet vibrating horizontally at A makes 20 oscillations in a minute how many oscillations would it make in the same time at B ? University of Bombay.

CHAPTER LXIV

MAGNETIC PROPERTIES OF MATERIALS

Intensity of magnetisation.—In order to study the magnetic properties of a given material, something more must be known than the magnetic moment of a bar of the material, for this evidently depends, amongst other things, upon the size of the bar. Dividing the magnetic moment by the volume, a quantity is obtained which gives the average intensity of magnetisation of the material. If the body be uniformly magnetised, the magnetic moment per cubic centimetre is the same whatever part of the body be chosen. It is called the **intensity of magnetisation** of the body. Thus, for a uniformly magnetised body,

$$\text{Intensity of magnetisation} = \frac{\text{magnetic moment}}{\text{volume}}$$

or,

$$\mathrm{I} = \frac{\mathrm{M}}{\mathrm{V}}.$$

There is another way of representing this quantity: imagine a uniformly magnetised bar (Fig. 748); let l be the length, and a the area of each end. Let σ be the amount of pole per unit area of each end. Then the amount of pole on each end is $a\sigma$, and the magnetic moment is $la\sigma$ (p. 781). Now the volume is la;

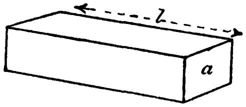


FIG. 748.—Uniformly magnetised bar.

$$\therefore \text{ Intensity of magnetisation} = \frac{la\sigma}{la} = \sigma.$$

Thus, the intensity of magnetisation may also be defined as **the amount of pole per unit area of face, the face being taken at right angles to the direction of magnetisation**; or,

$$\mathrm{I} = \sigma.$$

Although bodies are not, as a rule, magnetised uniformly, the above definitions of intensity of magnetisation hold for any volume sufficiently small for the magnetisation throughout it to be considered uniform.

Magnetic susceptibility.—Magnetisation is always acquired on account of the magnetic body being placed in a magnetic field, and the intensity of magnetisation acquired depends upon the strength of the field, and the nature of the body. The ratio of the intensity of magnetisation (I) produced, to the strength of the field (H) producing it, is called the **magnetic susceptibility** (k) of the material, thus,

$$\frac{I}{H}=k, \quad \text{or} \quad I=kH.$$

The magnetic susceptibility of most magnetic materials varies with the magnetising field in a complex manner. Its study will be left until later (p. 821).

Magnetic permeability.—We must now reconsider the equation given on p. 774 for the force between magnetic poles, namely,

$$\text{Force}=\frac{m_1 m_2}{d^2} \text{ dynes.}$$

This equation is strictly true only when the poles are situated in empty space, and very nearly true when they are situated in air, or any other non-magnetic material. If the poles are situated within a magnetic material, the force is quite different. It is, however, still proportional to both pole strengths and varies inversely as the square of their distance apart. A quantity (μ) is now introduced into the equation to render it correct.

Thus, the equation

$$\text{Force}=\frac{m_1 m_2}{\mu d^2} \text{ dynes}$$

applies to the force when the poles are situated in any material, where μ is called the **magnetic permeability** of the material. This quantity, like the susceptibility, is not constant for any material; its variations are studied on p. 821.

Field strength represented by lines of force.—On referring to the diagrams of lines of force (pp. 779, 780) it becomes obvious that the strength of field is greater where the lines are packed closely together, and weaker where they are spread apart. This suggests the possibility of representing the magnetic field quantitatively, as well as merely in direction, by means of these lines. It can be shown by reasoning, beyond the scope of the present book, that if a surface be taken round a magnet, and through every unit of area of this surface a number of lines of force be drawn, numerically equal to the strength of field at that unit of area, these lines when continued will, by their number per unit area, represent everywhere the strength of the field.

This quantitative representation of a magnetic field by means of lines of force is very useful, as it helps us to visualise the field itself. Thus, where there is one line of force per square centimetre the strength of field is unity; and where the strength of field is H, there are H lines of force per square centimetre, the area always being taken at right angles to the direction of the field at the place considered.

Hence the total number of lines of force crossing an area of A sq. cm. is AH, where H is the strength of field, provided that this is uniform. The total number of lines crossing any area is sometimes called a **magnetic flux.**

Magnetic induction.—We must now introduce a new term, **magnetic induction,** into our magnetic considerations, a term which has often been loosely and improperly employed.

Consider two magnetic poles, m_1 and m_2, d centimetres apart. The force between them, when in air or in a vacuum, is m_1m_2/d^2 dynes, and if m_2 is a unit pole, the force m_1/d^2 upon it is called the strength of field (H) due to the pole m_1 (p. 781). If, however, the medium in which the poles are situated has permeability μ, the force between the poles is $m_1m_2/\mu d^2$ dynes, and the strength of field (H) due to m_1 is $m_1/\mu d^2$. Thus the strength of field due to a pole depends upon the medium in which it is situated. It is most important, as we shall see later, to have some quantity which is fixed for a given pole and distance, independently of the medium. This quantity is called the **magnetic induction** (B), and we see from the above that it must be μ times the strength of field, or $\mathrm{B}=\mu\mathrm{H}$.

Now, for a distance d from a given pole,

$$\mathrm{H}=\frac{m}{\mu d^2}; \quad \therefore \quad \mathrm{B}=\mu\cdot\frac{m}{\mu d^2}=\frac{m}{d^2}.$$

B is therefore independent of the medium, thus fulfilling the condition required. Thus, **the definition of magnetic induction is that it is μ times the strength of magnetic field at any point** where H is defined as on p. 781 and μ as defined on p. 811.

Magnetic lines of induction.—In plotting lines of force (p. 778) the process always begins at the surface of the magnet, in fact the lines may be said to arise upon N poles and end on S poles. These lines, however, are continuous with others within the magnet, so that each line in its entire path forms a closed circuit (Fig. 749); but within the magnet they are not lines of force, that is to say, they do not by their number per unit area, represent the strength of field, or force that would be exerted on a unit pole. These are **magnetic lines of induction,** and may, or may not, lie wholly or partly within a magnetic material. When in air or other non-magnetic material they are also lines of force; that is, they represent the strength of field by their number per unit area, but do not do so within a magnetic material. The number of these lines per square centimetre, however, represents the magnetic induction at the place considered, whatever the nature of the medium may be. The relation of their number per unit area to the strength of field within a magnetic material will be found on p. 816.

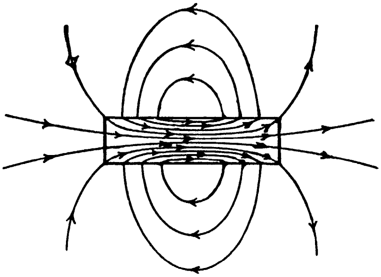


FIG. 749.—Lines of induction.

Gauss's law.—There is a convenient theorem due to Gauss which is derived from the inverse square law, and is of great service in solving magnetic problems. Its proof is beyond the scope of this work, but it may be stated in the following simplified manner. **The number of lines of magnetic induction arising upon any N pole or ending on a S pole is equal to 4π times the strength of the pole.** This applies to any pole, whatever may be the medium in which it is situated; but when it is situated in air, the lines of induction are also lines of force. Hence for a magnetic N pole of strength m, situated in air, the number of lines of force arising upon it is $4\pi m$.

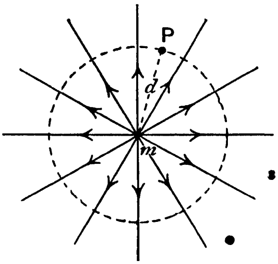


FIG. 750.—Magnetic field due to a single pole.

Application of Gauss's law to a single pole.—Consider a N pole of strength m, and let us find the strength of field at a distance d from it. Let P be a point at distance d cm. from m (Fig. 750). Through P describe a sphere with centre m. The magnetic field around a single pole is symmetrically distributed, that is, the same number of lines of force will pass through each square centimetre of the sphere. Now, from Gauss's law, the total number of lines is $4\pi m$, and since the area of the sphere is $4\pi d^2$ sq. cm., the number of lines per sq. cm.

is $4\pi m/4\pi d^2 = m/d^2$; and this we already know to be the strength of field in air at a distance d (p. 812).

Field due to a plane sheet of magnetic pole.—Whenever the distribution of magnetic field is symmetrical, Gauss's law can be applied to find the strength of magnetic field. Imagine a plane sheet of N pole, of infinite extent, the amount of pole per unit area being σ. The lines of force pass out uniformly in both directions from such a sheet.

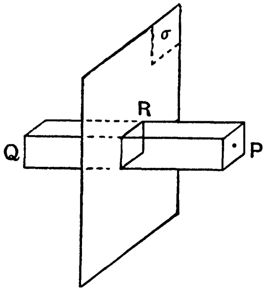


FIG. 751.—Magnetic field due to a plane polar sheet.

To find the strength of field at any point P (Fig. 751) draw a unit of area through P parallel to the plane polar sheet. Through the boundary of this unit area draw lines perpendicular to the sheet, so as to form a prism which cuts the sheet in unit area at R, and close the prism by another plane at Q. The amount of pole which lies within the prism is that upon the unit area at R, which is σ. Hence, by Gauss's law, $4\pi\sigma$ is the number of lines passing outwards from R. The field being everywhere perpendicular to the sheet, half the lines pass through the unit area at P, and the other half through Q. Hence the number of lines per square centimetre, or the strength of field at P, is $2\pi\sigma$.

It will be seen that the strength of field is independent of the distance of P from the sheet, provided that the sheet is so large that the lines pass out from it uniformly.

Field near the end of a bar magnet.—Consider the space between the poles of two bar magnets placed close together as in Fig. 752. If the faces are sufficiently close together, the magnetic field at a point P between them is uniform, and each polar face may, for all practical purposes, be considered to be infinite. Let I be the intensity of magnetisation of each magnet. The amount of pole per square centimetre of each face is then I (p. 810), and the strength of field at P due to the polar sheet N is $2\pi I$ (from above), and that due to S is also $2\pi I$. These, however, are both directed from N to S, so that the resultant strength of field at P is $4\pi I$, and this is the force on a unit pole when situated at P.

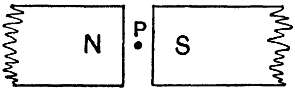


FIG. 752.—Magnetic field between two plane poles.

It will be noticed that the strength of magnetic field at P is independent of the distance apart of the polar faces N and S, provided that these faces are of such an extent that the field in the space between them is uniform.

Force between plane poles in contact.—In Fig. 752 the magnetic field due to either polar face is $2\pi I$, and when the poles are very close together, each is situated in the field due to the other. Thus, the field due to N is $2\pi I$, and each square centimetre of S has an amount of pole I upon it, and therefore experiences a force

$$2\pi I \times I = 2\pi I^2.$$

Hence if the two poles are in contact there is a force $2\pi I^2$ for each square centimetre of area in contact, causing them to adhere together.

It is not necessary that the two magnets in contact should be permanent magnets ; one may be a permanent magnet and the other a piece of soft iron. In this case the iron becomes magnetised in the field due to the magnet, the force between the two polar faces being the same as before, provided that the two intensities of magnetisation are the same. If they are not, the force per square centimetre will, of course, be $2\pi I_1 I_2$, where I_1 and I_2 are the respective intensities of magnetisation on the two sides of the plane of contact.

Since lines of induction are continuous, and for two plane parallel faces they are at right angles to the faces, it follows that the value of B must be the same on both sides of the gap. It will be seen on p. 816 that $B = 4\pi I$, so that the force between the faces may be represented by $B^2/8\pi$ instead of $2\pi I^2$. This result is of value in finding the lifting power of electromagnets upon masses of iron, and in the case of the telephone (Chap. LXXVIII.), where the force between a small electromagnet and an iron disc is used in the transmission of sounds.

Magnetic induction in iron.—We must now find the value of the induction (B) in the interior of a mass of iron. The strength of field there (H) is the force upon unit pole situated within the iron. This is due to external causes ; the magnetisation of the iron itself does not affect it, except in so far as the free poles at the ends of the magnet may modify it, because the molecular magnets constituting the iron consist of poles situated so close together that at any appreciable distance from them, their effect is zero. The effect of this external field (H) has been seen (p. 771) to cause an alignment of the molecular magnets, and consequently there will be lines of induction running the length of the magnet, due to these molecular magnets. These lines will, of course, leave at the N end of the magnet, since there they pass from the free N ends of the molecular magnets and will contribute to the external magnetic field. In the interior of the iron they pass immediately from the N ends of one molecular magnet to the S end of the next. This state of affairs is represented diagrammatically in Fig. 753, where the original field H is represented in dotted lines, and the induction due to the magnetisation of the iron by continuous lines.

In order to find the total value of the induction due to the magnetisation of the iron, consider a section of it in the inter-molecular spaces at P. If I be the intensity of magnetisation of the iron, then on each side of the section there is an amount of pole I units per square centimetre, N on one side and S on the other, and the number of lines crossing this square centimetre is $4\pi I$ (p. 814). Hence, the total number of lines of induction per square centimetre (B) is made up of H due to the original field, and $4\pi I$ due to the magnetisation of the iron, or $B = H + 4\pi I$.

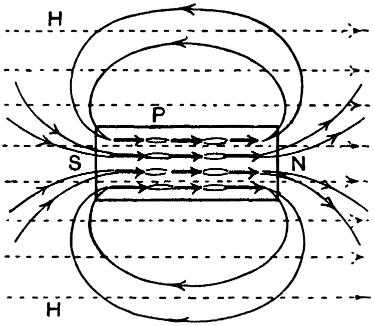


FIG. 753.—Lines of induction due to a magnetic material.

In the case of a permanent magnet there may not be any magnetising field, in which case $B = 4\pi I$.

The above expression may be written differently; for, on dividing through by H, we get

$$\frac{B}{H} = 1 + 4\pi\frac{I}{H},$$

or

$$\mu = 1 + 4\pi k,$$

μ being the magnetic permeability of the material, and k its magnetic susceptibility, as defined on p. 811.

The right-hand side of the equation $B = H + 4\pi I$ is represented by the lines drawn in Fig. 753, the two sets of lines being drawn in the same diagram. Within the iron we see that their resultant, or sum represents B. At external points the two sets are not in the same direction at every point, and in order to obtain their resultant, the two fields must be compounded by the ordinary law for the addition of vector quantities. This is not always easy, particularly for a rectangular bar, but the general arrangement is exhibited in Fig. 754. It will be noticed on examining this diagram of the resultant field, that the effect of placing the iron bar in a uniform field is to cause the lines of induction to converge upon it, producing a concentration of the lines at points such as A and B, and a separation at C and D. Thus, at A and B the field is strengthened by the presence of the specimen, while at C and D it is weakened. Within the iron the induction is greatest, but the strength of field is not increased. Within a medium of permeability μ, $H = B/\mu$, so that although B is great, μ is also great; in fact B depends upon μ, so that H has its original value, apart from any disturbance due to the poles at the ends of the specimen.

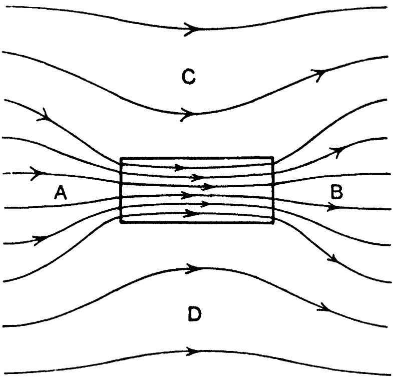


FIG. 754.—Resultant lines of induction for a magnetic body in a magnetic field.

Keepers.—The above discussion is only an approximation to the truth, for it will be seen that the poles at the ends of the specimen produce a field within it, of opposite direction to the original magnetising field H. This field **tends to demagnetise the bar,** with an effect which is greater, the shorter the bar. With long thin bars this demagnetising effect is small, but with short bars it becomes important. It is for this reason that permanent magnets are provided with soft iron **keepers** when not in use. Thus two bar magnets NS (Fig. 755) when not in use are packed in a box with the poles of opposite kind adjacent to each other. A soft iron keeper is placed at each end, and the poles $N'S'$ produced in these keepers will produce opposite fields to those of the bar magnets. Hence the **demagnetising** effect of the poles on the bars is removed by the **magnetising** field of the poles on the keepers.

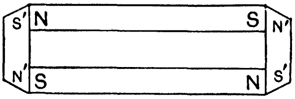


FIG. 755.—Keepers for a pair of bar magnets.

Measurement of magnetic susceptibility and permeability.—In order to measure the magnetic susceptibility and permeability of any material, it must be placed in a magnetic field of known strength, and must be of such a form that no poles are produced, or else that the poles, if produced, shall have as little demagnetising effect as possible. This is usually effected by using the material in the form of a long thin wire. The magnetising field is nearly always produced by means of a coil of wire through which an electric current is flowing. A straight coil of this kind (Fig. 756) produces a uniform magnetic field of strength $H = 0{\cdot}4\pi n \times$ (current) in its interior (Chap. LXXV). Hence the magnetising field (H) is known, and it only remains to find the magnetic moment of the specimen. This may be carried out by means of the magnetometer. AB and CD (Fig. 757) are two coils of length about 12 cm., each consisting of about 400 turns of No. 22 copper wire. They are placed one on either side of the magnetometer needle, and are so connected together that when an electric current flows through them, they produce equal and opposite effects upon the needle. This may be attained by moving one of them nearer to, or further from the needle, until there is no movement of the needle, whatever the current in the coils may be.

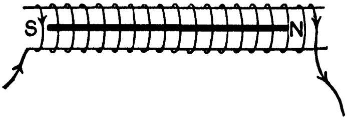


FIG. 756.—Magnetising coil.

The specimen to be experimented upon may be a piece of knitting needle about 8 cm. long and diameter 2·5 mm. When placed in one of the coils, it becomes a magnet (Fig. 756), and it is our object to determine its magnetic moment M, for when this is known we can calculate from it the intensity of magnetisation. It is first necessary to find the effective length of the magnet, l, and the strength of the earth's field H_1. This is effected by placing the specimen in the coil and magnetising it with the strongest current available, and then stopping the current, when the specimen will remain magnetised. Take the specimen out of the coil, and with its middle point at a distance d_1 cm. from the needle, the deflection θ_1 is obtained, and from the relation on p. 786 we have

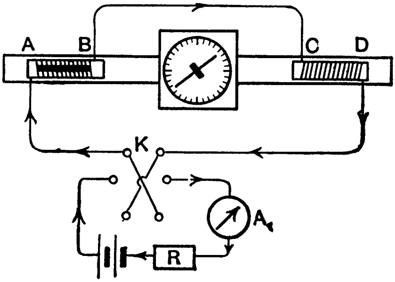


FIG. 757.—Magnetometer arranged to measure intensity of magnetisation.

$$\frac{M}{H_1} = \frac{(d_1^2 - l^2)^2}{2d_1} \tan \theta_1.$$

The specimen is now brought nearer to the needle, to a distance d_2 cm., and the new deflection θ_2 observed; when

$$\frac{M}{H_1} = \frac{(d_2^2 - l^2)^2}{2d_2} \tan \theta_2.$$

Hence, $$\frac{(d_1^2 - l^2)^2}{2d_1} \tan \theta_1 = \frac{(d_2^2 - l^2)^2}{2d_2} \tan \theta_2,$$

or,
$$\frac{d_1^2 - l^2}{d_2^2 - l^2} = \sqrt{\frac{d_1}{d_2} \cdot \frac{\tan\theta_2}{\tan\theta_1}},$$
and since d_1, d_2, θ_1 and θ_2 are all known, l may be found.

Next, the magnet is suspended in a vibration box or beaker, and its time of oscillation T found (p. 788).

Then,
$$\mathsf{T} = 2\pi\sqrt{\frac{\mathrm{I}_1}{\mathsf{MH}_1}}, \quad \text{or} \quad \mathsf{MH}_1 = \frac{4\pi^2\mathrm{I}_1}{\mathsf{T}^2},$$
where I_1 is the moment of inertia of the magnet (p. 789).

Combining this with the relation
$$\frac{\mathsf{M}}{\mathsf{H}_1} = \frac{(d_2^2 - l^2)^2}{2d_2}\tan\theta_2,$$
we have
$$\mathsf{H}_1^2 = \frac{4\pi^2\mathrm{I}_1}{\mathsf{T}^2} \cdot \frac{2d_2}{(d_2^2 - l^2)^2}\frac{1}{\tan\theta_2},$$
or
$$\mathsf{H}_1 = \frac{2\pi\sqrt{\mathrm{I}_1}}{\mathsf{T}(d_2^2 - l^2)}\sqrt{\frac{2d_2}{\tan\theta_2}},$$
whence H_1 is known.

Then in any further experiment on the deflection, at distance d,
$$\mathsf{M} = \frac{(d^2 - l^2)^2}{2d} \cdot \mathsf{H}_1 \tan\theta,$$

and, since the intensity of magnetisation I is M/la, where a is the area of cross section of the specimen,
$$\mathrm{I} = \frac{(d^2 - l^2)^2}{2d \,.\, l \,.\, a} \,.\, \mathsf{H}_1 \tan\theta,$$
or,
$$\mathrm{I} = \mathsf{K}_1 \tan\theta,$$
where
$$\mathsf{K}_1 = \frac{(d^2 - l^2)^2\mathsf{H}_1}{2dla}.$$

Every quantity in this last expression is known, so that the constant K_1 is found, and the deflection θ at once gives us the intensity of magnetisation of the specimen. With a specimen of the dimensions given, the demagnetising effect of its own poles is great (p. 817) and involves a large correction. If greater accuracy is required a long thin wire must be used and the reflecting magnetometer employed.

EXPT. 178.—**Determination of intensity of magnetisation.** Using a piece of steel knitting needle, find the deflections θ_1 and θ_2 at distances d_1 and d_2 and so obtain l (p. 818). Then find the time of vibration T and obtain H_1. Hence obtain the constant $\mathsf{K}_1 = \frac{(d^2 - l^2)^2}{2dla}\mathsf{H}_1$.

Replace the specimen in the coil AB (Fig. 757) and adjust the rheostat R until the largest current available is indicated by the ammeter A_1. Read the deflection θ. Then diminish the current and again read θ. Repeat this, taking a number of readings down to zero current. Reverse by means of the key K, and increase the current up to its greatest negative value; then reduce it again to zero by steps, taking observations of θ on the way. Then reverse again and proceed to the greatest positive current, noting the values of θ as before. Make a table of the observations as below. The first two columns give the values of the current and θ. Then fill in the third column with the values of tan θ, and the fourth with the values of I, that is, $K_1 \tan \theta$.

Current.	θ.	tan θ.	I.	H.

The values of H, the magnetising field, are put in the last column, being obtained from the relation $H = 0{\cdot}4\pi n \times$ (current) (p. 818).

Plot, on squared paper, the values of H and I. This will give a curve of the form *abcdefa* (Fig. 760) and shows a **cycle of magnetisation.**

If an unmagnetised piece of knitting needle exactly like the specimen be used, the observations may be started at zero instead of maximum current, so that the part O*a* of the curve is obtained; or, if preferred, the cycle of readings may be taken before the standardising observations are made, in which case only one specimen is required

A further experiment may be made upon a piece of soft iron, using the same value of K_1 as before, provided that the specimen has the same dimensions as the steel specimen used in making the standardising measurements.

It should be noted that the demagnetising effect of the poles upon the specimen (p. 817) has been neglected. With the dimensions of specimen given (p. 818) this introduces an error of about 10 per cent. in the value of H for the greatest readings, and as a first approximation is neglected. The more refined methods in which this correction is eliminated are beyond the scope of this book.

B-H curves.—The curve in Fig. 760 gives the relation between I and H. If that for B and H is required the values of B must be calculated from those of I, remembering that $B = H + 4\pi I$.

For many purposes the cycle of magnetisation (Fig. 760) is not required, but only the values of the induction with continually increasing magnetising field. Such a curve is given in Fig. 758 for soft iron. It will be noticed that for small values of H, B increases slowly, as from O to *a*. From *a* to *b* the curve is very steep, the value of B rising rapidly as H increases. After this the increase in B is slow, and eventually the part *bc* becomes very nearly a straight line, inclined at a small angle to the axis OH. The corresponding values of the permeability μ, obtained from the equation $B=\mu H$, are plotted on the same diagram. It will be seen that μ starts with a constant small value, rises to a maximum at *d*, and then falls gradually to a small value, constantly approaching the limiting value $\mu=1$ for very high magnetising fields.

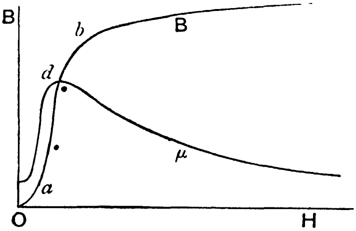


FIG. 758.—B-H and μ-H graphs.

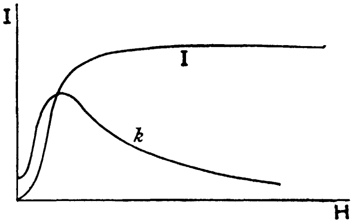


FIG. 759.—I-H and *k*-H graphs.

I-H curves: saturation.—On reducing the curve of B (Fig. 758) to that of I by the relation $B=H+4\pi I$, it will be seen that there is a general similarity between the two, but in this case the curve of I (Fig. 759) gradually becomes horizontal. It would only become strictly horizontal for infinite values of H. For the highest fields at which measurements have been made, $I=1610$ when $H=15530$, and the curve is very nearly horizontal. In this case the value of *k* has become very nearly zero. Since the curve of I eventually becomes horizontal for very high fields, the iron is said to have reached **saturation.** This is to be expected, from the molecular theory of magnetisation (p. 771), for if all the molecular magnets have been turned into the direction of the magnetising field, there is no possibility of producing further magnetisation. The relative magnetic properties of iron, steel, nickel, and cobalt are given in Fig. 763.

Hysteresis.—On examining a typical curve representing a cycle of magnetisation, several things may be noticed. As the magnetising field H increases, the intensity of magnetisation I increases, as shown from O to *a* (Fig. 760). On decreasing H, the value of I remains greater than when H was rising, and the path *ab* is followed. The value O*b* of the intensity of magnetisation when the magnetising field has dropped to zero is called the **residual magnetism.** This must not be confused with what is usually called permanent magnetism, which is the magnetisation persisting in a piece of steel although it may be subjected to a variety of treatment.

On reversing H and increasing its value to Oc, the value of I is brought to zero. This reverse field Oc required to reduce I to zero is called the **coercive force.** On further increasing the negative value of H, the path *cd* is followed. On diminishing and reversing, the completion *defa* of the curve is obtained. It will be seen that the descending branch of the curve always lies above the ascending branch. Further, the zero value of H is reached at an earlier part of the cycle than the zero value of I. Owing to this lagging of I behind H in the cycle, the phenomenon has been called **hysteresis,** from the Greek word ὑστερέω "to lag behind."

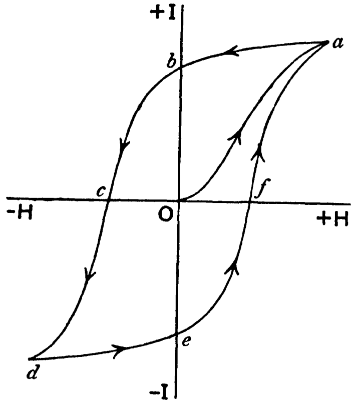


FIG. 760.—Cycle of magnetisation.

Iron, steel, nickel, cobalt.—The relative magnetic properties of iron and steel may be studied by reference to Fig. 761. Iron has a greater residual magnetism than steel, but a smaller coercive force. The curve representing the cycle in the case of iron is more upright and has a smaller area than that for steel. In Fig. 762 are cycles for nickel and cobalt.

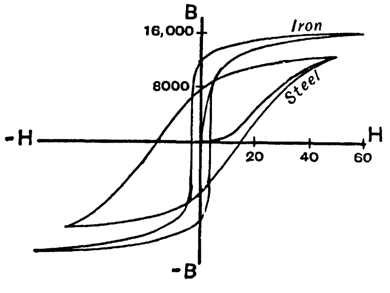


FIG. 761.—Cycles of magnetisation for iron and steel.

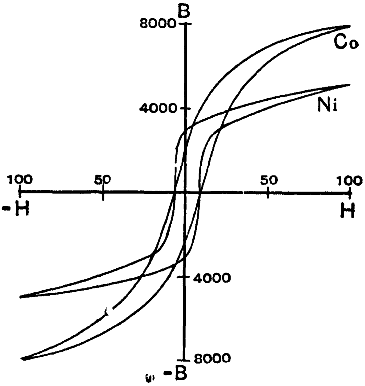


FIG. 762.—Cycles of magnetisation for nickel and cobalt.

In order to take a specimen through a cycle of magnetisation work must be done; for every cubic centimetre of the material, the work for a complete cycle is equal to the area, to scale, enclosed by the I–H curve. Thus, it will be seen that more work is required to take a specimen of steel through a magnetic cycle than in the case of a specimen having the same size and consisting of iron. As this work is converted into heat in the specimen, steel will become heated to a greater extent than iron when subjected to reversals of magnetisation. This fact is important in the design of electrical machinery, as this energy is wasted; consequently the parts subjected to rapid reversals of magnetisation are usually made of soft iron.

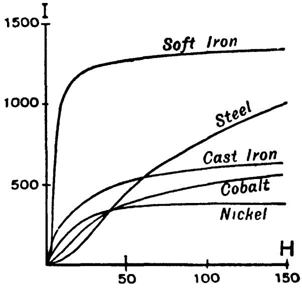


FIG. 763.—Magnetic properties.

Demagnetisation of steel.—It is frequently necessary to demagnetise a piece of steel. This can, of course, be performed by heating it up to a red heat and allowing it to cool, but a more satisfactory method is desirable. Heating the steel changes its character, and in the case of a delicate body like the hair-spring of a watch, the elastic properties of the body would be destroyed.

An examination of Fig. 760 would, at first sight, lead us to suppose that by stopping at the point *c* in the cycle, the body would then be demagnetised. But this is not the case, as there is still a magnetising field O*c*, and on removing this field, the body will be found to be magnetised. The only satisfactory method is to take the body through a succession of magnetic cycles, continually decreasing in magnitude, until the cycles become so small that the magnetic field is practically zero. Thus a watch whose hair-spring has been magnetised accidentally, may be very much improved, if not cured, by placing it inside a coil in which an alternating current is flowing. The current should be great at first, and then be gradually diminished to zero.

Ewing's molecular theory of magnetisation.—The molecular theory of magnetisation has been accepted for a long time, but in its simple form certain difficulties arise. For example, how is it that a magnetic field, however weak, does not set all the molecular magnets in its own direction, and so produce saturation? To get over this difficulty it must be supposed that the molecular magnets are not perfectly free to turn, and former investigators in the subject assumed that there is some friction opposing the turning of the molecular magnets. Sir James Ewing showed that the observed magnetic effects could all be explained, without introducing the idea of friction, by means of the magnetic effects of the neighbouring molecular magnets upon each other. He imitated a piece of magnetic material by means of a group of compass needles. The explanation may be simplified by considering a group of four such magnets, but it must be remembered that actually there must be an infinite number arranged in all possible ways.

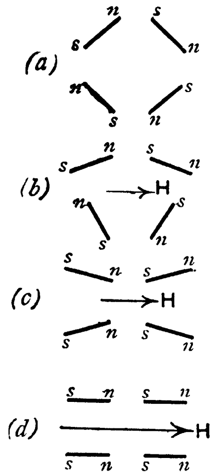
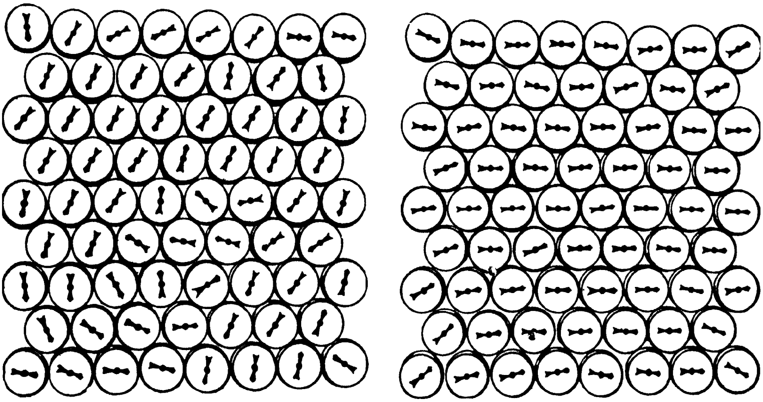


Fig. 764.—Magnetisation of a group of four magnets.

A group of four magnets arranged as in Fig. 764 (*a*) would set as shown when there is no magnetising field. The N and S poles of consecutive magnets would be so close together that there would be no external magnetic effect produced. This corresponds to the unmagnetised condition. A weak magnetising field H would rotate the magnets slightly into its own direction as in (*b*), but would not break up the group. On increasing H, a point will be reached at which the lower two magnets will swing round to the position shown in (*c*). Thus, a small increase in H will correspond to a large increase in the intensity of magnetisation, being the change indicated by the part *ab* of the curve in Fig. 758. Any further increase in H can only set the magnets slightly more in line, as in Fig. 764 (*d*), the part *bc* of the curve in Fig. 758 being realised.



(a) **Fig. 765.**—Ewing's model. *(b)*

Remembering that an actual piece of iron consists of a vast number of molecular magnets which arrange themselves in groups and lines of all degrees of stability, it will be seen that these groups will not all break up at the same time, and the flowing curve of Fig. 758 will be the result. Fig. 765 (*a*) is from a photograph of a group of compasses which are not subject to a magnetising field, and (*b*) the same group when subjected to a moderately strong magnetising field.

Paramagnetic and diamagnetic substances.—The three substances —iron, nickel, and cobalt—are so vastly more magnetic than any other substances that they are placed in a group by themselves; they are said to be **ferromagnetic.** The permeability of iron may be as great as 2000, that of nickel 300, and of cobalt 250. No other substance has permeability approaching these, in fact its value in all other cases is very nearly unity. Nevertheless, nearly all substances have feeble magnetic properties. The magnetic property of these substances is better expressed in terms of the susceptibility than permeability. Thus the magnetic susceptibility of platinum is $+1{\cdot}32\times10^{-6}$, of aluminium $+0{\cdot}65\times10^{-6}$, water $-0{\cdot}8\times10^{-6}$, copper $-0{\cdot}087\times10^{-6}$, and bismuth $-1{\cdot}4\times10^{-6}$.

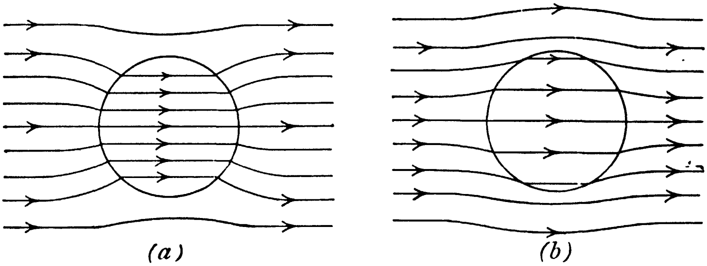

(*a*) (*b*)

FIG. 766.—Paramagnetic and diamagnetic substances.

For some substances the magnetic susceptibility is positive and for others negative. When it is positive the substance is said to be **paramagnetic**, and when negative the substance is **diamagnetic.** By employing the relation $\mu=1+4\pi k$, we see that the permeability of platinum is 1·000017 and of bismuth 0·99996. Thus **paramagnetic substances have a permeability greater than unity and diamagnetic substances a permeability less than unity.**

Fig. 766 (*a*) represents the distribution of the lines of induction for a sphere of a paramagnetic substance placed in a uniform magnetic field, and (*b*) represents the case of a diamagnetic substance.

Whether a substance is paramagnetic or diamagnetic may be determined by its behaviour in a strong magnetic field. Thus, a rod of the substance, suspended in a strong magnetic field, will set with its length in the direction of the field, if the substance is ferromagnetic or paramagnetic, but if the substance is diamagnetic the rod will set with its length at right angles to the field. Thus a piece of iron or platinum suspended in a magnetic field will set as shown in Fig. 767 (*a*), but a piece of bismuth as in (*b*).

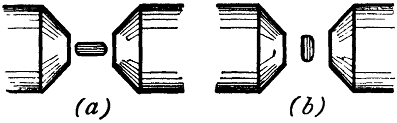


FIG. 767.—Para- and diamagnetism.

Further, a piece of a ferromagnetic or paramagnetic substance tends to move from the weaker to the stronger parts of a magnetic field, as has been seen by the manner in which iron filings cling to a magnet. On the contrary, a piece of diamagnetic substance tends to move from the stronger to the weaker parts of the field, but the effect is so small even in the case of the most diamagnetic substances that special arrangements are necessary in order to observe it.

* **The magnetic circuit.**—A magnetic field may be represented completely by means of lines of induction (p. 816). This gives rise to a means of calculating the magnetic field and induction in several important practical cases. Through the boundary of any area S_1 situated at *a* (Fig. 768) and at right angles to the direction of the induction, draw lines of induction. Since lines of induction are closed curves, they will enclose a tube *abcd* when produced in both directions. The lines crossing the area S_1 at *a* will also cross S_2 at *b*, S_3 at *c*, and so on, for no lines of induction pass into or out of the tube. Now the number of lines per unit area at *a* is B_1 (the value of the induction there), so that the whole number crossing S_1 is B_1S_1. Also the number crossing S_2 is B_2S_2, and so on. But as these are all the same,

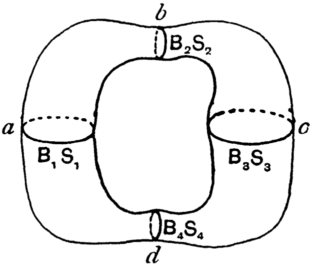


FIG. 768.—Magnetic circuit.

$$B_1S_1 = B_2S_2 = B_{\,}S_3 = B_4S_4.$$

In other words, the quantity BS is the same for every section of the tube. Such a closed tube of induction is called a **magnetic circuit**. It is the property of a magnetic circuit that the quantity BS, sometimes called the **magnetic flux**, or the total number of lines of induction, is the same for every section.

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* To be omitted until after reading Chapter LXXV.

The magnetic circuit may be made up of a number of parts, having different cross-sections and different permeabilities. Thus, consider the core of an electro-magnet (Fig. 769). The magnetic circuit will be very nearly that indicated by the dotted lines. Let the yoke have an effective length l_1 and cross section S_1, and be constructed of iron of permeability μ_1. The quantity $l_1/\mu_1 S_1$ is called the **magnetic resistance** of this part of the circuit. Similarly $l_2/\mu_2 S_2$ is the magnetic resistance of each limb, and $l_3/\mu_3 S_3$ that of each pole piece. The magnetic resistance of the air gap between the pole pieces is l_4/S_4, since the permeability of air is unity. Now **the magnetic resistance of the whole circuit is the sum of the magnetic resistances of its separate parts.** Thus, for the whole circuit

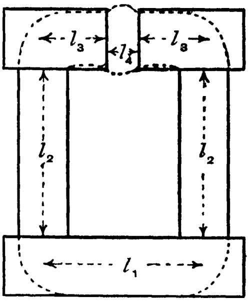

FIG. 769.—Core of an electro-magnet.

$$\text{Magnetic resistance} = \frac{l_1}{\mu_1 S_1} + \frac{2l_2}{\mu_2 S_2} + \frac{2l_3}{\mu_3 S_3} + \frac{l_4}{S_4}.$$

The **product of the total induction or magnetic flux and the magnetic resistance is called the magneto-motive force in the circuit,** usually written M.M.F.

Thus, (Magnetic flux) × (magnetic resistance) = M.M.F.,

or,
$$\text{Magnetic flux} = \frac{\text{M.M.F.}}{\text{magnetic resistance}}.$$

In the case of an electro-magnet the M.M.F. is due to an electric current flowing in coils surrounding the limbs (Fig. 933). **The product of the current in amperes (p. 843) and the total number of turns is called the number of ampere-turns**; further,

$$\text{M.M.F.} = \frac{4\pi \times (\text{ampere-turns})}{10}.$$

Thus, for a given electro-magnet, if the number of ampere-turns be known, the M.M.F. may be found. Then, knowing the magnetic resistance, the magnetic flux can be calculated. Dividing this by the area of the air gap, the value of B, or the strength of magnetic field in the gap can be found.

EXAMPLE.—An iron ring of circumference 50 cm. and cross-section 0·5 sq. cm. is wound with 400 turns of wire in which a current of 1·5 amperes flows. At one part of the ring is an air gap 2 mm. wide. Find the strength of magnetic field in this gap, if the permeability of the iron is 500.

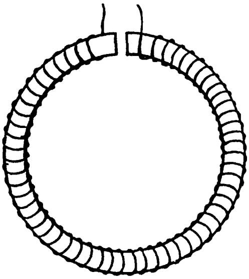

FIG. 770.—Circular magnetic circuit.

$$\text{Number of ampere turns} = 400 \times 1{\cdot}5 = 600;$$

$$\therefore \text{ M.M.F.} = \frac{4\pi \times 600}{10} = 240\pi.$$

Again, Magnetic resistance of the iron ring

$$= \frac{50}{500 \times 0{\cdot}5} = 0{\cdot}2,$$

$$\text{Magnetic resistance of the air gap} = \frac{0{\cdot}2}{0{\cdot}5} = 0{\cdot}4;$$

$$\therefore \text{ Total magnetic resistance} = 0{\cdot}2 + 0{\cdot}4 = 0{\cdot}6.$$

$$\text{Now magnetic flux} = \mathsf{BS} = \frac{\text{M.M.F.}}{\text{magnetic resistance}} = \frac{240\pi}{0{\cdot}6} = 1257.$$

This is also the value of BS for the air gap;

$$\therefore \mathsf{B} = \frac{1257}{0{\cdot}5} = 2514 \text{ C.G.S. units.}$$

But the permeability of the air is unity, so that the strength of field in the air gap is also 2514 C.G.S. units.

EXERCISES ON CHAPTER LXIV.

1. Define intensity of magnetisation. How is it measured, and how does it vary in iron with the magnetising force? L.U.

2. Define "intensity of magnetisation" and "magnetic susceptibility." A hollow iron mast 12 metres high, having external diameter 30 cm. and internal diameter 20 cm., is magnetised by the vertical component of the earth's magnetic field. Taking the intensity of this to be 0·40 unit, and the susceptibility of the iron to be 8·0, calculate the magnetic moment of the mast, and its effect upon the time of vibration of a compass needle placed 4 metres north of the foot of the mast, neglecting the effect of the pole at the top, and taking H = 0·2. L.U.

3. Give two separate definitions of intensity of magnetisation. Find an expression for the force between the poles of two bar magnets placed face to face and in contact.

4. Calculate the strength of field near a plane sheet of magnetic pole of strength 8·5 per sq. cm.

A gap is cut in a ring magnet, the intensity of magnetisation of which is 80. Find the strength of magnetic field in the gap.

5. Give a brief account of the molecular theory of magnetism.

6. What is a magnetic circuit?

An electro-magnet of ring form has a cross-section of 10 sq. cm., a length of 70 cm. measured along the circumference, and is excited by a current of 5 amperes flowing in 600 turns of wire. The length of the air gap being 1 cm., find the strength of magnetic field in this gap, if the permeability of the iron is 500.

7. Define intensity of magnetisation.

The maximum intensity of permanent magnetisation in a steel bar 10 cm. long by 1 cm. square has been found to be 225 C.G.S. units.

Find the tangent of the greatest deflexion of a magnetometer which such a magnet could cause if the centre of the needle were 30 cm. east of the centre of the magnet. ($H = 0{\cdot}18$ C.G.S.) L.U.

8. Define "magnetic moment" and "intensity of magnetisation."

Find the strength of magnetic field at a point 50 cm. from the middle of a piece of magnetised steel wire 5 cm. long and 2 mm. in diameter, if the intensity of magnetisation of the steel is 200, the point being situated on the axis of the magnet. L.U.

9. A cylindrical magnet of 1 sq. cm. section and with poles 20 cm. apart, suspended by a vertical thread, makes 20 complete vibrations in 88 secs. at a place where the earth's magnetic horizontal intensity is 0·25. If the moment of inertia of the magnet is 245, find the magnetic moment, pole strength and intensity of magnetisation. Univ. of Sydney.

10. A piece of iron wire 36 cm. long and 2 mm. in diameter is magnetised in the direction of its length by a field of strength 25 C.G.S. units. If the magnetic susceptibility of the material is 52, calculate the magnetic moment and the strength of field due to the wire, at a point 80 cm. from it and situated on the line bisecting it at right angles.

11. A bar of steel of length 23 cm., breadth 1·2 cm. and thickness 0·5 cm. is placed in a magnetic field of strength 7·5 units and parallel to its length. Find the magnetic moment of the bar if its permeability is 640.

12. Find the magnetic moment and strength of pole of a bar of iron of length 10 cm. and cross-section 0·5 sq. cm., if it is uniformly magnetised in the direction of its length to an intensity of 500.

13. Two long soft iron rods of area of cross-section 2·5 sq. cm. are placed end to end and in contact. They are situated in a long solenoid having 15 turns per cm. in which a current of 1·5 ampere is flowing. If the permeability of the iron is 150, what is the force required to separate the rods ?

CHAPTER LXV

THE ELECTRIC CURRENT

Effects of current.—In certain circumstances a wire may exhibit characteristic phenomena; thus, there may be a magnetic field associated with it, although the material of the wire is not itself a magnetic substance. Also the wire may become hot, without any apparent application of heat to it; and, further, if the wire be cut and the two ends dipped into a solution of sulphuric acid in water, bubbles of gas, oxygen and hydrogen, will appear upon the ends immersed. Without, for the moment, considering how the wire may be brought into this condition, we shall study these effects. When they are observed we say that **an electric current is flowing in the wire.**

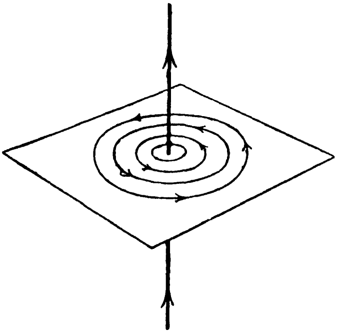

FIG. 771.—Magnetic lines of force due to a straight current.

Magnetic field due to an electric current.—In the case of a long straight wire carrying an electric current, the magnetic field in its neighbourhood is in the form of circles having their centres upon the wire and their planes perpendicular to it. The magnetic lines of force may be traced as on p. 778 by passing a straight wire through a piece of cardboard, and using a compass needle to find the direction of the magnetic field at various points. Or iron filings may be sprinkled upon the card; on tapping the card the filings will set themselves in approximate circles (Fig. 771).

If the wire be in the form of a circle, the form of the field in a plane at right angles to the circle is as shown in Fig. 772.

EXPT. 179.—**Magnetic field due to a circular current.** Pass a current round the circular coil (Fig. 772), which passes through a horizontal board. Cut holes in a piece of drawing paper to fit the coil and place it upon the board. Trace the magnetic lines of force by means of a compass needle as on p. 778. Note that the lines close to the limbs of the coil approximate to circles.

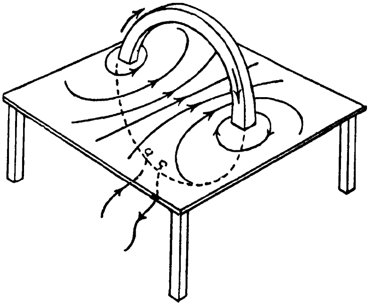

FIG. 772.—Magnetic field due to a circular current.

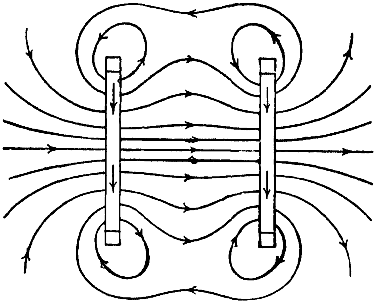

FIG. 773.—Magnetic field due to a double coil.

EXPT. 180.—**Magnetic field due to a double coil.** Pass a current in the same direction round two parallel circular coils (Fig. 773). Fit a piece of drawing paper to the coils, and plot the magnetic lines of force as in the last experiment. Note that for a considerable space in the middle of the field the lines are nearly parallel; that is, the magnetic field is very nearly uniform.

Direction of current.—In the preceding experiments there is nothing to indicate the direction in which the electric current is flowing in the wire or coil. In fact, the direction of flow is defined conventionally in the following manner. Let the wire be taken in the **right** hand with the fingers pointing in the direction of the magnetic lines of force; then the **thumb** points in the direction of the current (Fig. 774). This will be found to be in accordance with the arrows in Figs. 771 and 772.

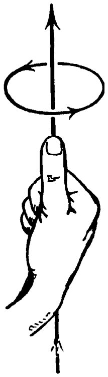

FIG. 774.—Direction of current.

EXPT. 181.—**Direction of magnetic field due to a current.** Join one end of a piece of copper wire to the zinc pole of a Daniell's cell, and the other end to the copper pole. The current then flows in the wire from the copper to the zinc. Place part of the wire horizontally with the current flowing from north to south and situated immediately over a compass needle. Note the direction of deflection of the N pole of the compass. Place the wire under the compass and again note the direction of deflection. Continue according to the following table, and enter the results as shown :

Wire.	Current flowing,	Position of wire.	Deflection of N pole of compass.
Horizontal	north to south	above compass	E
,,	south to north	,, ,,	
,,	,, ,,	below compass	
,,	north to south	,, ,,	
Vertical	downwards	N of compass	
,,	,,	S ,,	
,,	upwards	,, ,,	
,,	,,	N ,,	

Verify the fact that these results are in accordance with the rule for the direction of current given on p. 831.

Expt. 182.—**Magnetisation of iron by an electric current.** Take a cylindrical bar of soft iron, about 1 cm. diameter, and wrap round it about 20 or 30 turns of insulated copper wire. Pass a current through the wire. It will be found that the rod becomes a strong magnet while the current is flowing. Test with iron filings, and find the polarity of each end by means of a compass needle. Reverse the direction of the current and again find the polarity of the ends. Show that the polarity is in accordance with Fig. 773.

Expt. 183—**Heating effect of a current.** Connect the terminals of a storage cell by means of a piece of bare platinoid wire, No. 22, about a metre long, and note that the wire becomes warm. If the wire be shortened it becomes hotter. Replace the platinoid wire by a piece of iron wire, size No. 30. This wire also becomes very hot, and if shortened to a few centimetres becomes red hot and eventually fuses.

Sharpen two carbon rods to points and bind a fine piece of bare copper wire round each rod. Join each copper wire to one terminal of a battery of secondary cells. On touching the two carbon points together they will become red hot or even white hot at the tips.

Expt. 184.—**Chemical effect of a current.** Dip the ends of the two carbon rods of the last experiment into a vessel of water containing a few drops of sulphuric acid. Bubbles will form on each of the rods and will rise up and burst at the surface of the liquid. Note that at the carbon rod connected to the **black pole** of the battery the bubbles are about twice as copious as those on the rod connected to the **red pole.** If the bubbles be collected in inverted test tubes, it will be found that the gas liberated at the carbon rod connected to the black pole is hydrogen, while that from the rod connected to the red pole is oxygen.

Dip the rods into a solution of copper sulphate. Copper is deposited on the carbon attached to the black pole of the battery, while bubbles appear upon that attached to the red pole.

Measurement of electric current.—Like all other physical quantities an electric current may be measured by the effect it produces. Thus for its measurement, the magnetic effect, the heating effect, and the electrolytic or chemical effect are all available, and any one of them might be chosen to define the magnitude of the current. If the current be taken as proportional to the heat produced in a given time, very grave difficulties would be encountered in its future study. This is consequently abandoned. Use of the chemical effect involves the measurement of time, as the effect goes on continually, being greater for a large than for a small time. On the other hand, for a given current the magnetic field is constant, being independent of time. It is therefore chosen as the most satisfactory property of the current for the purpose of measurement. We will therefore consider that **an electric current is proportional to the strength of the magnetic field which accompanies it.**

Unit of current.—In order to define a unit of current in terms of a strength of magnetic field, it is desirable to take such a shape for the conductor carrying the current that the strength of field is proportional to the length of the conductor as well as to the current. This can only be the case when the current is in the form of a circle, and the point at which the magnetic field is considered is the centre of the circle.

Unit current is such that when flowing in an arc of a circle of unit radius, the arc having unit length, the magnetic field at the centre has unit strength.

Thus, if ABC (Fig. 775) be a circle of 1 cm. radius, and AB be an arc of this circle having length 1 cm., the magnetic field due to the current in this arc AB has unit strength at the centre O.

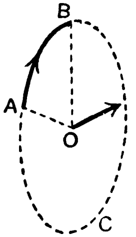

FIG. 775.—Magnetic field due to unit current.

Of course, it is not usual to attempt to produce a current flowing in a portion of a circle such as AB; the circuit consists in practice of a complete circle, the length of which is 2π cm. when the radius is 1 cm. The strength of field due to the complete circle is therefore 2π units, in fact, we may define a unit current as a current of strength such that when flowing in a complete circle of 1 cm. radius, it produces a magnetic field of strength 2π at the centre.

Strength of magnetic field at the centre of any circular current.—It has now been seen that the strength of magnetic field is proportional to the current (i) flowing in a circle, and to the length of the arc (l), and it remains to be seen how it depends upon the radius of the circle. This will not be proved experimentally here, but it has been conclusively shown that, other things being equal, the strength of field varies inversely as the square of the radius. Thus, the complete expression is,

$$\text{Strength of magnetic field, } H \propto \frac{li}{r^2}.$$

Since, however, $H = 1$, when $i = 1$, $l = 1$, and $r = 1$, from the definition of unit current (p. 833), we may now write $H = li/r^2$.

Again, for one complete circle, $l = 2\pi r$, or for a circle consisting of n complete turns of wire, $l = 2\pi nr$, hence,

$$H = \frac{2\pi nri}{r^2} = \frac{2\pi ni}{r}$$

where r is in centimetres and i in units of current.

The tangent galvanometer.—From the preceding discussion we see that the measurement of electric current depends upon the measurement of magnetic field. For the measurement of magnetic field the magnetometer was devised (p. 784) and this instrument may now be adapted for the purpose of measuring current. It is then called the **tangent galvanometer**, owing to the tangent law which we saw to hold in the case of the magnetometer.

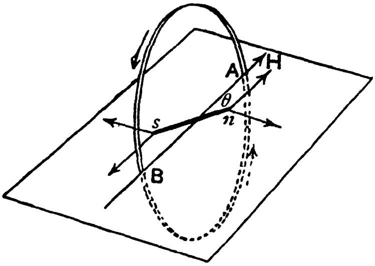

FIG. 776.—The tangent galvanometer.

A vertical circular coil AB (Fig. 776) carries the current, and this produces a magnetic field of strength $2\pi ni/r$, at the centre, the direction of field being at right angles to the plane of the circle. The plane of the circle is therefore placed in the magnetic meridian, so that a suspended magnet ns experiences a couple twisting it out of the meridian, while the earth's horizontal component of magnetic field H tends to retain it in the meridian. This is exactly the condition that held in the case of the magnetometer (p. 784), and the suspended magnet will come to rest at some angle θ to the meridian (Fig. 777) when the restoring and deflecting couples are equal.

The restoring couple is $\mathrm{H}m \sin\theta$, where m is the magnetic moment of the suspended magnet; also,

$$\text{Deflecting couple} = \frac{2\pi n i}{r} m \cos\theta;$$

$$\therefore \frac{2\pi n i}{r} m \cos\theta = \mathrm{H}m \sin\theta,$$

or,
$$i = \frac{\mathrm{H}r}{2\pi n} \tan\theta.$$

Thus, if H, r, and n be known and θ be observed, the strength of current, i, can be calculated.

Even when H is not known the tangent galvanometer may be usefully employed for comparing currents, in which case the equation is usually written,
$$i = k \tan\theta.$$

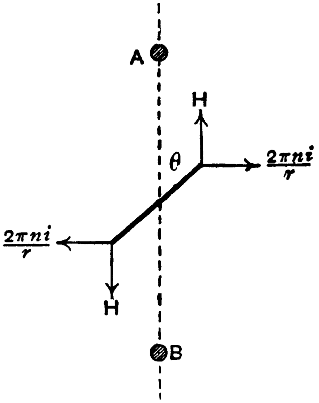

FIG. 777.—Diagram for the tangent galvanometer.

The quantity k is called the **reduction factor of the galvanometer.**

When two currents i_1 and i_2 are to be compared, the deflections θ_1 and θ_2 which they produce are observed; then

$$i_1 = k \tan\theta_1,$$
$$i_2 = k \tan\theta_2;$$
$$\therefore \frac{i_1}{i_2} = \frac{\tan\theta_1}{\tan\theta_2}.$$

Use of the tangent galvanometer.—A common form of the galvanometer has a coil consisting of a number of turns wound upon a circular frame, the ends of the coils being attached to the terminals upon the base of the instrument. Three levelling screws are provided and enable the instrument to be adjusted so that the suspended needle may be at the centre of the coil. The deflections are observed by means of a long light pointer, attached to the magnet, the end of the pointer moving over a large horizontal circular scale as in the magnetometer (Fig. 727). To avoid parallax in making the readings, the scale is pasted upon a piece of plate-glass mirror, in order that the eye may be placed so that the pointer and its image in the mirror coincide. The eye is then normally over the scale and the reading is free from error due to parallax.

It should be noted that the suspended magnet must be small, so that the magnetic field due to the coil may be considered to be uniform over the whole of the magnet, and its value, therefore, equal to that at the centre of the coil. An inspection of Fig. 772 will show that the magnetic field is sensibly uniform over a small space only at the centre of the coil, so that, should the magnet be too large, it is not in a uniform field and the poles of the magnet are no longer situated in a magnetic field whose value is $2\pi ni/r$.

Adjustments of the tangent galvanometer.—(i) The fibre supporting the needle must be free from torsion, otherwise there is a disturbing couple due to this cause. If the magnet is fairly strongly magnetised and the fibre sufficiently thin, this disturbing effect may be eliminated by removing the magnet and replacing it by a small piece of wire of equal weight. When this comes to rest, replace the magnet.

(ii) The needle must be at the centre of the scale. This adjustment is effected by means of levelling screws, and is sufficiently accurate when the needle is in the centre of the hole in the middle of the plate glass, and the pointer is symmetrically situated with respect to the scale.

(iii) The plane of the coil must be in the meridian. The whole instrument is rotated until the ends of the pointer are at the zero marks on the scale.

(iv) The pointer may not be at right angles to the magnetic axis of the magnet, in which case the plane of the coil is not in the magnetic meridian, and the tangent law does not hold good. This error may be eliminated by suspending the magnet and pointer the other way up, and seeing whether the pointer still comes to rest in the same position. When this is the case the pointer and needles are at right angles (p. 795), but if it comes to rest in a new position, one must be twisted with respect to the other, and another trial must be made. Fig. 778 shows how this may be done. The system is suspended by a loop A from a hook attached to the fibre. When the system is reversed it is suspended from the loop B. The framework, made of copper wire, is sufficiently flexible for the twisting of the magnet with respect to the pointer to be performed.

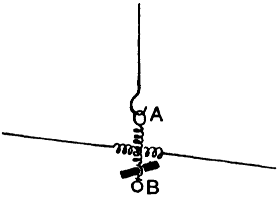

FIG. 778.—Suspension of the needle of a tangent galvanometer.

When this adjustment is completed, (iii) must be performed again.

Method of reading.—(i) Having made the above adjustments, pass the current through the coil; the deflection should not be less than 10° or greater than 60°. Both ends of the pointer must be observed, in order to correct for any want of centring of the needle with respect to the scale.

(ii) The current is now reversed and the deflections on the other side of the zero observed. This eliminates any small want of symmetry of the needle with respect to the coil.

These measurements should be repeated with each of the currents to be measured or compared, the results being recorded as under :

	Deflections.					
	E end of pointer.	W end of pointer.	Current reversed.		Mean θ.	tan θ.
			E.	W.		
1st current -						
2nd current -						
.						

EXPT. 185.—**Measurement of current by the tangent galvanometer.** Make the adjustments described on p. 836. Pass the current from a Daniell's cell through the tangent galvanometer, making the readings of deflection and recording them as above. Count the number of turns in the coils and measure their mean diameter. Calculate the current from the equation $i = Hr \tan \theta/2\pi n$, using a previously determined value of H, the earth's field.

EXPT. 186.—**Measurement of H by means of the tangent galvanometer.** For this purpose some other method of measuring the current is necessary. An ammeter of some form to be described later (p. 870) may be used. Connect up as in Fig. 779, G being the tangent galvanometer, A the ammeter, R an adjustable resistance, B a battery and K a key which reverses the current in the tangent galvanometer without reversing it in the rest of the circuit. Starting with a small current, observe θ, and record as above, writing down the value of the current as observed by the ammeter in the current column. Then repeat with a slightly larger current, proceeding until ten values of the current have been employed. Plot on a curve the values of i and tan θ, when it will be found that the points lie very nearly on a straight line. Draw a straight line with a ruler to lie evenly amongst the points. The equation $i = Hr \tan \theta/2\pi n$ may then be written, $H = 2\pi n i/r \tan \theta$, and the values of n and r measured as before.

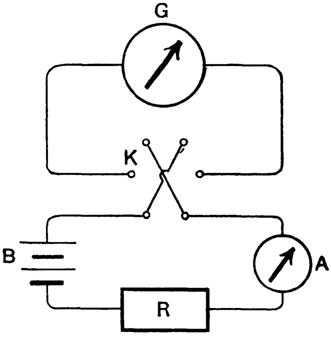

FIG. 779.—Use of the tangent galvanometer.

The value of $i/\tan\theta$ may be taken from the graph, but it must be noticed that the current as recorded by the ammeter must be divided by 10, for a reason given later (p. 843). In this way the value of H is obtained.

EXERCISES ON CHAPTER LXV.

1. Describe how you would show that an electric current is flowing in a given wire, giving a method for finding its direction.

2. Explain why it is necessary that the suspended magnet in a tangent galvanometer should be of small dimensions.

3. Calculate the strength of field at the centre of a circular coil of 30 turns of diameter 18 cm. due to a current of 5 c.g.s. units flowing in it.

4. A circular coil of 10 turns and radius 8 cm. is placed with its plane at right angles to the magnetic meridian. If a suspended magnet at its centre makes 18 vibrations per minute with the current in one direction and 30 vibrations per minute when the direction of the current is reversed, what is the strength of the current, given that the field due to the coil is greater than H? (Take $H = 0{\cdot}18$.)

5. Describe the principle of the tangent galvanometer. Why is the deflection not proportional to the current?

6. Explain why the plane of the coil of the tangent galvanometer must be in the magnetic meridian.

What is the relation between the strength of the electric current and that of the magnetic force due to it at the centre of the coil?

7. Two tangent galvanometers, A and B, are identical in construction, except for the number of turns in the coil. They are connected in series and a current is sent through them. The deflection in A is 45°, and that in B is 31°. Calculate the ratio of the numbers of turns in the two instruments. ($\tan 31° = 0{\cdot}60$.) L.U.

8. Describe with the aid of diagrams the construction of a tangent galvanometer, give the theory of the instrument, and show that it may be used to determine the strength of a current in absolute measure.

The coil of a tangent galvanometer has a radius of 15 cm. and contains 50 turns. Assuming that it is only used to measure currents which give a deflection less than 60° and greater than 1°, determine the range of current in amperes for which it is available. L.U.

9. A current flowing through a tangent galvanometer consisting of 10 turns of wire of radius 8 cm. produces a deflection of 45° when the instrument is in a position where $H = 0{\cdot}18$ dyne per unit pole. What alterations would you make in the instrument so that it would give this same deflection for a current of one thousandth of an ampere? L.U.

10. A current of 3 amperes is flowing in a coil consisting of 5 turns of wire each of 10 cm. radius.

Calculate the magnetic field at the centre of the coil, stating the units in which it is expressed. If a magnetic pole of strength *m* is placed at the centre of the coil, what force acts on the coil when the current is flowing? L.U.

11. Define the C.G.S. unit of current.

A circular coil of wire of 30 turns and radius 20 cm. is placed with its plane vertical and at right angles to the magnetic meridian. When no current flows in the coil, a magnetic needle at its centre makes 15 vibrations per minute; but when a current flows, the needle is reversed in direction and makes 50 vibrations per minute. Taking the horizontal component of the earth's magnetic field as 0·2, calculate the current.

12. Describe, and explain the use of, the tangent galvanometer.

If in a given instrument a current of ·5 ampere causes a deflection of 45°, what currents would be indicated by deflections of 30° and 60° respectively?

CHAPTER LXVI

POTENTIAL DIFFERENCE, RESISTANCE, AND WORK

Analogy between an electric current and the flow of liquid in a tube.—There are certain important points of resemblance between the electric current and the flow of water in a tube. These arise from the fact that in both cases some motive power is necessary to maintain the flow, and the energy of the supply eventually becomes heat in the circuit. The student is particularly warned against pushing the analogy too far. For example, in the case of an electric current there is no fluid passing along the wire, and, on the other hand, the flow of water in a tube which is not horizontal is partly due to the weight of the water. Also, when the water is flowing it possesses kinetic energy derived from the source supplying the energy to maintain the current. If, however, we confine our attention to narrow horizontal tubes the analogy serves a useful purpose.

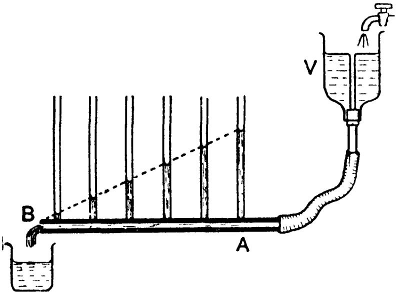

FIG. 780.—Flow of water in a horizontal narrow tube.

When a steady flow of water is maintained in a narrow horizontal tube into which vertical tubes are fixed to act as pressure gauges (Fig. 780), it will be noticed that the pressure falls uniformly along the tube. It is a maximum at the end A, and falls uniformly to the pressure of the atmosphere at the open end B. This fall of pressure is due to frictional resistance. By raising or lowering the vessel V, the flow of water may be altered, but the current of water, as measured by the amount passing through the tube in unit time, is proportional to the difference of pressure between the ends of the horizontal tube.

In the case of the electric current, the quantity analogous to difference of pressure is called **potential difference,** or p.d. Further, **the potential difference is proportional to the current,** and for a uniform conductor, the potential difference over equal lengths of conductor is the same.

There is a further resemblance between the two cases, for the work done in maintaining both currents is converted into heat, the **quantity of heat produced in unit time being in both cases proportional to the flow and to the difference of pressure, or potential difference.** The measurement of potential difference is deferred until later.

For convenience, the analogous quantities in the two cases are collected into the following table :

Flow of Liquid.	Electric Current.
(i) Flow, or volume of liquid crossing any section of pipe per second.	Electric current.
(ii) Flow $\propto$ difference of pressure between ends.	Current $\propto$ potential difference between ends of conductor.
(iii) Uniform fall of pressure along pipe.	Uniform fall of potential along conductor.
(iv) Quantity of liquid = flow × time.	Quantity of electricity = current × time.
(v) Amount of work converted into heat per second $\propto$ flow × difference of pressure.	Amount of work converted into heat per second $\propto$ current × p.d.

Unit of potential difference.—Analogy (v) in the above table serves to define the unit of potential difference. The relation of p.d. to current depends, of course, upon the nature of the conductor. For a given conductor, any current may flow, depending upon the p.d. between its ends. Thus, for a long and thin conductor, the p.d. corresponding to a given current will be greater than for a thicker conductor of the same material, and the rate of working to maintain this current is greater in the former case. We will define our unit of potential difference, as **that potential difference which, when maintaining unit current, performs one unit of work per second.** Measuring all our potential differences in terms of this unit, the analogy (v) may now be expressed as follows :

$$\text{Rate of working in conductor} = \text{current} \times \text{p.d., ergs per second}$$
$$= i \times e, \text{ ergs per second,}$$

or, total work performed in time t seconds

$$= \text{current} \times \text{p.d.} \times t$$
$$= i \times e \times t \text{ ergs.}$$

Ohm's law.—The relation between the current in a conductor and the potential difference between its ends is indicated in analogy (ii). It was first given by G. S. Ohm, and is known as **Ohm's law.** It may be stated as follows :

For a given conductor, the ratio of the potential difference between its ends to the current flowing in it is constant; or, p.d./current = constant.

For this law to hold, the conductor must remain under constant conditions ; for example, its temperature must not vary. A satisfactory proof of Ohm's law is beyond the scope of this work.

Resistance.—The name **resistance** is given to the ratio of p.d. to current for any conductor ; thus,

$$\frac{\text{p.d.}}{\text{current}} = \text{resistance.}$$

or,

$$\frac{e}{i} = r, \quad e = ir.$$

A conductor therefore has unit resistance if the p.d. between its ends is unity when unit current flows in it.

Since the rate of working in any circuit is known from the above equation, in terms of the p.d. and current, it may now be expressed in terms of any two of the three quantities—p.d., current, and resistance.

Thus, $$\text{Rate of working} = ei = i^2r = \frac{e^2}{r} \text{ ergs per second.}$$

This work performed in maintaining the current appears as heat in the conductor. With a small current, the amount of heat produced per second may be so small that it leaks away by conduction, etc., so rapidly that the temperature of the wire is not appreciably raised. It may happen, however, that the rate of production of heat is so great that the conductor is considerably raised in temperature, as in the case of the electric incandescent lamp. In fact, pieces of fine wire, called **fuses**, are generally placed in lighting and power circuits, which, for the limiting safe current, are raised to such a temperature that they melt, and so break the circuit. Such fuses are generally pieces of tin wire, since tin melts at a fairly low temperature, but they are sometimes made of copper.

The amount of heat produced in a circuit in a given time may be found from a knowledge of the fact that $4 \cdot 2 \times 10^7$ ergs (approximately) when converted into heat become 1 calorie (p. 355).

Thus, Heat produced in circuit

$$= \frac{ei}{4 \cdot 2 \times 10^7} = \frac{i^2 r}{4 \cdot 2 \times 10^7} \text{ calories per second.}$$

It will be noticed that for a circuit of given resistance, the heat produced per second is proportional to the square of the current and not simply to the current. This is the reason why the heating effect was not chosen to define the unit of current (p. 833).

Practical units.—The units defined above have been chosen on account of their simple relation to the mechanical units of force and work. For practical purposes these units are of inconvenient size, and others are therefore selected which can be obtained easily from the absolute units, and represent, by convenient numbers, the ordinary quantities to be measured.

The ampere.—One tenth of the absolute unit of current is taken as the practical unit and is called the **ampere**, after Ampère, who did a great amount of work in the study of electric and magnetic phenomena. We shall in future use I to denote a current when measured in amperes, and i when measured in absolute units. Thus, for any given current $\mathrm{I} = 10i$, since the number of amperes is ten times the number of absolute units in its measurement.

Hence the magnetic field (H) at the centre of a circular coil (p. 834) is given by

$$\mathsf{H} = \frac{2\pi n i}{r} = \frac{2\pi n \mathrm{I}}{10r}.$$

The volt.—The absolute unit of potential difference is an extremely small quantity; hence the practical unit is taken to be 100000000, or 10^8 times as great. It is called the **volt**, after Volta, the discoverer of the 'voltaic' pile.

Thus, $e = 10^8 \mathsf{E}$, where E is a given p.d. measured in volts, and e the same measured in absolute units.

The Ohm.—Having chosen two of our new units of convenient size, we are no longer at liberty to choose the others, but must derive them from those already fixed. Thus, for a conductor to have one practical unit of resistance, there must be a p.d. of 1 volt between its ends when the current in it is 1 ampere. This unit of resistance is called the **ohm.** Thus if R is the resistance of a conductor in ohms,

$$\frac{\mathsf{E}}{\mathrm{I}}=\mathsf{R}, \quad \text{or} \quad \mathsf{E}=\mathrm{I}\mathsf{R}.$$

But, $\mathsf{E}=e\times 10^{-8}$ and $\mathrm{I}=10i$;

$$\therefore\ \mathsf{R}=\frac{\mathsf{E}}{\mathrm{I}}=\frac{e\times 10^{-8}}{10i}=\frac{e}{i}\times 10^{-9}=r\times 10^{-9}.$$

Hence, $r=10^9\mathsf{R}$, and a given resistance has 10^9 as many absolute units in it as it has ohms. Thus the ohm is equal to 10^9 absolute units of resistance.

Hence there are two equations which may be used in the calculation, of current, potential difference, and resistance, namely, $\mathsf{E}/\mathrm{I}=\mathsf{R}$, when the ampere, volt, and ohm are the units employed, and $e/i=r$ when the absolute units are used. The former is by far the more common method.

The watt.—On p. 842 it was seen that the unit of p.d. is so chosen that the product of p.d. and current is the rate of working in a circuit, expressed in ergs per second. It is desirable to give a name to the rate of working in a circuit when the current is 1 ampere and the p.d. 1 volt. This is called the **watt.**

Thus,

$$\begin{aligned}\text{Rate of working} &= \mathsf{E}\times\mathrm{I}\text{ watts}\\ &= \mathrm{I}^2\times\mathsf{R}\text{ watts.}\end{aligned}$$

Now for given current and p.d., $\mathrm{I}=10i$ and $\mathsf{E}=e\times 10^{-8}$;

$$\begin{aligned}\therefore\ \text{rate of working} &= e\times 10^{-8}\times 10i\text{ watts}\\ &= ei\times 10^{-7}\text{ watts}\\ &= ei\text{ ergs per second.}\end{aligned}$$

Hence it follows that the given rate of working, namely ei ergs per second, is only $ei\times 10^{-7}$ watts, so that 1 watt $=10^7$ ergs per second.

It likewise follows that,

$$\text{Rate of working}=\mathsf{E}\times\mathrm{I}\times 10^7\text{ ergs per second.}$$

Again, since $4{\cdot}2\times 10^7$ ergs, when converted into heat, give rise to 1 calorie,

$$\begin{aligned}&\text{Rate of production of heat in conductor}\\ &\quad=\frac{\mathsf{E}\mathrm{I}\times 10^7}{4{\cdot}2\times 10^7}\\ &\quad=\mathsf{E}\mathrm{I}\times 0{\cdot}24\text{ calories per second}\\ &\quad=\mathrm{I}^2\mathsf{R}\times 0{\cdot}24\text{ calories per second.}\end{aligned}$$

It is useful to note (p. 172) that since 1 horse-power is 33,000 foot-pounds per minute, by converting foot-pounds to ergs, we find that

$$1 \text{ horse-power} = 746 \text{ watts.}$$

Hence to find the horse-power expended on any circuit we have

$$\begin{aligned}\text{Rate of working} &= \mathrm{EI} \text{ watts}\\ &= \frac{\mathrm{EI}}{746} \text{ horse-power.}\end{aligned}$$

EXAMPLE 1.—Find the heat produced in a conductor in five minutes, when a current of 3·5 amperes is flowing, the p.d. between the ends of the conductor being 20 volts.

$$\begin{aligned}\text{Rate of working} &= 3{\cdot}5 \times 20 \text{ watts}\\ &= 3{\cdot}5 \times 20 \times 0{\cdot}24 \text{ calories per second.}\\ \therefore \text{ Total heat produced} &= 3{\cdot}5 \times 20 \times 0{\cdot}24 \times 300 \text{ calories}\\ &= 5040 \text{ calories.}\end{aligned}$$

EXAMPLE 2.—Find the horse-power required to maintain a current of 3 amperes in a 220 volt incandescent lamp.

$$\begin{aligned}\text{Rate of working} &= 3 \times 220 \text{ watts}\\ &= \frac{3 \times 220}{746}\\ &= 0{\cdot}885 \text{ horse-power.}\end{aligned}$$

Units of work and energy.—The absolute unit of work being the erg, while the practical unit of rate of working is the watt, or 10^7 ergs per second, it is useful to employ a practical unit of work corresponding to the watt. This is the work done when a rate of working of 1 watt is maintained for 1 second, and is called the **joule**. Thus the watt is equivalent to 1 joule per second, while the joule itself is equal to 10^7 ergs. Hence for any conductor carrying current,

$$\text{Work done} = \mathrm{E} \times \mathrm{I} \times t \text{ joules,}$$

where t is the time for which the current flows, measured in seconds.

For the commercial supply of electrical energy, the joule is too small for convenience, and another derived unit is employed. This is the **kilowatt-hour**, and is the work done when a rate of working of 1 kilowatt, or 1000 watts, is maintained for 1 hour. The kilowatt-hour is the legal unit for the supply of electrical energy and is called the **Board of Trade unit.**

$$\begin{aligned}1 \text{ kilowatt-hour} &= 1000 \text{ watts for 1 hour}\\ &= 1000 \times 3600 \text{ joules}\\ &= 3{\cdot}6 \times 10^6 \text{ joules.}\end{aligned}$$

EXERCISES ON CHAPTER LXVI.

1. Define the units of potential difference and resistance.

2. State Ohm's law and give an account of what you mean by the resistance ' of a conductor.

3. Find the potential difference required to maintain a current of 2·5 amperes in a conductor of 23 ohms resistance, and the rate of working in the conductor.

4. State the relation of the ampere and the volt to the absolute units of current and p.d., and find the number of ergs performed in a circuit in 10 minutes when a current of 0·3 ampere flows in a conductor when the p.d. between its ends is 40 volts.

5. Find the horse-power required to maintain a 30 candle-power incandescent lamp of 1·5 watts per candle-power, and the current in the lamp when on a 200 volt circuit.

6. A wire conveying an electric current of 0·5 ampere has a resistance of 1·6 ohm per metre. If 75 cm. of this wire be placed in 650 grams of water, find the rise of temperature in 15 minutes.

(1 joule = 0·24 calorie.)

7. Find an expression for the heat developed by an electrical current in a wire.

The current through a resistance coil which is connected to the terminals of a 100 volt circuit, raises the temperature of 1 litre of water 10 degrees in one minute. What is the resistance of the coil? L.U.

8. How would you concentrate the production of heat in one part of an electric circuit carrying an electric current? Illustrate your answer by reference to an electric lamp and its leads. State definitely the laws which relate to the production of heat in the circuit. L.U.

9. How is the heat produced in a conductor related to the potential difference between the ends of the conductor and the current flowing in it?

If the heating effect in a certain resistance box endangers the constancy of the coils when the energy used in them exceeds 0·0001 watt per ohm, find the limiting safe voltage applied to the box when the resistance 1·5 ohm is being used, and also when 2500 ohms resistance is being used. L.U.

10. What is the law of heat developed by an electric current?

A glow lamp is immersed in a litre of water, and when the current is switched on the temperature of the water rises at the rate of one degree in 5 minutes. If the current through the lamp is $\frac{1}{2}$ ampere, what is the voltage of the lamp, and what is its resistance?

(42 million ergs., or 4·2 joules = 1 calorie.) L.U.

11. What are the laws relating to the development of heat in a conductor during the passage of an electric current through it?

A current of electricity of 2 amperes is passed for 1 hour through an electric lamp the resistance of which is 52 ohms. Calculate the amount of energy dissipated as heat, and the difference of potential between the terminals of the lamp. L.U.

12. The lighting of a room requires 300 candle-power and the lamps supplied have an efficiency of 1·5 watts per c.p. What is the cost of lighting the room for 24 hours if the cost of supply is 3d. per Board of Trade Unit ?

13. The lighting installation of a building consists of 120 incandescent lamps each having a resistance of 65 ohms and requiring a p.d. of 100 volts. Find the rate of working in kilowatts and in horse-power required to maintain them incandescent.

14. If a piece of wire carrying a current is immersed in 600 grams of water and the rise in temperature is 5° C. per minute, calculate the current, if the resistance of the wire is 2·5 ohms.

15. Twenty metal filament lamps, each of 32 candle power and taking 1·4 watts per candle, are installed in a house and supplied at 250 volts. What is the total current taken when the lamps are all lighted, and the cost per hour if electricity is supplied at fourpence per Board of Trade unit? C.G.

CHAPTER LXVII

ELECTRICAL CIRCUITS: ELECTROMOTIVE FORCE

Conductors in series.—Conductors are said to be arranged **in series** when one current flows through all of them, as in Fig. 781. The current enters at A and leaves at D, and is the same in all the conductors. Calling this current I amperes, the potential difference between A and B is IR_1 volts. Similarly, the p.d. between B and C is IR_2 volts, and between C and D is IR_3 volts.

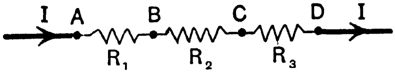

FIG. 781.—Conductors in series.

If R be the total or effective resistance between A and D, the p.d. between A and D is IR.

Hence, $$IR = IR_1 + IR_2 + IR_3,$$

or $$R = R_1 + R_2 + R_3.$$

Thus, **for conductors in series, the effective resistance is the sum of the separate resistances**, and for any number of conductors,

$$R = R_1 + R_2 + R_3 + R_4 + \ldots \qquad (1)$$

Conductors in parallel.—When several conductors are joined between two points so that the current divides between them, they are said to be **in parallel**, as in Fig. 782. The current I, entering at A, divides into three parts, I_1, I_2 and I_3 which unite again at B.

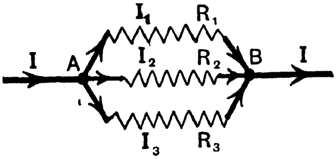

FIG. 782.—Conductors in parallel.

$$I = I_1 + I_2 + I_3.$$

Now taking the conductors separately we have,

$$\text{p.d. between A and B} = I_1R_1 = I_2R_2 = I_3R_3 = \text{say } e;$$

$$\therefore \; I_1 = \frac{e}{R_1}, \; I_2 = \frac{e}{R_2} \quad \text{and} \quad I_3 = \frac{e}{R_3}.$$

Again, if R be the equivalent resistance between A and B,

$$\text{p.d. between A and B} = IR = e;$$

$$\therefore\ I = \frac{e}{R},$$

where I is the total current. Since this is equal to the sum of the three currents I_1, I_2 and I_3,

$$\frac{e}{R} = \frac{e}{R_1} + \frac{e}{R_2} + \frac{e}{R_3},$$

or,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3},$$

Thus, **when conductors are in parallel, the reciprocal of the effective resistance is the sum of the reciprocals of the separate resistances,** and for any number of conductors,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \ldots\ldots \qquad (2)$$

To find the **current in each branch,** notice that $I_1 = \frac{e}{R_1}$, and that $I = \frac{e}{R}$.

Thus,

$$I_1 = I \cdot \frac{R}{R_1}, \qquad (3)$$

or, the **current in any branch is equal to the main current multiplied by the combined resistance and divided by the resistance in that branch.**

When there are only two conductors in parallel, the current divides into two parts, inversely as the resistances of the two branches. Thus:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2};$$

$$\therefore\ R = \frac{R_1 R_2}{R_1 + R_2}.$$

Now,

$$I_1 = I\frac{R}{R_1}, \quad \text{and} \quad I_2 = I\frac{R}{R_2};$$

$$\therefore\ I_1 = I \cdot \frac{R_2}{R_1 + R_2}, \quad \text{and} \quad I_2 = I \cdot \frac{R_1}{R_1 + R_2};$$

or

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}. \qquad (4)$$

Conductance.—The reciprocal of the resistance is sometimes called the **conductance.**

Thus,

$$\text{Resistance} = \frac{\text{p.d.}}{\text{current}},$$

$$\text{Conductance} = \frac{\text{current}}{\text{p.d.}};$$

$$\therefore \text{ Resistance} = \frac{1}{\text{conductance}},$$

or

$$R = \frac{1}{K}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where K is taken to be the conductance of the conductor. This enables us to write the relation for conductors in parallel in a simpler manner.

For

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots .$$

But

$$\frac{1}{R} = K, \quad \frac{1}{R_1} = K_1, \quad \frac{1}{R_2} = K_2, \text{ etc.};$$

$$\therefore \; K = K_1 + K_2 + K_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Or, **the effective conductance of a number of conductors in parallel is equal to the sum of the separate conductances.**

Specific resistance, or resistivity.—In dealing with conductors of different materials, it is desirable to define in some way the specific properties of each material. Two conductors of exactly the same dimensions will not have the same resistance if they are made of different materials. **The specific resistance of any material is the resistance of a conductor of this material, having unit length and unit area of cross-section.** The shape of the cross-section is immaterial, provided that it is uniform throughout the length of the conductor.

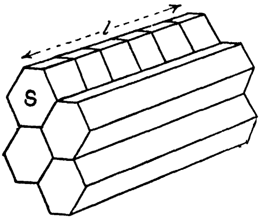

FIG. 783.—Conductor of uniform cross-section.

Let the rod S in Fig. 783 be supposed to have a length of 1 centimetre and an area of cross-section of 1 square centimetre. If its resistance be S, this is the specific resistance, or **resistivity**, of the material of the rod. The section has been drawn hexagonal to emphasise the fact that it need not necessarily be square. If l of these unit blocks be placed in series the resistance is lS (p. 848). Thus a conductor of length l and unit cross-section has resistance $l\mathrm{S}$, where S is the resistivity of the material.

To find the effect of varying the cross-section, imagine several similar rods of unit cross-section placed in parallel. If there are 4 such rods, as in Fig. 783,

$$\frac{1}{\mathrm{R}}=\frac{1}{l\mathrm{S}}+\frac{1}{l\mathrm{S}}+\frac{1}{l\mathrm{S}}+\frac{1}{l\mathrm{S}}=\frac{4}{l\mathrm{S}},$$

or, if there are a rods, a being then the area of cross-section of the whole conductor,

$$\frac{1}{\mathrm{R}}=\frac{a}{l\mathrm{S}},$$

or

$$\mathrm{R}=\frac{\mathrm{S}l}{a}. \qquad (7)$$

Thus, **the resistance of a uniform conductor is its resistivity multiplied by the length of the conductor and divided by the area of cross-section.** The method of measuring resistivity is given on p. 889.

TABLE OF RESISTIVITIES.

	Resistivity at 0° C.	Coefficient of increase of resistivity.
Aluminium - -	$3{\cdot}00\times10^{-6}$	0·00423
Copper - - -	$1{\cdot}59\times10^{-6}$	0·00428
Gold - - -	$2{\cdot}17\times10^{-6}$	0·00377
Iron - - -	$8{\cdot}85\times10^{-6}$	0·00625
Mercury - - -	$9{\cdot}407\times10^{-5}$	0·000879
Platinum - -	$1{\cdot}17\times10^{-5}$	0·003669
Silver - - -	$1{\cdot}54\times10^{-6}$	0·00400
Manganin - -	$4{\cdot}76\times10^{-5}$	0·000018
Platinoid - -	$4{\cdot}24\times10^{-5}$	0·00025
Platinum silver -	$2{\cdot}26\times10^{-5}$	0·000344

EXAMPLE 1.—Find the resistance of a copper wire of length 180 metres and diameter 0·5 mm., given that the resistivity of copper is 0·0000016.

$$\text{Area of cross-section} = \pi\times0{\cdot}025^2 \text{ sq. cm.};$$

$$\text{length} = 18000 \text{ cm.}$$

$$\therefore\ \text{resistance} = \frac{\mathrm{S}l}{a}$$

$$=\frac{0{\cdot}0000016\times18000}{\pi\times0{\cdot}025^2}$$

$$=\underline{14{\cdot}7} \text{ ohm.}$$

EXAMPLE 2.—On maintaining a p.d. of 100 volts between the two faces of a sheet of badly conducting material it is found that a current of 0·25 ampere flows through it. Find the resistivity of the material if the area of the sheet is 120 sq. cm. and its thickness 2 mm.

$$\text{Resistance of sheet} = \frac{100}{0{\cdot}25}\text{ohms.}$$

Now, $$\mathsf{R} = \frac{\mathsf{S}l}{a}, \quad \text{or} \quad \mathsf{S} = \frac{\mathsf{R}a}{l},$$

$$a = 120 \text{ sq. cm. and } l = 0{\cdot}2 \text{ cm.}$$

$$\therefore \quad \mathsf{S} = \frac{100 \times 120}{0{\cdot}25 \times 0{\cdot}2}$$

$$= \underline{240000} \text{ ohms per unit conductor.}$$

Conductivity.—The reciprocal of the resistivity is called the **conductivity** of any material. It is thus the conductance of a conductor, made of the material, whose length is 1 cm. and area of cross-section 1 sq. cm. The conductivity of various materials may thus be calculated from their resistivities given on p. 851.

Electromotive force.—Up to the present, only currents in inactive conductors have been considered. By an inactive conductor is meant one which carries the current, but does not in any way contribute to the energy required to maintain the current. In fact, energy is always dissipated in such a conductor, at a rate measured by the product of the current and the p.d. between the ends of the conductor. In a circuit made up entirely of such inactive conductors no current would flow. To produce a current some source of electrical energy in the circuit is required, in other words, an **electromotive force** is necessary.

Since a current only flows in complete circuits, and there is a drop of potential along the conductors in which there is a current, it follows that there must be a step up in the potential somewhere, for the potential cannot drop all along a complete circuit. This step up in the potential occurs at some place, or places, where energy of some other form is converted into electrical energy. It may be due to a voltaic cell or battery, to a dynamo, or to a variation of temperature at different parts of the circuit, or to certain other causes.

In all cases, however, the total rate of performing work in the conductors comprising the circuit must be equal to the rate at which energy is given to the circuit from the source of electrical energy.

When unit current is flowing in a circuit, the rate at which electrical energy is drawn from the source and dissipated in the circuit is called the Electromotive Force (e.m.f.) in the circuit. Hence, for a circuit in which the electromotive force is e and the current i,

$$\text{Rate of working} = ei \text{ ergs per second.}$$

Or, in practical units,

$$\text{Rate of working} = \mathsf{EI} \text{ watts.} \quad \dots\dots(8)$$

Thus, electromotive force is measured in the same units as potential difference, that is, the practical unit is the volt. The distinction between the two quantities may be made clearer by considering the hydraulic analogy (p. 841) to apply to a complete circuit. Let the horizontal tube (Fig. 780) be bent round and joined upon itself so that the water in it forms a complete circuit. There will then be no flow, as there is no source of energy in the circuit. Now imagine a pump, centrifugal or otherwise, introduced at some point of the circuit. The rate of supply of energy by the pump to the water in the tube in maintaining a circulation corresponding to unit current, is analogous to the electromotive force in the electrical circuit. If any two points in the circuit be considered, there is a difference of pressure between them, and this difference of pressure represents, for unit flow, the rate of dissipation of energy between these two points. This is analogous to the potential difference between two points in a conductor in the case of an electric current.

Ohm's law applied to a whole circuit.—Consider a circuit consisting of a number of conductors in series, say R_1, R_2 and R_3. Let I be the current in the circuit, and E the electromotive force at some place in the circuit, maintaining the current. Then, for the whole circuit, the rate of working is EI watts. Again, for the separate conductors, the rates of dissipation of energy are $\mathsf{I}^2\mathsf{R}_1$, $\mathsf{I}^2\mathsf{R}_2$ and $\mathsf{I}^2\mathsf{R}_3$ watts (p. 844).

Hence,
$$\mathsf{EI} = \mathsf{I}^2\mathsf{R}_1 + \mathsf{I}^2\mathsf{R}_2 + \mathsf{I}^2\mathsf{R}_3,$$

or,
$$\frac{\mathsf{E}}{\mathsf{I}} = \mathsf{R}_1 + \mathsf{R}_2 + \mathsf{R}_3. \quad \dots\dots(9)$$

Thus, for the whole circuit we might say that the ratio of electromotive force to current is constant, and this constant is identified as the total resistance of the circuit

Or, for practical purposes, we may use the equation

$$\frac{E}{R_1+R_2+R_3}=I, \qquad (10)$$

to calculate the current in any circuit when we know the electromotive force in it and the resistances of its several parts.

Diagram of circuit.—The various quantities applying to an electrical circuit may be represented conveniently upon a diagram of potential differences and resistances. Thus, let the circuit (Fig. 784 (*a*)) consist

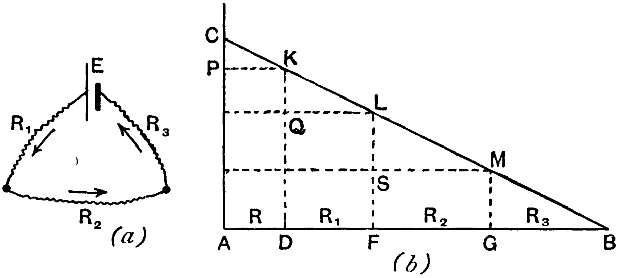

FIG. 784.—Diagram of an electrical circuit.

of a cell of electromotive force E and internal resistance R, in series with resistances R_1, R_2 and R_3. Set off to scale along AB (Fig. 784 (*b*)) the resistances in order, and the e.m.f. E along AC. Then the current in the circuit is $\frac{E}{R+R_1+R_2+R_3}$. Join the points C and B; then the current I in the circuit is represented by CA/AB.

Note also that MG represents the p.d. between the ends of the resistance R_3, for

$$\begin{aligned} MG &= R_3(CA/AB) \\ &= R_3 I. \end{aligned}$$

Similarly, LS and KQ represent to scale the p.d.'s between the ends of the resistances R_2 and R_1 respectively.

Note that CP does not represent the p.d. between the terminals of the cell. It is the p.d. corresponding to the resistance of the cell and the current; but, between the terminals, the source of electromotive force exists. The p.d. between the cell terminals is represented on the diagram by AP, or DK, because this is the p.d. for the external resistance $R_1+R_2+R_3$ and this is not complicated by the existence of any source of electromotive force in these conductors.

Potential difference between the terminals of a cell.—It was seen on p. 854 that the electromotive force of a cell is not in general the same thing as the potential difference between its terminals. Thus, in Fig. 784 (*b*), the e.m.f. is represented by AC, and the p.d. between the terminals by DK. Consider, however, the case of a cell placed in series with a very high resistance. The p.d. between the terminals of the cell being KD, this is less than the e.m.f. of the cell by the small amount CP. On very greatly increasing the external resistance, the current becomes less, but the p.d. between the terminals of the cell becomes DK′ (Fig. 785). As the external resistance approaches infinity, the line CB′ becomes horizontal, taking the position CB″. In this limiting condition the p.d. between the terminals is DK″ and is equal to AC the e.m.f. of the cell. When the external resistance is infinite, the current is zero, and the cell is said to be **on open circuit.** Thus **when the cell is on open circuit the p.d. between its terminals is equal to the e.m.f. of the cell.**

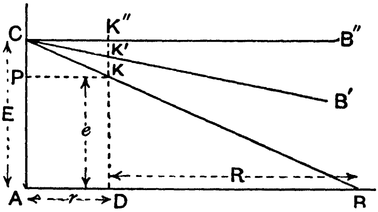

FIG. 785.—Potential difference between the terminals of a cell.

Thus, if E is the electromotive force of a cell or dynamo, and r its internal resistance, R the external resistance of the circuit, and e the potential difference between the terminals of the cell or dynamo,

$$\text{Current in circuit} = \frac{E}{r+R};$$

$$\therefore \text{ p.d. between terminals} = \frac{E}{r+R} \cdot R = e;$$

$$\therefore \ ER = er + eR,$$

$$\frac{E-e}{e} = \frac{r}{R}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The voltage E of the cell may therefore be considered to consist of two parts,—e, that required to maintain the current in the external resistance R, and $E-e$, that required to maintain the current in the cell or dynamo itself. Further, these voltages are proportional to the resistances of the corresponding parts of the circuit.

Maximum current obtainable from a cell or dynamo.—Obviously the greatest current is obtainable when the resistance external to the source of electromotive force is zero. Then $I = E/r$, where r is the internal resistance.

A Daniell's cell (p. 911) has an e.m.f. of about 1·1 volts and its internal resistance is usually of the order of half an ohm. Hence the greatest current that such a cell can produce is 1·1/0·5 = 2·2 amperes. When great currents are required, a number of these cells may be arranged in parallel. If, say, four cells be joined in parallel (Fig. 786), the combined internal resistance is only one-quarter of that of one cell, say 0·125 ohm. Hence the greatest current such a set of cells can produce is 1·1/0·125 = 8·8 amperes.

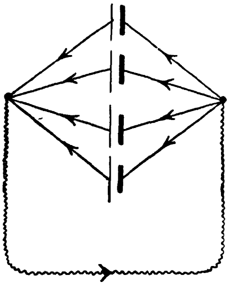

FIG. 786.—Cells in parallel.

For the production of large currents by means of cells, it is usual to employ secondary cells. These have an e.m.f. of about 2·1 volts, and a very small internal resistance. For an internal resistance of 0·01 ohm, the maximum current would be 210 amperes. Such a large current is rarely necessary, and is probably not allowable, since the cell would in all likelihood be injured by it.

In the case of dynamos, the internal resistance is usually very small, so that large currents may be obtained, and the heating in the dynamo produced by this current is small.

EXAMPLE.—The terminals of a cell of e.m.f. 1·5 volts and internal resistance 3 ohms are joined by two conductors in parallel having resistances 10 and 15 ohms respectively. Find the p.d. between the terminals, and the current in each conductor.

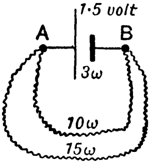

FIG. 787.

Let A and B (Fig. 787) be the terminals.

Combined external resistance, R, is given by

$$\frac{1}{R}=\frac{1}{10}+\frac{1}{15};$$

$$\therefore\ R=6 \text{ ohms};$$

$$\therefore\ \text{Current}=\frac{1\cdot5}{3+6}=\frac{1\cdot5}{9}\text{ ampere};$$

$$\therefore\ \text{p.d. between terminals}=\frac{1\cdot5}{9}\times6=\underline{1}\text{ volt.}$$

$$\text{Current in }10\,\omega\text{ conductor}=\frac{1\cdot5}{9}\times\frac{6}{10}\quad\text{(p. 849)}$$

$$=\underline{0\cdot1}\text{ ampere.}$$

$$\text{Current in }15\,\omega\text{ conductor}=\frac{1\cdot5}{9}\times\frac{6}{15}=\underline{0\cdot0667}\text{ ampere.}$$

Insulation Resistance.—The materials used for protecting wires, cables, and parts of electrical machinery from leakage of current from them, must have very high resistivity, and are called **insulators.**

Among these may be noticed guttapercha ($S = 2 \times 10^9$), mica ($S = 9 \times 10^{15}$), ebonite ($S = 2 \times 10^{15}$), paraffin wax ($S = 3 \times 10^{18}$), and quartz ($S = 1{\cdot}2 \times 10^{14}$). The values of the resistivity (S) given are only approximate.

In dealing with cables it should be noticed that any leakage of current takes place through the insulation from the conductor inside to the conducting sheath outside, so that the total or effective resistance decreases as the length of the cable increases. Thus the effective insulation resistance varies inversely as the length of the cable, the sections being in parallel.

Temperature Coefficient of Resistance.—It may be taken as a general rule that metallic conductors increase in resistivity with rise in temperature, while for electrolytes (Chap. LXX.), carbon, and most insulators, the resistivity falls with rise of temperature. For the metals the equation

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

gives the resistance R_t at $t°$ C., in terms of the resistance R_0 at 0° C. for a very wide range of temperature, where α and β are constants for each metal. The constant β is so small that for moderate ranges of temperature, say from 0° to 100° C., it may be neglected, and the equation becomes

$$R_t = R_0(1 + \alpha t).$$

The quantity α is called the **coefficient of increase of resistivity,** and its value is given for several metals in the table on p. 851.

It will be seen later (Chap. LXXVIII.), that the resistance of a carbon filament lamp is about half of the value for the lamp when cold. The resistivity of guttapercha falls very rapidly with rise of temperature, falling about 50 per cent. in value for every rise of 5° C. at ordinary temperatures. The insulation resistance of mica changes very little with temperature.

EXERCISES ON CHAPTER LXVII.

1. Explain the terms 'series' and 'parallel' in connection with electric circuits.

Show that if n similar conductors are arranged in series the combined resistance is n^2 times as great as when they are arranged in parallel.

2. Conductors of 5, 10, and 15 ohms resistance respectively are arranged in parallel. Find their effective resistance.

3. A current of 3 amperes flows through three conductors in parallell whose resistances are 1, 2, and 3 ohms respectively. Find the current in each conductor.

4. Find the resistance of the wire that must be joined in parallel with a wire of 10·5 ohms, in order that the combined resistance shall be 10 ohms.

5. Calculate the specific resistance of the material of a wire of diameter 0·5 mm. if a length of 6 metres of it has a resistance of 1·5 ohm.

6. Find the resistance of 20 metres of copper wire of diameter 0·1 mm.

7. Find the number of calories produced in 5 minutes in a copper wire of diameter 0·5 mm. and length 150 cm. in which a current of 4 amperes is flowing.

8. Explain what is meant by the internal resistance of a battery.
One battery A has an e.m.f. of 2 volts and an internal resistance of 1 ohm. Another battery B has an e.m.f. of 2½ volts and internal resistance 2 ohms. A wire is found of such resistance that the current passing is the same whether it connects the poles of one battery or the other. Find the current and the resistance. Sen. Camb. Loc.

9. How does the difference of potential between the poles of a battery change when the poles are connected by a wire ?
A battery whose resistance is 5 ohms and voltage 8 is put in series with a resistance of 6 ohms. Calculate the current strength and the potential difference at the poles of the battery.

10. State the law of production of heat in the circuit of a battery.
A battery has an internal resistance of 4 ohms and its poles are connected by two wires in parallel, of resistances 3 and 5 ohms resepctively ; compare the quantities of heat generated in a given time in the battery and in the pair of wires.

11. State Ohm's law, defining the meaning of all the electrical quantities concerned.
A battery of cells with an electromotive force of 6 volts is connected in series with a coil of wire of resistance 12 ohms. An electrostatic voltmeter connected to the battery terminals indicates only 4 volts. Explain this result, and state what information can be deduced from it. L.U.

12. Four cells each of 1·5 volts e.m.f. and 2 ohms internal resistance, are used to send a current through a single wire of 2 ohms resistance. The cells are arranged (*a*) all in series, (*b*) in two parallel groups of two in series, and (*c*) all in parallel. Calculate the current in the wire in each case. L.U.

13. A battery of 20 volts e.m.f. and 4 ohms resistance is joined in parallel with another of 20 volts and 3 ohms to send a current through an external resistance of 10 ohms.
Calculate the current through each battery. L.U.

14. Two cells of e.m.f. 1·1 volts and 1·3 volts, and internal resistances 0·4 ohm and 0·6 ohm respectively, are connected in series with a galvanometer whose resistance is 4 ohms. Calculate the current with the cells (*a*) helping, and (*b*) opposing each other.

15. Two cells, each of e.m.f. 1·5 volts and resistance 5 ohms, are joined in series with a resistance box and a resistance coil of 10 ohms. What resistance must be unplugged in the box in order that the difference of potential between the ends of the 10 ohm coil may be 0·1 volt ?
Madras University.

16. A battery of e.m.f. 3 volts and internal resistance 5 ohms drives a current through an external resistance of 30 ohms, causing a certain heating effect in the external resistance.

What alteration must we make in the external resistance in order that the heating effect in it may be double of that in the original case ?

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17. The poles of a given constant battery are joined by a wire of 1 ohm resistance, and the potential difference between them is then 1 volt. A second wire of 3 ohms resistance is now joined in parallel with the first, and the potential difference between the poles of the battery falls to 0·9 volt.

Find the electromotive force and the internal resistance of the battery.

L.U.

18. Define the ampere, the volt, and the watt.

A battery supplies current to 250 incandescent lamps in parallel, the resistance of each lamp being 300 ohms. If the potential difference between the lamp terminals is 120 volts, but rises to 122 volts when 100 lamps are switched off, calculate the internal resistance of the battery, assuming that of the leads to be negligible. Also find the watts absorbed by each lamp in the first case, assuming that the resistance of each lamp remains constant. L.U.

19. The terminals of a battery of 3 cells connected in series, each of electromotive force 1·5 volts and internal resistance 2 ohms, are connected by two conductors in parallel whose resistances are 2 and 3 ohms respectively. Find the potential difference between the terminals of the battery, and the current in each of the parallel conductors. L.U.

20. The current for 150 incandescent lamps, each having 120 ohms resistance when running in parallel on a p.d. of 100 volts is maintained by a dynamo. If 10 per cent. of the power produced by the dynamo is wasted in the leads, what must be the output of the machine in horse-power ?

21. Give the meaning of the term ‘ temperature coefficient of resistance.’ What is the effect of rise of temperature upon the resistivity of (*a*) copper, (*b*) manganin, (*c*) carbon, (*d*) mica, and (*e*) guttapercha ?

22. The insulation resistance of a cable between two stations A and C is 15000 ohms. If the insulation resistance between A and an intermediate station B is 20000 ohms, find the insulation resistance of the section between B and C.

CHAPTER LXVIII

GALVANOMETERS, AMMETERS, AND VOLTMETERS

Sensitiveness of galvanometers.—The only galvanometer considered up to the present has been the tangent galvanometer. This is used for the measurement of current, or for the comparison of currents. Modifications, however, are necessary in order to meet the variety of purposes to which current measurers are put, and these modifications are made along two distinct lines. For some purposes it is necessary to make the instrument much more sensitive than the tangent galvanometer, as when extremely small currents are to be detected or measured. The name **galvanometer** is generally confined to this type of instrument, and the deflection does not as a rule indicate any fixed scale of current; the scale of the instrument has to be **calibrated** when necessary.

When comparatively large currents are to be measured, and rapidity of reading is desirable, instruments indicating the current directly in amperes upon a fixed scale are employed. These are called **ammeters.**

Referring to the equation for the tangent galvanometer (p. 835) we saw that

$$i=\frac{\mathsf{H}r}{2\pi n}\tan\theta, \text{ in absolute units,}$$

or,

$$\mathrm{I}=\frac{10\mathsf{H}r}{2\pi n}\tan\theta, \text{ in amperes.}$$

Thus, in order to make the instrument more sensitive, that is, to give readable deflections for smaller currents, the quantity $\mathsf{H}r/n$ must be made as small as possible. This may be done by increasing n, the number of turns of wire in the coil, by diminishing r, the radius of the turns, and by diminishing H, the controlling magnetic field. Also, improved methods of measuring θ, the deflection, are employed, so that much smaller values can be measured accurately than is possible with the pointer and scale of Fig. 791.

The term **sensitiveness**, or **sensibility**, of a galvanometer is the deflection produced by 1 micro-ampere, that is, 10^{-6} ampere. This point will be dealt with later (p. 865). The current required to produce a deflection of one scale division is called the **figure of merit** of the galvanometer.

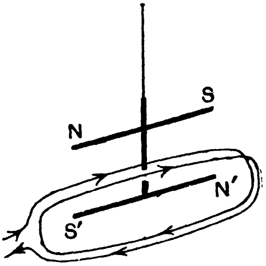


FIG. 788.—Astatic magnetic system.

Astatic couple.—In all sensitive galvanometers, the number of turns of wire in the coil is much greater than in the tangent galvanometer, and these turns are of smaller radius. There is a method of reducing the effect of the controlling magnetic field that further increases the sensitiveness. Two magnets NS and N′S′ (Fig. 788) are attached to the same vertical rigid stem, and thus form a couple which, when suspended, experiences a very small directive influence by the earth's magnetic field. If the magnets are vertically over each other and have exactly equal magnetic moments, the couples acting on them due to the earth's field are always equal and opposite, so that there is no resultant couple tending to make them turn towards the meridian. This perfect equality cannot be obtained, and, in fact, is not desirable. But when the two magnets are nearly alike, the directive effect of the earth's magnetic field is much less than when one magnet alone is used. A given current then produces greater deflection and the sensitiveness is increased. The coil may be wound so that it surrounds one of the magnets (Fig. 788), or two coils wound in opposite directions (Fig. 789) may be used. It will be seen that both coils produce deflection in the same direction, so that the sensitiveness is further increased.

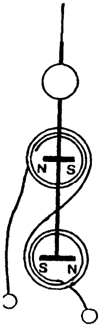


FIG. 789.—Galvanometer with astatic magnets.

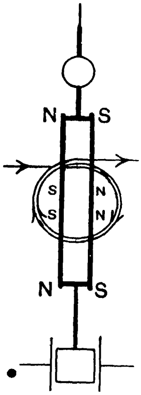


FIG. 790.—Magnetic system of the Broca galvanometer.

In some modern forms of galvanometers, as in the case of the "Broca" type, an astatic arrangement of magnets is employed, but with the magnets vertical (Fig. 790). The vertical magnets

NssN and SNNS have poles ss and NN at their middle parts. The coil is arranged with these middle poles at its centre.

In addition to rendering the galvanometer more sensitive, the astatic arrangement of magnets makes it less liable to disturbances by varying external magnetic fields. This is important, as in towns, the proximity of electric trams and railways causes considerable disturbances in the magnetic field.

Controlling magnets.—The controlling magnetic field can be varied by means of a **controlling magnet** which may be attached to the instrument as in Fig. 795, or in the case of a simple galvanometer (Fig. 791) may be placed on the bench near it.

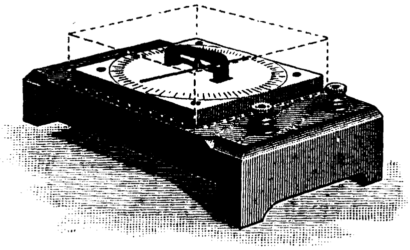


FIG. 791.—Simple galvanometer.

The controlling magnet has two uses. By means of it the controlling field may be strengthened or weakened. The former makes the galvanometer less sensitive and the latter makes it more sensitive. To strengthen the controlling field, the magnet is so placed that its field is in the same direction as the earth's field ; to weaken it, the magnet must be reversed pole for pole. The second use of the controlling magnet is to bring the suspended magnet into its zero position, so that the deflection is 0° when there is no current. This is effected by a slight rotation of the controlling magnet.

Calibration of galvanometer scale.—It must be noted that in designing a galvanometer for increased sensitiveness the tangent law has been abandoned. The current is no longer proportional to the tangent of the deflection (compare p. 835), since the coil is no longer large in comparison with the magnet. When the deflection is small, say less than 5°, the current is very nearly proportional to the deflection. If, however, larger deflections are used, the scale of the galvanometer must be specially calibrated. This is done by passing a series of known currents through the galvanometer, and noting the deflection in each case. The results are then plotted in the form of a graph, which is preserved for future use, and enables the current for any observed deflection to be found.

EXPT. 187.—**Calibration of a simple galvanometer.** Connect a simple galvanometer G, a secondary cell E, an adjustable resistance box R (p. 880), and a reversing key ABCD, in series, with a resistance of 2100 ohms in the box (Fig. 792).

Neglecting the resistances of the cell and galvanometer, we have

$$\text{Current} = \frac{2{\cdot}1}{2100}$$

$$= 0{\cdot}001 \text{ ampere}$$

$$= 1 \text{ milliampere.}$$

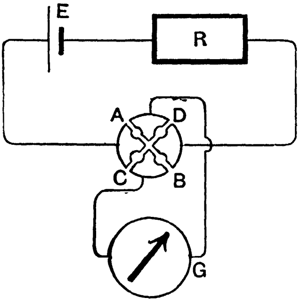


Fig. 792.—Calibration of a galvanometer.

Read both ends of the pointer of the galvanometer, when the plugs in the key are in A and B. Then remove the plugs to C and D; this reverses the current in the galvanometer. Read both ends of the pointer again. The mean of the four readings is the value of the deflection corresponding to a current of 1 milliampere.

Repeat with a resistance of 1050 ohms. The current is then 2 milliamperes. Continue, with the resistances given in the table, recording the readings in the form indicated below.

Resistance.	Current.	Readings of ends of pointer.				Mean deflection.
2100 ohms	1 milliampere					
1050 ,,	2 milliamperes					
700 ,,	3 ,,					
525 ,,	4 ,,					
420 ,,	5 ,,					
350 ,,	6 ,,					
300 ,,	7 ,,					
262 ,,	8 ,,					
233 ,,	9 ,,					
210 ,,	10 ,,					

The readings may be extended to values above 10 milliamperes, or below 1 milliampere, according to the sensitiveness of the galvanometer.

Plot the current and mean deflection on squared paper in the form of a graph.

Expt. 188.—**Use of a controlling magnet.** Connect up a circuit as in Expt. 187, and adjust the resistance until a deflection of 40° is obtained. This resistance must not then be altered. Stop the current by removing one of the plugs from the key. Place a bar magnet upon the bench at a distance of 30 cm. from the needle of the galvanometer (Fig. 793) with its N pole pointing north.

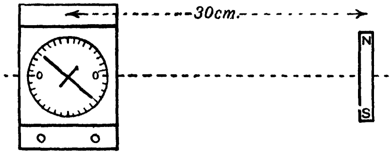


Fig. 793.—Experiment with controlling magnet.

Rotate the magnet slightly until the pointer is again at zero. Put in the plug-key and observe the deflection, taking the four readings, as in the last experiment. Then stop the current and reverse the magnet so that its S pole points north. Start the current and again observe the deflection. Repeat, with the magnet at 25 cm., then at 20 cm., and so on, continuing until, with the N pole pointing north position, the galvanometer needle swings round. For the N pole pointing south position, the readings may be continued until the magnet lies underneath the galvanometer. Readings should be recorded as in the last experiment.

Draw two graphs connecting deflection and distance of magnet. Note that the current has been the same throughout, so that for one direction of the magnet, the galvanometer becomes more sensitive as the magnet approaches; for the other direction it gets less sensitive.

Mirror or reflecting galvanometer.—For measuring deflection, the pointer (Fig. 791) is very clumsy, the error of observation being of the order of half a degree. This error is at least 1 per cent., and may be a higher percentage of the deflection. It may be greatly reduced by the following modification. A small mirror M, usually concave, of 1 metre radius of curvature, is attached to the suspended system of magnets (Fig. 789). A beam of light from the lamp L falls upon M and is reflected to the scale S. Figure 794 is partly diagrammatic, the supports of the lamp and scale not being shown. These vary a great deal in design, but the essentials are nearly constant in type. On the front lens L is a vertical scratch, and this being at a distance of 1 metre from the mirror M an image of the lens and scratch is produced upon the semi-transparent scale S, which is also at a distance of a metre from M. The lens L merely serves as a condenser for the light from the incandescent electric lamp situated within the holder.

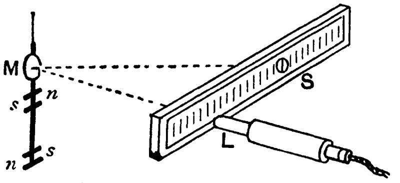


FIG. 794.—Lamp and scale.

The advantages of this optical method of measuring the deflection are fourfold: (i) the pointer, being a beam of light, is weightless, and therefore does not add to the inertia of the suspended system, or to the strength necessary in the suspending fibre; (ii) the angular deflection of the beam of light is twice that of the mirror (p. 557), thus doubling the observed deflection; (iii) the beam of light is much longer than is allowable for a mechanical pointer; and (iv) there is no parallax to avoid, the image falling upon the scale itself.

A good type of **reflecting galvanometer** is shown in Fig. 795.

In adjusting the lamp and scale for use, it is first necessary to direct the beam of light upon the mirror; then to withdraw, or push in, the lens in its sliding tube, until the spot is sharply focussed upon the scale; and lastly by means of the controlling magnets, or otherwise, to bring the spot of light to the middle of the scale. The scale itself should be symmetrically situated with respect to the galvanometer and beam of light. If it is not situated with its zero on the normal to the face of the galvanometer, or is not itself in a plane at right angles to the normal, the deflections on opposite sides of zero, for the same current will be unequal. Hence, it is necessary to pass a small current through the galvanometer and to observe the deflection; then to reverse the current and observe the deflection on the other side of zero. If these are not equal, adjust the scale until, on making the two observations, the deflections are equal.

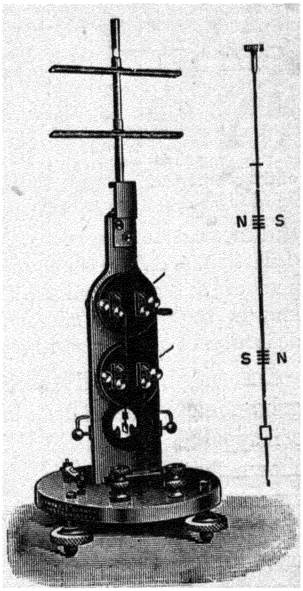


FIG. 795.—Reflecting galvanometer.

The scale is generally divided in millimetres, the convention being to express the sensitiveness of a galvanometer in terms of millimetres deflection with the scale at a distance of 1 metre from the mirror, for a current of 1 micro-ampere. Or, the **figure of merit** is the current in micro-amperes that will produce a deflection of 1 millimetre with the scale at a distance of 1 metre. It does not follow that the galvanometer with the highest sensitiveness, expressed in this way, is always the best. The time of swing of the suspended system and the resistance of the galvanometer have also to be considered; but these considerations are too complicated for a full discussion here.

A millimetre deflection, with the scale at 1 metre, corresponds to an angular deflection of 0·001 radian, or 0·0573 degree of the beam of light, or half this, that is 0·0286°, for the suspended system. Readings with a pointer cannot be determined accurately to less than 1°, and thus the accuracy of observation has been increased 1/0·0286, or roughly 30 times, by substituting the beam of light for the pointer. This is quite distinct from any accuracy gained in the design of the electrical parts of the galvanometer.

EXPT. 189.—**Sensitiveness of a reflecting galvanometer.** After adjusting the lamp and scale, as described on p. 865, connect the reflecting galvanometer G in series with a Daniell's cell E, and a high resistance R (Fig. 796), which should include coils of at least a million ohms. With plugs in A and B of the reversing key, adjust the resistance until about 10 mm. deflection is produced. Reverse the current by placing the plugs in C and D, and observe the reverse deflection. Repeat with other resistances and record in a table, for each case, the resistance, the current in micro-amperes calculated from the relation $I = 1000000E/R$, taking the e.m.f. of the Daniell's cell as 1·1 volts, also the deflections and mean deflection. Plot in the form of a graph the current and mean deflection, and from the graph obtain the figure of merit and the sensitiveness of the galvanometer. If the resistance of the galvanometer be known, the p.d. in microvolts between its terminals corresponding to deflection of one division may also be found.

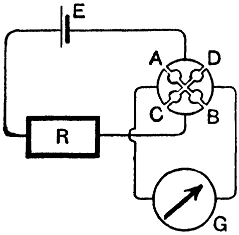


FIG. 796.—Determination of the sensitiveness of a galvanometer.

Suspended coil galvanometer.—In recent years suspended magnet galvanometers have been largely replaced by those of the suspended coil type. In Chap. LXXV. it will be shown that when a coil carrying a current is suspended in a magnetic field, it experiences a couple whose magnitude is proportional to the current. The coil CC (Fig. 797) consists of a number of turns of fine wire and is usually rectangular, but sometimes circular, in shape. It is suspended between two massive soft iron pole pieces N and S fixed to a permanent magnet which is commonly of the horseshoe type. The coil is suspended by a fine phosphor-bronze strip F, which also serves to bring in the current, which leaves by the loosely coiled phosphor-bronze strip G. When the current passes, the couple rotates the coil until the opposite couple due to the twist in the strip

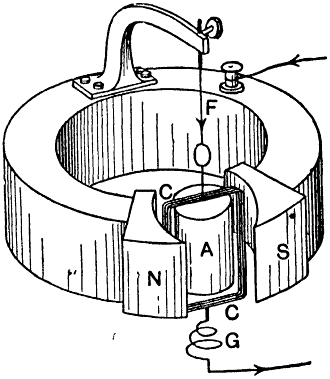


FIG. 797.—Suspended coil galvanometer.

F brings it to rest. The control in this case is therefore mechanical. and not magnetic, as in the suspended magnet galvanometer.

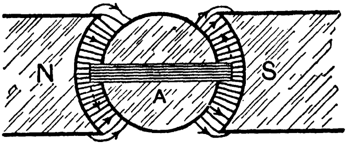


FIG. 798.—Magnetic field of the suspended coil galvanometer.

The ends of the soft iron pole pieces are hollowed out and between them is situated a soft iron cylinder A (Fig. 798). Thus the sides of the suspended coil move in the cylindrical space between the cylinder and the pole pieces. In this space the magnetic field is radial, and as the coil rotates, the sides of the coil remain situated in a magnetic field of constant strength. With this arrangement the couple, and therefore the current, is proportional to the deflection. Hence such galvanometers, if properly designed, have a **linear scale**, that is, the deflection is proportional to the current over quite a large range. Without the cylinder A the field would no longer be radial, so that when the coil rotates it would be differently situated with respect to the magnetic field. The law connecting deflection and current would then be more complex.

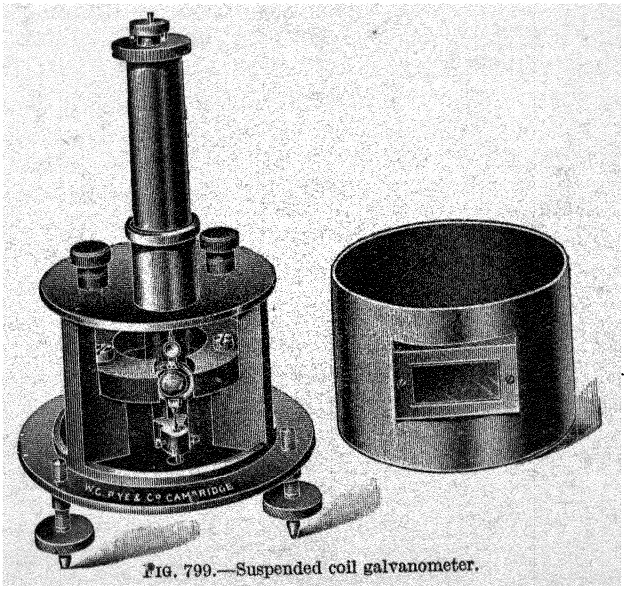


FIG. 799.—Suspended coil galvanometer.

In Fig. 799 a suspended coil galvanometer is illustrated whose form is similar to that of Fig. 797, the chief difference being that one coil is circular while the other is rectangular. Another type of suspension is shown in Fig. 800. The magnet is vertical and the coil is circular. The most important difference between this and the previous type is in the suspension, which is bifilar. The current comes down one phosphor-bronze strip, passes round the coil, and goes up the other strip.

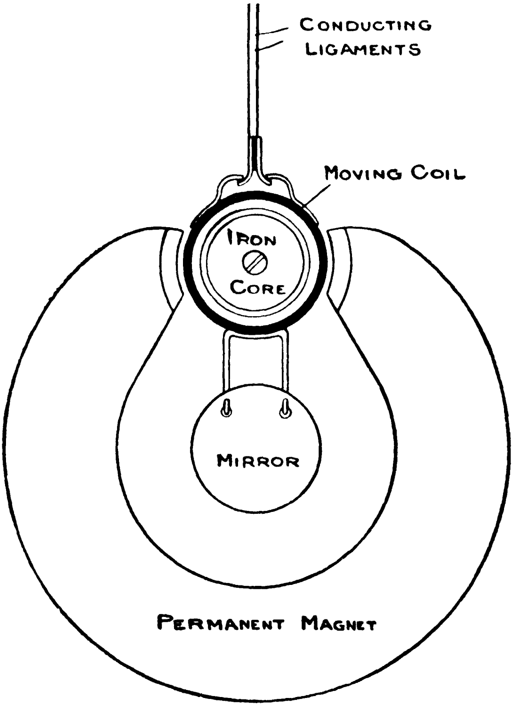


Fig. 800.—Suspended coil galvanometer with bifilar suspension.

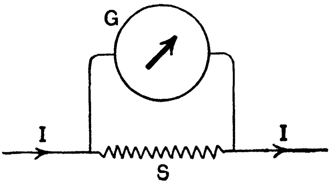


Fig. 801.—Galvanometer shunt.

Shunts.—Another method of varying the sensitiveness of a galvanometer, according to the use to which it is to be put, consists in placing a conductor in parallel with it to carry part of the current. Such a conductor is called a **shunt**. If S be the resistance of the shunt (Fig. 801) and G that of the galvanometer, the current I in the main circuit divides into two. If the current in the galvanometer is I_G, and that in the shunt I_S, then,

$$I_G = I\frac{S}{G+S}, \quad \text{and} \quad I_S = I\frac{G}{G+S}.$$

With the older types of galvanometer, shunts were supplied by the makers, frequently in boxes of three, and having resistances $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of that of the galvanometer. By placing the plug in the suitable place the corresponding shunt is used.

Thus with the resistance $S = G/99$, the current in the galvanometer is given by

$$I_G = I\frac{\frac{G}{99}}{G+\frac{G}{99}} = \frac{I}{100}.$$

One-hundredth of the main current now flows in the galvanometer and the sensitiveness is thus reduced to one-hundredth of its value when unshunted. Similarly, with the resistance G/9 its sensitiveness is one-tenth and with G/999, one-thousandth.

Universal shunt.—Owing to the inconvenience of requiring a box of shunts for each galvanometer, the **universal shunt** is now generally employed. Its advantage lies in the fact that it can be attached to any galvanometer. This instrument consists of a very high resistance, AB (Fig. 802), which is connected to the terminals of the galvanometer. AB should have at least 100 times the resistance of the galvanometer so that it does not appreciably reduce its sensitiveness.

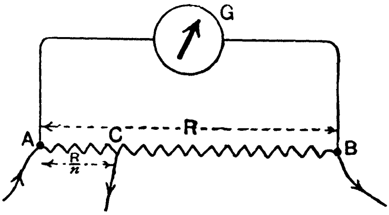


FIG. 802.—Universal shunt.

The main current enters at A, and if it leaves at B, the current in the galvanometer is $IR/(G+R)$. If now the point at which the current leaves be transferred from B to C, the resistance of AC being R/n, the two circuits in parallel between A and C have resistances of R/n and $(G+R-R/n)$ respectively. Hence the current in the galvanometer is given by

$$I_G = \frac{I\frac{R}{n}}{\left(G+R-\frac{R}{n}\right)+\frac{R}{n}} = I\frac{R}{n(G+R)}.$$

That is, by transferring the external connection from B to C the current in the galvanometer is reduced to one-*n*th of its original value.

By fixing a number of points on AB corresponding to C, having different values for *n*, the shunt may be used to give corresponding degrees of sensitiveness to the galvanometer. In Fig. 803 the connections of a common form of universal shunt are given, where the current may be $\frac{1}{1000}, \frac{1}{300}, \frac{1}{100}, \frac{1}{30}, \frac{1}{10}$, or 1, according to which of the terminal blocks the rotating arm CD is brought into contact with.

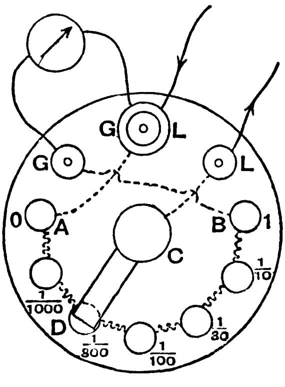


FIG. 803.—Universal shunt box.

It must be remembered that if the shunt be changed during an experiment, the effective resistance of the circuit is changed. This, however, is not usually of great importance, and may be allowed for, or compensated by extra resistances in series, if necessary.

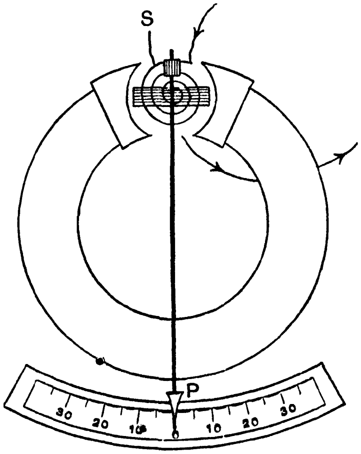


FIG. 804.—Milliammeter.

Ammeters.—The distinction between a galvanometer and an **ammeter**, or amperemeter, is that the latter is provided with a fixed scale calibrated to read current directly in amperes, or some simple fractions of an ampere. Most **milliammeters** are moving coil galvanometers with a pointer moving over a scale calibrated to read thousandths of an ampere. A typical arrangement is illustrated in Fig. 804. The permanent magnet, with its pole pieces and the soft iron cylinder, is arranged exactly as described on p. 866 for the galvanometer. The coil, however, is pivoted between needle points, and the control is exerted by the spiral spring S, which in some instruments also serves as one of the leads for bringing the current to the moving coil. The long, light, balanced pointer P is carried by the moving coil, and serves to indicate the deflection upon the scale. The scale is so graduated that the readings indicate milliamperes. In the form shown, the zero is at the middle of the scale, so that currents, in either direction, up to 35 milliamperes can be measured. An excellent form of suspension is given in Fig. 805, where the actual suspension is made by a needle point resting upon an agate cup situated at the centre of a soft iron sphere. The coil in this case is circular instead of

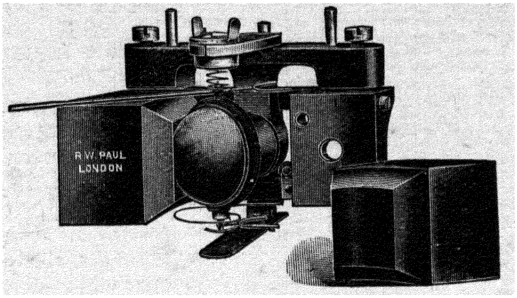


Fig. 805—Unipivot milliammeter.

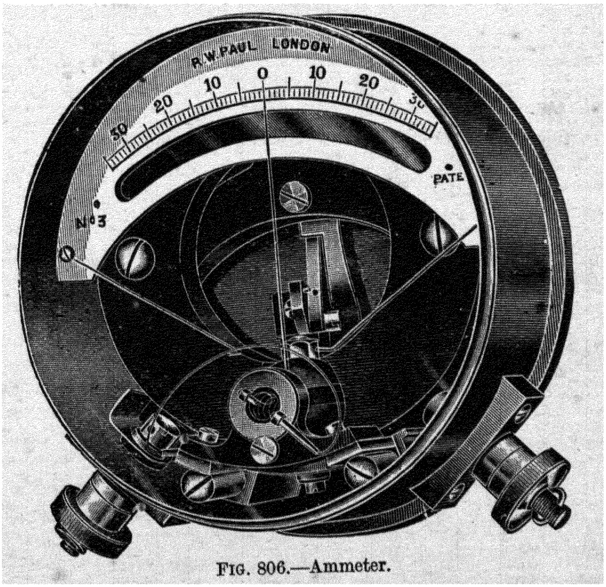


Fig. 806.—Ammeter.

rectangular. As there is only one point of suspension this is called the **unipivot** form.

The same form of instrument is employed for measuring greater currents, but the main current is carried by a shunt, the resistance of the shunt being so adjusted that the scale reads directly in amperes.

If the resistance of the milliammeter be, say, 10 ohms, and it is required that the given scale shall give amperes instead of milliamperes, the shunt must carry $\frac{999}{1000}$ of the main current while the coil carries $\frac{1}{1000}$. Hence the shunt must have $\frac{1}{999}$ of the resistance of the coil, which in this case will therefore be $\frac{10}{999}$ ohm. This method of determining the range of the instrument by means of a shunt is convenient, as it enables the galvanometer part to be constructed of some standard form, the only variation in pattern being that of the shunt, which is usually placed inside the case of the instrument. An ammeter of this type, designed to read up to 35 amperes, is shown in Fig. 806.

Hot-wire ammeter.—The galvanometer type of ammeter described above depends upon the mutual action of an electric current and a magnetic field. There are also **hot-wire ammeters** which are designed to make use of the heating effect of a current, and these may be calibrated so that the numbers on the scale give amperes direct. The main current I passes through a shunt S (Fig. 807) placed in parallel with a fine platinum-iridium wire W, through which a small fraction of the current passes. A fibre is attached at A, and after taking a turn round the axle B of the pointer, is attached to a stretched spring C. This keeps all the wires taut. On the current passing, W is heated and therefore expands. Owing to its sagging, the spring C is able to pull the fibre CBA forward and in so doing rotates the axle B, thus moving the pointer over the scale. The value of the scale deflections must first be fixed by comparison with some standard ammeter, so that they will afterwards indicate amperes. When small currents are to be measured, that is, currents below 0·3 ampere, the shunt may be dispensed with, but for higher reading instruments the shunt is necessary.

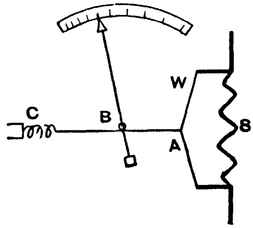


FIG. 807.—Hot-wire ammeter.

Hot-wire instruments are very liable to change of zero due to slipping of the axle and fibre, and also to change of temperature of the frame. The latter is usually compensated as well as possible by constructing the framework which supports the wire W of such material, or materials, that its coefficient of expansion shall be the same as that of the wire. Change of temperature of the entire instrument does not then tend to tighten or slacken the wire. For some purposes the hot-wire ammeter has this advantage, that owing to the heating in the wire being independent of the direction of the current, the instrument will give a reading when the current is alternating. Owing to the rapid reversals in direction, an alternating current would not produce any deflection upon the galvanometer type of ammeter. Its value, as indicated by the hot-wire ammeter, is called the **virtual current.**

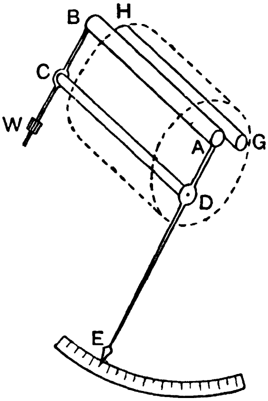


FIG. 808.—Soft iron ammeter of the repulsion type.

Soft iron ammeters.—Another type of ammeter frequently used is that in which soft iron is magnetised by the current passing in a coil or solenoid. There are two forms of **soft iron ammeters.** In one of them (Fig. 808) two parallel soft iron rods AB and GH are magnetised by the current flowing in the solenoid, to the axis of which they are parallel. To prevent confusion, this solenoid is only represented by a dotted outline in the diagram. Whatever may be the direction of the current in the solenoid, the poles at A and G are of the same kind, and there is a repulsion between them. Similarly, there is repulsion between B and H. These repulsions rotate the framework ABCD which is pivoted on jewelled points at C and D. The pointer DE then indicates the current upon the scale E, whose divisions are fixed by comparison with some standard ammeter. The control in this case is due to gravity, the small weight W being adjusted in position until the pointer is on the zero of the scale when there is no current.

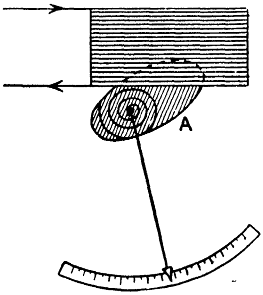


FIG. 809.—Soft iron ammeter.

Another form of the soft iron instrument is shown in Fig. 809. An oval-shaped piece of soft iron sheet A is pivoted at a point to the left of the axis of the solenoid carrying the current. When the current flows, the soft iron is magnetised in such a manner that the force between it and the coil tends to draw the iron into the coil. As A is mounted eccentrically, this force causes it to rotate. It is shown with a spring control in Fig. 809, but some forms of the instrument have a gravity control, as in Fig. 808. The coil is not circular but is flat, just sufficient clearance for the soft iron sheet A being allowed.

In the soft iron instruments, the current passes directly through the solenoid, no shunt being used. For heavy currents, a few turns of thick wire are used. For smaller currents the size of the wire is diminished and the number of turns increased. In both the forms described, the direction of the deflection is independent of the direction of the current, so that the instrument can be used to measure alternating currents.

Voltmeters.—Nearly all the types of ammeter described above may be modified for use as **voltmeters**. Thus the milliammeter (Fig. 804) may usually be calibrated so that the scale reading indicates the p.d. in millivolts between its terminals, without any alteration in the design of the instrument. In the case of instruments for measuring larger voltages considerable modification is necessary. One of the best forms of voltmeter is that in which a high resistance is placed in series with a milliammeter. Suppose that the milliammeter has a resistance of 10 ohms. Now let a resistance of 990 ohms be placed in series with it. The total resistance being 1000 ohms, 1 milliampere corresponds to a p.d. of 1 volt between the extreme terminals. Hence the scale now reads directly in volts instead of milliamperes.

Another device sometimes employed consists in using a divided resistance (Fig. 810). The high resistance CE (say 1000 ohms) is joined between the instrument terminals AB. Between points C and D, such that CD has a resistance of 1 ohm, the millivoltmeter V is placed. Then for 1 millivolt between C and D, the p.d. between A and B is 1 volt, and the readings upon V then indicate volts between A and B. Other ranges may be obtained by varying the resistances CE and CD.

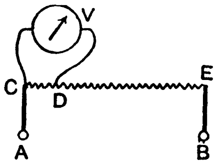


FIG. 810.—Divided resistance for a voltmeter.

The hot-wire instrument may be used as a voltmeter by placing a suitable high resistance in series with it. The soft iron instrument (p. 873) may also be employed for the same purpose by making the coil of a great many turns of very fine wire, to obtain a high resistance and at the same time sufficient sensitiveness.

One of the most satisfactory forms of voltmeter is the electrostatic voltameter described in Chap. LXXII.

Comparison of ammeters and voltmeters.—It will have been noticed already that an ammeter must have a low resistance and a voltmeter a high resistance. There are two reasons for this. In the first place the instrument must not appreciably disturb the current in the circuit to which it is applied, and in the second place there must not be an excessive heating in the instrument due to the current flowing in it. Thus, if it is required to measure the current in a lamp AB (Fig. 811), the ammeter is placed **in series** with it. If the ammeter had a high resistance, the total resistance in the lamp circuit would then be appreciably raised, and the current correspondingly diminished. Also, since the heat developed in the ammeter is proportional to I^2R, where R is its resistance, the resistance must be made as small as possible so that the heating may be negligible. For example, for an ammeter to measure 10 amperes, the resistance should not exceed 0·01 ohm. With this resistance the rate of expenditure of energy in the ammeter would be 1 watt.

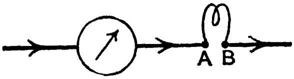


FIG. 811.—Use of an ammeter.

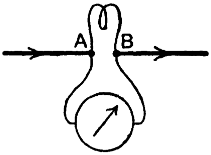


FIG. 812.—Use of a voltmeter.

On the other hand, to measure the p.d. between the terminals AB of a lamp (Fig. 812) the voltmeter must be joined **in parallel** with the lamp. If its resistance were low, a large fraction of the current would now flow through the voltmeter and the current in the lamp would be disturbed. Also the heating in the voltmeter would be considerable. Thus, if the p.d. between A and B be 100 volts, the resistance of the voltmeter should be at least 1000 ohms. This would mean a heating due to 10 watts, which is still rather large. The resistance of the voltmeter should be greater if possible. For this reason the electrostatic voltmeter (p. 949) is very advantageous, as its resistance is infinite, and hence on a continuous current circuit no current at all flows in it.

EXERCISES ON CHAPTER LXVIII.

1. Explain the principle of a shunt and calculate the length of wire of resistance 1·5 ohm per metre required to make a one-hundredth shunt for a galvanometer of 5 ohms resistance.

2. Explain the use of a shunt in connection with a galvanometer.

A galvanometer of resistance 90 ohms is shunted by a resistance of 10·3 ohms. What further resistance must be placed in parallel with the galvanometer and shunt so that $\frac{1}{10}$ of the main current shall pass through the galvanometer ? L.U.

3. A galvanometer whose resistance is G is provided with a shunt whose resistance is S. If the current in the circuit is C, prove that the current through the galvanometer will be $SC/(G+S)$.

The coil of an ammeter has a resistance of 100 ohms, and a difference of potential of 120 millivolts between the terminals gives a deflection from one end of the scale to the other. What resistances must be provided as shunts in order that the instrument may register currents (*a*) from 0 to 5 amperes, (*b*) from 0 to 100 amperes ? L.U.

4. You are provided with a sensitive galvanometer of 600 ohms resistance, and are asked to measure a current, the strength of which is known to be not more than three amperes. If the largest current that can be indicated by the unshunted galvanometer is 1/1000 ampere, what resistance would you apply as a shunt ? If the shunted galvanometer then indicates a current of 0·0009 ampere, what is the actual current in the main circuit ? L.U.

5. A galvanometer whose resistance is 300 ohms is so shunted that only 0 01 of the total current flows in it. What is the resistance of (*a*) the shunt, and (*b*) of the galvanometer and shunt combined ? Allahabad University.

6. Describe a method of measuring the resistance of a battery.

When two batteries, A and B, are joined in turn to a galvanometer it is found that A gives the greater current ; but when another galvanometer is employed B gives the greater current. Explain how this may occur. L.U.

7. Describe any form of galvanometer suitable for detecting very small electric currents, explaining particularly the way in which the sensitiveness is secured. L.U.

8. A galvanometer having a resistance of 40 ohms gives a deflection of one scale division for a current of 1/1001 ampere. Find the magnitude of the resistance required, and show how it must be connected, to change the galvanometer into : (*a*) an ammeter reading 1 ampere per scale division, (*b*) a voltmeter reading 1 volt per scale division. Bombay University.

9. A current circuit consists of a cell of e.m.f. 1·5 volts and internal resistance 1 ohm, a coil of resistance 10 ohms, and a galvanometer of resistance 50 ohms in series. What is the current through the galvanometer ? What is the current in the galvanometer when a shunt of 5 ohms is connected across its terminals ? L.U.

10. An instrument of resistance 4 5 ohms reads milliamperes. Find what resistance must be placed (*a*) in series with it in order to convert it into an instrument reading volts, and (*b*) the resistance in parallel with it in order that it shall read amperes.

11. An instrument of resistance 15 ohms, gives a reading of 25 when a current of 32 milliamperes flows in it. Find the resistance that must be placed in series with it in order to convert it into a voltmeter.

12. Describe the construction of a moving coil galvanometer and explain its action.

How can it be adapted—

(i) for use as a voltmeter ?
(ii) for measuring currents of widely varying magnitude ?

13. Describe the hot wire ammeter, and also some method for adapting it for use as a voltmeter.

14. Mention the principles on which the action of different types of voltmeters is based, and state the conditions for which each type is most suitable. C.G.

15. Describe, with sketches, some form of voltmeter suitable for a 220-volt continuous current circuit. Show how this voltmeter, and also an ammeter, would be connected to this circuit. C.G.

CHAPTER LXIX

MEASUREMENT OF ELECTROMOTIVE FORCE AND RESISTANCE

Resistance by ammeter and voltmeter.—The most direct method of measuring the resistance of a conductor is to determine the potential difference between its ends by means of a voltmeter, while the current in it is observed by means of an ammeter. This method, although not capable of great accuracy, is rapid and convenient. The only difficulty likely to arise is due to the fact that instruments of suitable range may not be available. For a conductor such as an incandescent lamp, on a 100 volt supply, the voltmeter must, of course, have a range of 100 volts and the ammeter should read up to 2 or 3 amperes. For conductors through which heavy currents must not be passed, milliammeters and millivoltmeters will be required.

EXPT. 190.—**Resistance of an incandescent lamp.** Connect the lamp in series with an ammeter A (Fig. 813), including a rheostat or adjustable resistance R in the circuit. Connect a voltmeter V across the lamp terminals. First with the resistance of R zero, read the current and p.d. Then increase R, and again read the current and p.d. Continue the process until the lamp no longer emits light. Tabulate the results, making four columns, one each for the current, the p.d., the power in watts and the resistance in ohms, remembering that watts = p.d. × current, and ohms = p.d./current. Plot a graph showing the relation between watts and ohms.

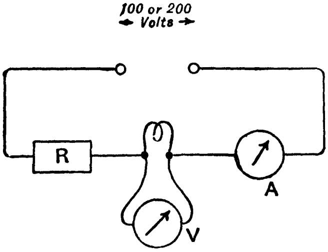


FIG. 813.—Measurement of resistance.

Repeat the experiment with another lamp. If the first one was a carbon filament lamp, now use a metal filament lamp, and *vice versa.*

Expt. 191.—Resistance of a coil. Join the coil in series with a Daniell's cell and a milliammeter. Connect a millivoltmeter to the ends of the coil as in Fig. 813. Using the readings of the instruments, calculate the resistance of the coil from the relation $R = E/I$. Note that if the ranges of the instruments are unsuitable, an extra resistance may have to be included in series with the coil.

Standard resistances.—For the purposes of electrical measurement it is necessary that the instruments used should all be compared with some fixed standards. The determination of these fixed standards in accordance with the definitions in Chapter LXVI is not an easy matter, and its description is beyond the scope of this book. From experiments it has been found that the ohm is the resistance of a column of mercury of length 106·300 cm., of uniform cross-section and having a mass of 14·4521 grams, when its temperature is 0° C. This corresponds to a cross-section of 1 square millimetre; but since the mass of the mercury is much more easily measured with accuracy than the bore of the tube in which it is contained, the mass rather than the cross-section is defined. This definition of the ohm has been rendered legal in this country.

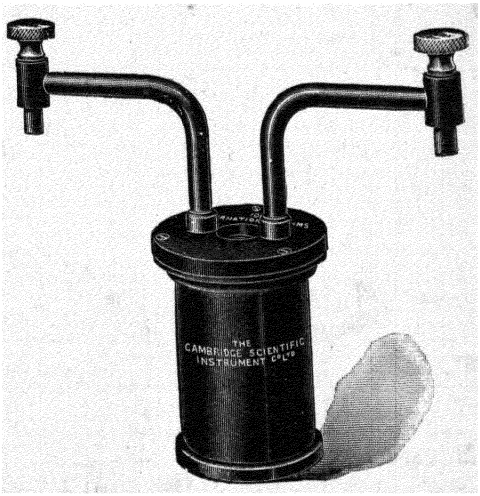


Fig. 814.—Standard ohm coil.

For practical use, several makers supply 1 ohm standards which have been standardised at the National Physical Laboratory. One form of standard ohm is shown in Fig. 814. The wire comprising the conductor is of platinum-silver and is inclosed in a brass case. The ends of the wire are soldered to stout brass terminals which are well amalgamated at the ends; when in use they rest in mercury cups. This makes certain that the resistance of the contacts is very small. A hole running down the axis is provided, for the insertion of a thermometer. The whole is then inserted in an oil bath to ensure constancy of temperature. The certificate issued with the coil states the temperature at which it is correct.

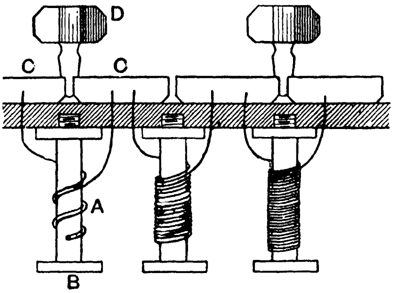


FIG. 815.—Arrangement of the coils in a resistance box.

Resistance boxes.—It would be very inconvenient in experimental work if the resistances employed had to be made up of coils of the form shown in Fig. 814. They are therefore joined together in boxes. Each coil is wound upon a bobbin B (Fig. 815) attached to an ebonite sheet. The wire is first doubled and then wound upon the bobbin, the free ends being soldered, one to each of the brass blocks. Between each pair of brass blocks a hole, slightly conical, is bored, into which fits a brass plug attached to an ebonite handle D. When all the plugs are out, the coils are all in series and the total resistance in the box is the sum of the separate resistances of the coils. By inserting any plug, the corresponding coil is short circuited, so that when all the plugs are in, the total resistance is practically zero. By making the coils to have respectively 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000 and 4000 ohms, any resistance from 0 to 11110 may be used. In some cases fractions of an ohm are also supplied. In Fig. 816 is shown a view of such a resistance box. There are many forms of resistance box.

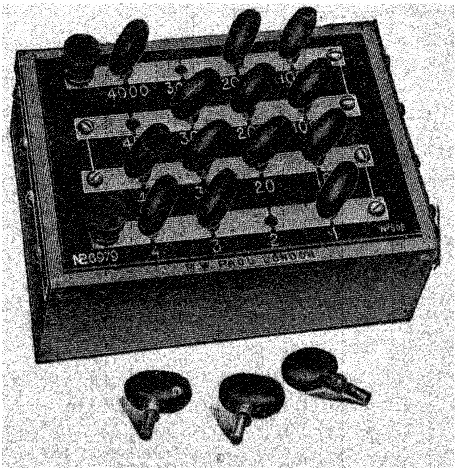


FIG. 816.—Resistance box.

Comparison of electromotive forces.—The simplest method of comparing the e.m.f.'s of two cells is to find the current which each would produce in a given circuit. On placing one of the cells, of e.m.f. E_1, in series with a resistance box R and a galvanometer G (Fig. 817) the current is $E_1/(G+R)$, and the deflection observed may be written θ_1. On replacing the cell E_1 by another cell whose e.m.f. is E_2, the current is $E_2/(G+R)$ and the deflection is θ_2. If the deflections are proportional to the currents, $E_1/E_2 = \theta_1/\theta_2$, or if a tangent galvanometer be employed, $E_1/E_2 = \tan\theta_1/\tan\theta_2$. Or, again, if a simple galvanometer, which has previously been calibrated, is used, the currents may be taken from the calibration curve (p. 863) and then $E_1/E_2 = I_1/I_2$.

Another method is to obtain the same deflection with each cell in turn, by varying the resistance R. Then, since the current in each case is the same, $E_1/(G+R_1) = E_2/(G+R_2)$. When R_1 and R_2 are great in comparison with the resistance G of the galvanometer,

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}.$$

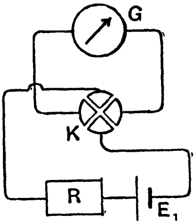


FIG. 817.—Comparison of e.m.f.'s.

EXPT. 192.—**Comparison of e.m.f.'s (constant resistance).** Connect the two cells to be compared, in turn, in series with a galvanometer and resistance box, using a reversing key K (Fig. 817), so that the current in the galvanometer may be reversed. Adjust R, when the cell is a Daniell's cell, until the deflection is a reasonable amount, and read the galvanometer deflection as in Expt. 187. Replace the Daniell's cell by another cell, say a Leclanché cell, and again read the deflection. If the galvanometer is of the tangent form, calculate the ratio of the e.m.f.'s from the relation $E_1/E_2 = \tan\theta_1/\tan\theta_2$. If it is a calibrated simple galvanometer, find the currents I_1 and I_2 corresponding to the deflections θ_1 and θ_2 (p. 863) and use the relation $E_1/E_2 = I_1/I_2$. Taking the e.m.f. of the Daniell's cell as 1·1 volt, find the e.m.f. of the other cell.

EXPT. 193.—**Comparison of e.m.f.'s (constant deflection).** Connect up the circuit as in Expt. 192, noting the deflection and recording the resistance R_1. Replace the cell by the other with which it is to be compared, and adjust the resistance until the same deflection as before is obtained. Calling this resistance R_2, we have

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}.$$

Sum and difference method.—In both the above methods, the resistances of the cells and of the galvanometer have been assumed to be negligible. A method which is independent of these resistances consists in joining the two cells in series with the galvanometer and resistance box, (i) both acting in the same direction, and (ii) acting in opposition. The resistance in the circuit is then the same in both cases, so that

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{I_1}{I_2},$$

or

$$\frac{E_1}{E_2} = \frac{I_1 + I_2}{I_1 - I_2}.$$

With the tangent galvanometer this equation becomes

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}.$$

EXPT. 194.—**Comparison of e.m.f.'s (sum and difference).** Connect two cells, say a Daniell's and a dry cell, in series with a tangent galvanometer and a resistance box, employing a reversing key as in Fig. 817. Find the mean deflection, reading both ends of the pointer with the current first in one direction and then in the other. Let the mean deflection be θ_1. Reverse the Daniell's cell, and repeat the experiment, obtaining the deflection θ_2. Then,

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}.$$

Taking the e.m.f. of the Daniell's cell as 1·1 volt, find that of the dry cell.

Comparison of resistances.—A method closely resembling that of Expt. 192 may be used for comparing resistances, provided that these are not small. The circuit is made up as in Fig. 817 but with the unknown resistance R_1 in series, R being at first zero. The deflection is noted, and the unknown resistance is removed. The coils in the box R are then unplugged until the deflection, and therefore the current, is the same as before. The e.m.f. being constant, the total resistance is therefore the same as before; hence the unknown resistance removed from the circuit must be equal to the resistance due to the standard box.

For the measurement of very high resistances, of the order of several million ohms, another substitution method is frequently employed. The resistance is placed in series with a sensitive reflecting galvanometer and a cell, and the deflection observed. The unknown resistance R_1 is then replaced by a standard resistance of 100000, or 1000000 ohms, R_2, and the deflection again observed. Since the deflections are proportional to the current, and the current is inversely proportional to the resistance,

$$\frac{R_1}{R_2}=\frac{\theta_2}{\theta_1}.$$

With these high resistances, it is clear that the resistances of the cell and galvanometer may certainly be ignored.

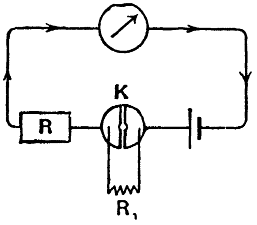


FIG. 818.—Measurement of resistance.

EXPT. 195.—**Measurement of resistance (simple substitution).** Connect the resistance box R, the galvanometer and cell in series (Fig. 818). Join the unknown resistance R_1, which may be an incandescent lamp (cold), or a resistance coil, to the terminals of the plug key K. Observe the deflection when all the plugs are in the resistance box R so that its resistance is zero. If the deflection is too great, it may be reduced by placing a piece of platinoid wire across the galvanometer terminals as shunt (p. 868) or by placing a bar magnet as a controlling magnet near the galvanometer (Expt. 188, p. 863). Now place the plug in the key K, to short circuit the resistance R_1. Remove plugs from the box R until the original deflection is restored. The resistance of the box is now equal to the resistance of R_1.

EXPT. 196.—**To make a high resistance.** Obtain a piece of ground glass about 2 cm. × 10 cm. Make a thick black coating of graphite over a considerable area at each end by rubbing with a lead pencil, and then draw a firm line AB (Fig. 819) with the pencil, to connect the blackened areas. Upon each blackened area lay a piece of tinfoil, and upon this a piece of fine copper wire C bent into a flat spiral, to serve as a conductor to bring the current to the conducting line AB. Place a piece of plain glass upon the ground glass and clamp the two together. After testing to see whether the carbon line has a suitable resistance (Expt. 197) the edges may be cemented with sealing wax to make the arrangement permanent.

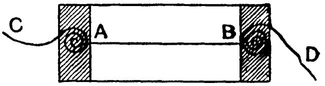


FIG. 819.—High resistance.

EXPT. 197.—**High resistance (simple substitution).** Make a circuit of the high resistance R made in Expt. 196, a cell E_1, and a reflecting galvanometer G, employing the reversing key K (Fig. 817). Observe the deflection, reverse the current, and again observe the deflection; take θ_1 to be the mean of the two. Replace R by a standard megohm (10^6 ohms) and repeat, taking the mean deflection to be θ_2.

Then,
$$\frac{R}{10^6}=\frac{\theta_2}{\theta_1}.$$

or
$$R=\frac{\theta_2}{\theta_1}\times 10^6 \text{ ohms}.$$

Resistance of galvanometer.—The most satisfactory method of measuring the resistance of a galvanometer is to clamp the moving part, and treat the instrument as an ordinary conductor, measuring its resistance by the method on p. 888 or that on p. 889. There are, however, several methods of finding the resistance, in which no other galvanometer is used.

With an instrument of resistance 5 to 20 ohms, a simple circuit is made of the galvanometer, a resistance box and a Daniell's cell (Fig. 817). The deflection θ_1 being observed, with 10 ohms resistance in the box, this is now changed to 20 ohms and θ_2 observed. Then, since the e.m.f. in the circuit is the same in both cases,

$$\frac{G+10}{G+20}=\frac{\theta_2}{\theta_1},$$

from which G can be found. The resistance of the cell has been neglected ; but in the case of a Daniell's cell, if this is not known accurately, it may be taken as about 1 ohm. The value of G obtained will, of course, include the battery resistance. It will be seen that this method is not of great precision.

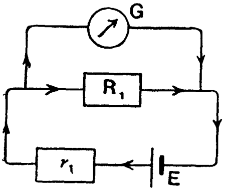


FIG. 820.—Measurement of galvanometer resistance.

When the resistance of the galvanometer is over 20 ohms the following method may be used. A high resistance r_1 (Fig. 820) is placed in series with the cell and a moderate resistance R_1 in parallel with the galvanometer. The current in the battery circuit is then

$$I_B = \frac{E}{r_1 + \frac{R_1 G}{R_1 + G}},$$

and that in the galvanometer is obtained by multiplying this by $\frac{R_1}{R_1 + G}$; it is therefore

$$I_G = \frac{E}{r_1 + \frac{R_1 G}{R_1 + G}} \cdot \frac{R_1}{R_1 + G}.$$

The two resistances are then changed to r_2 and R_2, which give the same current in the galvanometer and hence the same deflection.

Then, $$I_G = \frac{E}{r_1 + \frac{R_1 G}{R_1 + G}} \cdot \frac{R_1}{R_1 + G} = \frac{E}{r_2 + \frac{R_2 G}{R_2 + G}} \cdot \frac{R_2}{R_2 + G};$$

$$\therefore \frac{R_1}{(R_1 + G) r_1 + R_1 G} = \frac{R_2}{(R_2 + G) r_2 + R_2 G},$$

from which $$G = \frac{R_1 R_2 (r_1 - r_2)}{R_1 r_2 - R_2 r_1}.$$

Since the resistance of the cell is always small in comparison with r_1 and r_2 it may be neglected without introducing any sensible error.

EXPT. 198.—**Resistance of a galvanometer (simple deflection).** Connect the galvanometer, cell and resistance box as in Fig. 817. Find the mean deflection with resistance 10 ohms in the box, and again with 20 ohms. Then, as on p. 883, $(G + 10)/(G + 20) = \theta_2/\theta_1$ if the galvanometer is direct reading and $(G + 10)/(G + 20) = \tan\theta_2/\tan\theta_1$ if it is a tangent galvanometer, otherwise the values of I_1 and I_2 must be obtained from a calibration curve (p. 863).

EXPT. 199.—**Resistance of galvanometer (shunt method).** Connect two resistance boxes, a cell and a galvanometer, as in Fig. 820. Adjust the resistances to give a reasonable deflection. Change them to two other values which give the same deflection, and calculate the galvanometer resistance G, from the equation,

$$G=\frac{R_1R_2(r_1-r_2)}{R_1r_2-R_2r_1}.$$

Repeat with another value of the deflection, and take the mean value of G.

Wheatstone's bridge.—By far the most accurate and convenient method of comparing resistances is that due to Wheatstone. This method makes use of the principle of the divided circuit, which in this case is known as **Wheatstone's bridge.** Consider the two circuits in parallel ACB and ADB (Fig. 821). The current I entering at A divides into two parts, whose values are inversely as the resistances of the two branches. Hence the current in ACB is

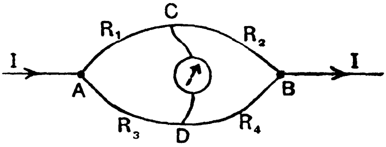


FIG. 821.—Wheatstone's bridge.

$$I_C=I\frac{R_3+R_4}{R_1+R_2+R_3+R_4},$$

and that in ADB is $$I_D=I\frac{R_1+R_2}{R_1+R_2+R_3+R_4}.$$

Thus the p.d. between A and C is

$$E_{AC}=R_1I\frac{R_3+R_4}{R_1+R_2+R_3+R_4},$$

and that between A and D is

$$E_{AD}=R_3I\frac{R_1+R_2}{R_1+R_2+R_3+R_4}.$$

If E_{AC} and E_{AD} are equal, there is the same p.d. between A and C as between A and D, and hence there is no p.d. between C and D. If C and D are then connected by a conductor there will be no current in this conductor. The condition for this is, that

$$R_1I\frac{R_3+R_4}{R_1+R_2+R_3+R_4}=R_3I\frac{R_1+R_2}{R_1+R_2+R_3+R_4},$$

or $$R_1(R_3+R_4)=R_3(R_1+R_2),$$

$$R_1R_3+R_1R_4=R_1R_3+R_2R_3;$$

$$\therefore\ R_1R_4=R_2R_3,$$

or $$\frac{R_1}{R_2}=\frac{R_3}{R_4}.$$

Thus the four resistances form a proportion when there is no current between C and D, although a current is flowing through the system from A to B. This condition is found experimentally by connecting a galvanometer between C and D. With a sensitive galvanometer, very small currents, and therefore minute potential differences, between C and D may be detected. For this reason very accurate balances between the resistances may be obtained. Since the deflection is zero for a perfect balance, this is called a **zero or null method.**

It should be noticed that if the current enters at C and leaves at D, the galvanometer being connected to A and B, the same condition will hold for zero deflection, that is,

$$\frac{R_1}{R_2}=\frac{R_3}{R_4}.$$

Expt. 200.—**Wheatstone's bridge.** Screw four double terminals A, B, C and D into a board (Fig. 822). Connect C and D to a simple galvanometer, and A and B to a cell.

(1) Connect the ends of a piece of platinoid wire 1 metre long to A and B and clamp its middle point in the terminal C. Connect a similar piece of platinoid wire to A and B, and find by trial the point of it which, when touched to the terminal D, produces no deflection upon the galvanometer. Measure the lengths of the parts AD and DB. Show that the ratio of lengths AC/CB is equal to the ratio AD/DB.

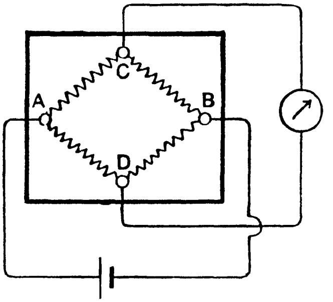


Fig. 822.—Experiment for demonstrating the Wheatstone's bridge relation.

(2) Make AC = 20 cm. and CB = 80 cm. and repeat. Show that the ratios are again equal.

(3) Make AC = 30 cm. and CB = 70 cm. and repeat.

(4) Replace ADB by a piece of fine copper wire and repeat, showing that the ratios must be equal whatever the actual resistances may be.

(5) Make AC = 60 cm. and CB = 30 cm. of platinoid wire. In AD place an unknown resistance coil of 2 or 3 ohms. In BD place a piece of platinoid wire and adjust its length by trial until the galvanometer deflection is zero, so that a balance is obtained. Knowing the ratio of CB to AC, and therefore the ratio of the resistances DB and AD, find the length of platinoid wire which has the same resistance as the coil in AD.

The metre bridge.—For the purposes of practical measurements of resistance, the Wheatstone's bridge is constructed in several convenient forms. One of the commonest forms is the **metre bridge**, so called because a wire AB one metre long (Fig. 823) is stretched between two fixed copper strips, the wire comprising two arms of the Wheatstone's bridge. The parts AC, DE and FB are stout copper strips, which have negligible resistance. If two conductors are placed in the gaps at D and E, a cell connected to C and F, and a galvanometer to L and to a movable point M on the wire, it will be seen that the four resistances R_1, R_2, R_3, and R_4 comprise a Wheatstone's bridge. The point of contact M on the wire is found, which gives zero galvanometer deflection. Then, $R_1/R_2 = R_3/R_4$. But if the wire is uniform, $R_3/R_4 = L_1/L_2$, where L_1 and L_2 are the corresponding distances of M from A and B. Hence for a balance, $R_1/R_2 = L_1/L_2$. L_1 and L_2 are measured by means of a scale fixed alongside of the wire, and hence the ratio of R_1 to R_2 becomes known. If R_1 is a standard coil, the resistance of R_2 in ohms can be calculated.

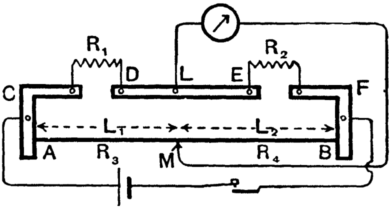


FIG. 823.—Diagram of the metre bridge.

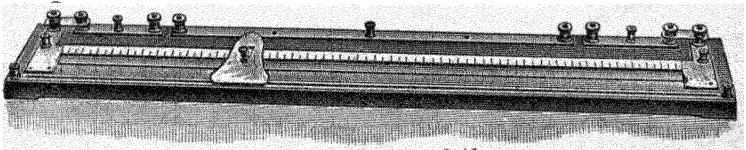


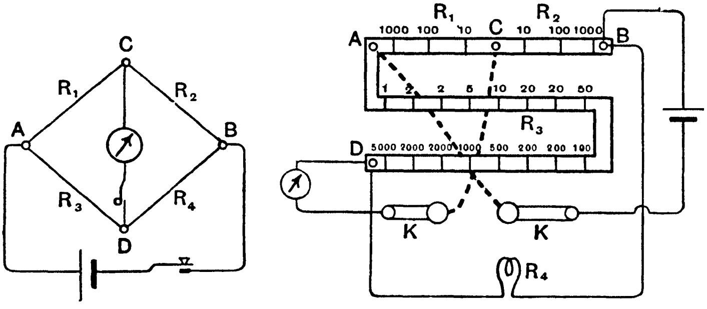
FIG. 824.—The metre bridge.

It is usual to employ a tapping key in the cell circuit so that the current only flows while the test is being made. The contact at M is usually a tapping contact, and **the practice should always be followed of making the battery circuit before the galvanometer circuit.** The reason for this will be understood later (Chap. LXXVI.). Also the cell and galvanometer may be interchanged (p. 886). For some reasons it is desirable to connect the cell to the wire contact M, and the galvanometer to the fixed points C and F, as will be explained in Chap. LXXIX.

The actual form the metre bridge takes in practice is shown in Fig. 824. The sliding contact and heavy terminals should be noted.

EXPT. 201.—**Resistance by metre bridge.** Place a resistance box in the gap D of the metre bridge (Fig. 823), and in E put a coil of unknown resistance. Having connected up the cell and galvanometer as shown, unplug the 10 ohm coil of the box and move the slider M until, on making contact, there is no galvanometer deflection. Observe the lengths L_1 and L_2, and calculate the resistance in E from the relation, Resistance $= 10\ L_2/L_1$. If this resistance is less than 10 ohms, repeat the experiment, using respectively 5 ohms, 2 ohms and 1 ohm in the box at D. If the resistance is more than 10 ohms, repeat, using 20, 50 and 100 ohms at D. In each case calculate the resistance at E and notice that the most accurate result is obtained when the balance is such that the point M is near the middle of the bridge wire.

The post-office box.—Any three resistance boxes may be used as the three arms, R_1, R_2 and R_3 of the Wheatstone's net (Fig. 825 (*a*))


C
R1
R2
A
B
R3
R4
D
R1
R2
1000 100 10 C 10 100 1000
A
B
1 2 2 5 10 20 20 50
R3
D 5000 2000 2000 1000 500 200 200 100
K
K
R4

(*a*) (*b*)

FIG. 825.—The post-office box

where R_4 is the unknown resistance to be determined. Such sets of resistances are frequently made all in one box; one such arrangement is shown diagrammatically in Fig. 825 (*b*). By comparison of (*a*) and (*b*) it will be seen that R_1 or AC consists of three coils of respectively 10, 100 and 1000 ohms resistance. Similarly, CB consists of three such coils. These are known as the **ratio arms**, and it will be noticed that ratios from $\frac{1000}{10}$ to $\frac{10}{1000}$ can be obtained. The arm AD is an ordinary set of resistances which, by unplugging, can be adjusted to have any resistance from 1 ohm to 11110 ohms. The fourth arm DB is the unknown resistance, which is therefore connected to D and B. The two tapping keys K are also included in the box, one being in the cell circuit and the other in that of the galvanometer. The connections of these keys vary with different patterns, and must be made out by the student for each box used. The whole set is known as the **post-office box**. It will be seen that for any given ratio of R_1 to R_2, the resistance R_3 may be varied until a balance is obtained, when the unknown resistance R_4 can be calculated. When R_4 is small it is advisable to make

$$R_1/R_2 = 1000/10 = 100/1,$$

and when R_4 is large to make

$$R_1/R_2 = 10/1000 = 1/100,$$

so that full use of the adjustable arm R_3 may always be made. In this way resistances ranging from 0·1 ohm to 1000000 ohms may be measured to within an error of 1 per cent., and intermediate resistances to 0·1 per cent.

EXPT. 202.—**Resistance by post-office box.** Connect up the post-office box, as in Fig. 825, using an unknown resistance coil as R_4. Beginning with 100 ohms in each ratio arm, find the resistance R_3 which produces a balance, and calculate R_4. If this is very small, or very large, alter the ratio arms suitably and redetermine R_4. In all cases the bridge is most sensitive when all four arms are of the same order of resistance. Measure the resistances of several coils and of an incandescent lamp.

EXPT. 203.—**Specific resistance.** Set up the post-office box, as in Fig. 825, using a piece of platinoid wire, at least a metre long, as resistance R_4. Measure its resistance as in Expt. 202. With a metre scale find the length of the wire between the terminals, and with a micrometer gauge determine its diameter in several places and take the mean. Calculate the area of cross-section a of the wire, and, calling its length l, and resistance R, find the specific resistance of the platinoid from the relation

$$S = \frac{Ra}{l} \quad \text{(p. 851).}$$

Repeat the experiment, using a piece of fine copper wire, and find the specific resistance of the copper.

EXPT. 204.—**Temperature coefficient of resistance.** Take leads P and Q (Fig. 826) from a coil of fine copper wire to the terminals B and D of the post-office box (Fig. 825). The fine copper wire is immersed in paraffin oil contained in a test-tube. The test-tube is surrounded by water contained in a beaker, so that its temperature can be raised gradually. A thermometer, passing through the stopper, enables the temperature of the oil, and therefore of the copper wire, to be observed. Starting at the temperature of the room, measure the resistance of the coil. Then raise the temperature about 5° C., and after making sure that this has become steady, measure the resistance again. Repeat this at intervals of about 5°, up to 100° C. Record the results in the form of a table, and draw a graph connecting temperature and resistance. The points should be very nearly on a straight line. Draw a straight line with a ruler to lie evenly amongst them, and produce it backwards so that the resistance of the wire at 0° C. can be found. Calling this R_0, and R_{100} the resistance at 100° C., calculate the coefficient of increase of resistance α from the relation $R_{100} = R_0 (1 + \alpha t)$. It is from methods such as this that the temperature coefficients of the metals given on p. 851 have been found.

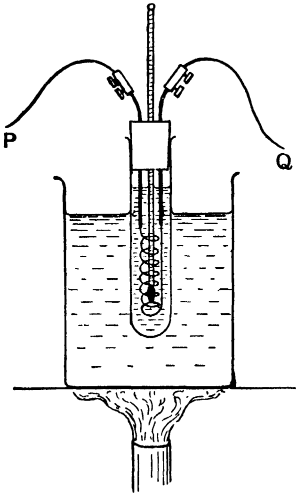


FIG. 826—Measurement of temperature coefficient of resistance.

The potentiometer.—Provided that a wire is uniform and that a steady current flows in it, there is a uniform **fall of potential along it; that is, the potential difference between any two points on it is proportional to the length of wire between those two points.** This is made use of in the potentiometer for the comparison of electromotive forces and potential differences. A current produced by a cell C, preferably a secondary cell (p. 916) flows in a uniform wire AB (Fig. 827). A cell E has one terminal connected to A, and the other through a galvanometer to a movable contact D. When the cell is on open circuit, the p.d. between its terminals is equal to its e.m.f. (p. 855). For some point D upon the wire, the p.d. between A and D is equal to that between the cell terminals on open circuit. If the terminal be then connected to the wire at D, nothing will happen in the cell circuit, as the p.d. between A and D due to the current in AB is exactly the same as that due to the cell E. No change is then made by making contact at D, and the galvanometer will not be affected. If the correct point

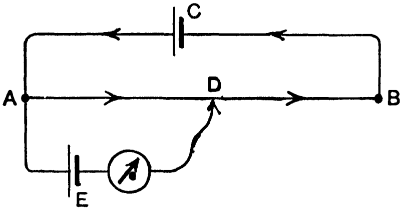


FIG. 827.—Diagram of the potentiometer.

be not found, a current will flow in one direction or the other through E and the galvanometer on making contact. Hence for a correct balance, the e.m.f. of E is equal to the p.d. between A and D and this is proportional to the length of wire AD, which equals, say, l_1. Let E_1 be the e.m.f. of E. Replace this cell by another of e.m.f. E_2, and find the length of wire l_2 for a balance.

Then, $$\frac{E_1}{E_2}=\frac{l_1}{l_2}.$$

Thus the two e.m.f.'s are compared.

This method is excellent for several reasons. It is a null method (p. 886), and therefore does not depend upon measuring a galvanometer deflection. The length of the wire AB may be several metres, and since the adjustment is easily made to within 1 millimetre, considerable accuracy is obtainable; also the cell E is not producing current when the adjustment is correct, and its internal resistance is thus immaterial.

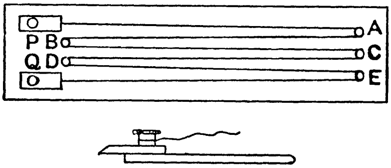


FIG. 828.—The potentiometer.

A convenient and simple form of potentiometer may be constructed by pasting a piece of paper square-ruled in centimetres and millimetres upon a board. After placing nails or screws at the points ABCD and E (Fig. 828), and terminals soldered to copper strips at P and Q, a piece of platinoid wire is passed round the screws and soldered to the copper strips at P and Q. The millimetre squares serve to measure lengths of wire from P or Q. A convenient form of contact maker is also shown. It is a wooden handle having a brass strip, filed to a sharp edge, screwed to it, a terminal being previously attached.

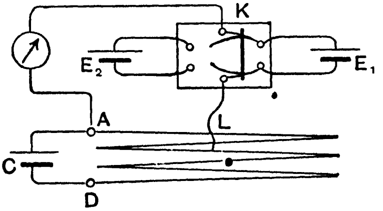


FIG. 829.—Comparison of e.m.f.'s by potentiometer.

EXPT. 205.—**Comparison of e.m.f.'s by potentiometer.** Connect a secondary cell C (Fig. 829) in series with the potentiometer wire ALD. If E_1 and E_2 are the cells whose e.m.f.'s are to be compared, join one to each side of the rocking key K, and join the galvanometer and terminal A of the potentiometer to one of the middle terminals of K, the movable contact wire L being joined to the other. With the rocker of K on one side, find the length l_1 of potentiometer wire, as measured from A which gives a balance. Rock the key to the other side and find the length l_2 for the other cell. Then $E_1/E_2 = l_1/l_2$. If E_2 be a Daniell's cell, then $E_1 = 1 \cdot 1\, l_1/l_2$ volts. Repeat, placing another cell in the position E_1 and find its e.m.f. in the same way.

Range of usefulness of the potentiometer.—Another great advantage of the potentiometer consists in the very great range of electromotive forces that it may be used to compare. It may be made so sensitive that e.m.f.'s of the order of millivolts or less may be compared by means of it. Also, by using auxiliary resistances with it, e.m.f.'s up to thousands of volts may be measured.

In order to make the potentiometer more sensitive, all that is necessary is to reduce the current in it, so that the p.d. over the whole wire, instead of being about 2 volts, is only a small fraction of a volt. This may be done by placing a resistance in series with it, in the manner employed in measuring thermal electromotive forces (Chap. LXXIX.).

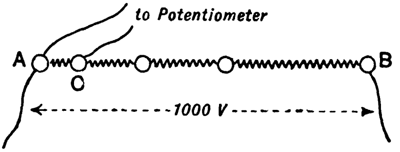


FIG. 830.—Subdivided resistance.

For the measurement of high electromotive forces, a **subdivided resistance** must be used. Thus, the resistance AB (Fig. 830) may be 10,000 ohms, and if the e.m.f. to be measured is of the order of 1000 volts, it may be applied to the points A and B. The p.d. between two points A and C, such that the resistance of AC is one-thousandth of AB is then of the order of 1 volt and may be measured by the potentiometer. The e.m.f. applied to AB may then be obtained by multiplying by 1000.

Zero error of the potentiometer.—It may happen that the resistance of the joint and terminal at P (Fig. 828) is not negligible, and it will then follow that lengths of wire measured by the scale are not strictly proportional to the resistances between the terminal and the movable point of contact. The proper correction to be applied to the scale readings may easily be found, and is called the **zero error** of the instrument. Thus, if AB and BC (Fig. 831) be two resistances in series in which a steady current is flowing, the p.d. between A and C is the sum of the p.d.'s between AB and BC.

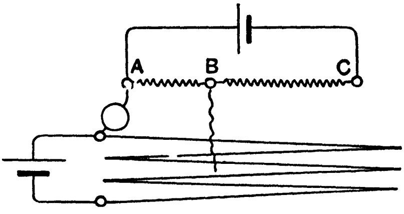


FIG. 831.—Measurement of the zero error of potentiometer.

On finding lengths of potentiometer wire l_1, l_2 and l_3 corresponding to these three p.d.'s, $l_3 = l_1 + l_2$ if there is no zero error. If, however, there is a zero error a, each length must have a added to it in order to make it proportional to the p.d. being measured. The relation between the three is then

$$l_3 + a = (l_1 + a) + (l_2 + a),$$

or,

$$a = l_3 - l_1 - l_2.$$

a is the correction that must be added to the potentiometer readings. It may be either positive or negative.

EXPT. 206.—**Zero error of potentiometer.** Place two resistance boxes AB and BC (Fig. 831) in series with a Daniell's cell; with 50 ohms in AB and 100 ohms in BC, measure the length of wire l_1 for the p.d. between A and B, l_2 for that between B and C, and l_3 for that between A and C. Then if a be the zero error of the potentiometer, $a = l_3 - l_1 - l_2$.

EXPT. 207.—**Comparison of resistances by potentiometer.** For the comparison of low resistances the potentiometer is convenient. Join the two low resistances AB and CD in series, and pass a current through them (Fig. 832). If the resistances are 0·1 ohm or less, at least 1 ampere should be employed. Find the length of potentiometer wire l_1 corresponding to the p.d. between A and B. Then remove the connections from A and B to C and D and again find the corresponding length of potentiometer wire.

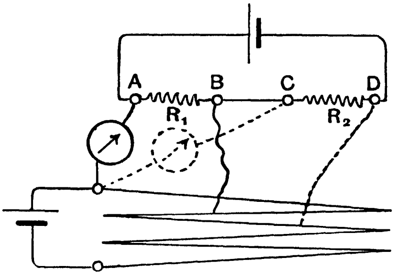


FIG. 832.—Comparison of resistances.

Then,

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}.$$

If the resistance of either AB or CD in ohms be known, that of the other can be found.

Current measurement by potentiometer.—Currents of any magnitude can be measured by means of the potentiometer, both accurately and conveniently. Hence this method is frequently employed for the calibrating of ammeters. The current I to be measured is caused to flow through a standard low resistance R. This should have a resistance of 0·1 ohm for a current of 10 amperes, 0·01 ohm for a current of 100 amperes, and so on, so that the p.d. between its terminals is of the order of 1 volt. The arrangement is shown in

Fig. 833. With the key K on one side, the length of potentiometer wire l_1 for the p.d. over R is found. Then the rocker of the key is placed over to the other side and the length of wire l_2 for the standard cell E is found. Since the p.d. between the terminals of R is IR, we have

$$\frac{IR}{E}=\frac{l_1}{l_2},$$

or,
$$I=\frac{E}{R}\cdot\frac{l_1}{l_2}.$$

In this way the current I is found in terms of the two standards E and R. For rough measurements E may be a Daniell's cell whose e.m.f. is taken as 1·1 volt, but for more exact measurements the cadmium cell (p. 914) or the Latimer-Clark cell (p. 913) should be used.

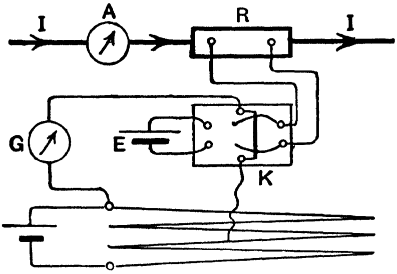


FIG. 833.—Current by potentiometer.

EXPT. 208.—**Calibration of an ammeter by means of a potentiometer.** Place the ammeter A in series with the standard resistance R. A rheostat or an incandescent lamp should be placed in the current circuit, the lamp being used when 100 volt supply is employed. Measure the length l_1 of wire corresponding to the p.d. between the terminals of R, and the length l_2 for the cell E, as above. Increase the current step by step, each time making the above two observations and reading the ammeter. Record the results in a table, making a fourth column for the current calculated from the relation $I=El_1/Rl_2$. After reaching the end of the ammeter scale, plot the current and ammeter readings in the form of a graph. If the ammeter is graduated to read amperes, the error for each observation should be found and plotted against the ammeter readings, thus giving a curve for future use with this ammeter.

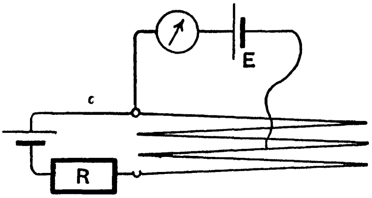


FIG. 834.—Calibration of a potentiometer.

EXPT. 209.—**Calibration of a potentiometer to read directly in volts.** Connect up the potentiometer and standard cell E as in Fig. 834, including an adjustable rheostat R in the potentiometer circuit. If the standard cell is a Daniell, place the movable contact at the point 110 cm. upon the wire. Adjust the rheostat R until a balance is obtained. Then 110 cm. of wire corresponds to 1·1 volt, so that 1 cm. corresponds to 0·01 volt. This calibration is convenient when a number of voltages are to be measured, as the instrument is now direct reading. When great accuracy is required a better form of standard cell than the Daniell must be used. For the cadmium cell the length of wire should be 101·8 cm., and for the Latimer-Clark cell 145·3 when 100 cm. of wire is to correspond to 1 volt.

Resistance box potentiometer.—A very convenient form of potentiometer may be constructed of two similar resistance boxes. The two boxes AB and CD are joined in series with a secondary cell E (Fig. 835). One of the cells whose e.m.f. is required is placed at E_1 in series with the galvanometer G. At first all the plugs are taken out of the box CD while all the plugs are in the box AB. The adjustment is made by removing plugs from AB and placing them in the corresponding positions in CD until the galvanometer deflection is zero. Thus the resistance in the circuit ABCD remains constant, and the current in it, therefore, remains constant also. The e.m.f. E_1 is then proportional to the resistance R_1 in AB, when the adjustment is correct. E_1 is now replaced by E_2 and the corresponding resistance R_2 in AB, for a balance is found. Then

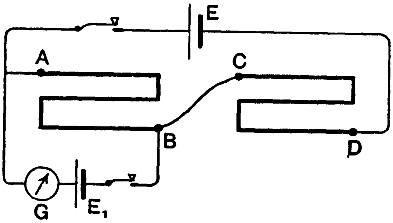


FIG. 835.—Resistance box potentiometer.

$$\frac{E_1}{E_2}=\frac{R_1}{R_2}.$$

Measurement of the internal resistance of a cell.—The potentiometer affords a means of measuring the internal resistance of a cell. On measuring the p.d. between the terminals of the cell, first on open circuit, and then when it is producing current in an external resistance of known value, the internal resistance of the cell can be calculated. These two quantities are represented by E and e in equation (11) (p. 855), where r is the internal resistance of the cell and R the external resistance joined between its terminals. The absolute values of E and e are not required, the lengths of potentiometer wire employed when the balance is obtained in the two cases serving instead. Calling these lengths L and l, equation

$$\frac{E-e}{e}=\frac{r}{R}, \text{ becomes } \frac{L-l}{l}=\frac{r}{R}.$$

Since all the quantities except r are known, r can now be calculated.

EXPT. 210.—**Internal resistance of a cell.** Connect up the potentiometer and galvanometer as in Fig. 827, using a secondary cell for C, and the cell (a Daniell's cell) whose internal resistance is required for E. Obtain the length AD = L when the cell E is on open circuit. Now join a resistance box across the terminals of E, using 20 ohms as the resistance in the box. Measure the length of wire l which now gives a balance.

Calculate the internal resistance r of the cell from the equation

$$\frac{L-l}{l}=\frac{r}{20}.$$

Repeat the experiment using values of R equal to 15, 10, 5, 2 and 1 ohms. and in each case calculate the value of r.

Repeat the experiment with another form of cell, such as a dry cell, or a Leclanché.

EXERCISES ON CHAPTER LXIX.

1. Enunciate Ohm's Law and explain its meaning.

A battery is connected to a tangent galvanometer of resistance 9 ohms and produces a deflection of 60°. An extra resistance of 7 ohms is then placed in the circuit and the deflection falls to 45°. Calculate the resistance of the battery. Sen. Camb. Loc.

2. Two voltaic cells A and B are connected in series, and form a simple circuit with a galvanometer. The current indicated is 2·4 amperes. The cell A is then reversed so as to oppose B, and the current observed is 0·6 ampere in the same direction as before. Calculate the ratio of the electromotive forces of A and B. L.U.

3. Explain the use of a potentiometer in measuring (*a*) potential difference, (*b*) current. What apparatus would you want in each case? L.U.

4. Explain how the e.m.f.'s of a number of cells can be compared by the aid of a potentiometer.

The terminals of a cell with an e.m.f. of two volts and with a negligible internal resistance, are joined by two coils in series. One coil has a resistance of 1 ohm, while the other has a variable resistance which may be denoted by R. What must be the value of R in order that a Leclanché with an e.m.f. of 1·58 volts can be connected in parallel with the 1 ohm coil, in such a way that no current flows through the Leclanché? Give a diagram of the arrangement. L.U.

5. Explain the Wheatstone's bridge method of comparing resistances.

If the resistance of a wire of length 120 cm. and diameter 0·4 mm. is found to be 2·5 ohms, what is the specific resistance of the material? L.U.

6. Two cells, when connected in series, give rise to a deflection of 45° on being joined to the terminals of a tangent galvanometer, and to a deflection of 30° when connected so as to oppose each other. Compare the electromotive forces of the cells. L.U.

7. What do the terms "electromotive force" and "internal resistance" mean as applied to a voltaic cell?

If the difference of potential between the poles of such a cell when no current is flowing is 1·4 volts, and this becomes reduced to 1·1 volts when the poles are joined by a wire of 5 ohms resistance, find the internal resistance of the cell. L.U.

8. Describe some method of measuring the resistance of a galvanometer. A galvanometer has a shunt of resistance 30 ohms, and the rest of the circuit comprises a cell and a resistance of 450 ohms. If the shunt is changed to 20 ohms and the external resistance decreased from 450 ohms to 350 ohms, the galvanometer indicates the same current as before. Find the resistance of the galvanometer.

9. A cell of e.m.f., 1·5 volts and resistance 2 ohms, maintains a current in an external resistance of 6·5 ohms., find the p.d. between the terminals of the cell.

10. A circuit consists of a cell of e.m.f. 1·72 volts and resistance 1·5 ohms, together with an external resistance of 4·2 ohms. Calculate the amount of heat developed in the external resistance and in the cell in one minute.

11. A cell having an e.m.f. of 1 6 volts and internal resistance 0·5 ohm is placed in series with a tangent galvanometer whose coil consists of 15 turns of radius 18 cm., and the deflection is observed to be 12°. Calculate the resistance of the galvanometer, taking $H = 0{\cdot}18$.

12. A piece of wire, 165 cm. long and 0·82 mm. in diameter, is used as an unknown resistance with the post office box. If the ratio arms have resistances 1000 : 10 and the third (Fig. 825) is 85 ohms when a balance is obtained, find the specific resistance of the wire.

13. Explain how the internal resistance of a cell may be determined. A length of potentiometer wire of 155 cm. balances the e.m.f. of the cell on open circuit, and a length of 135 cm. when the cell has a conductor of resistance 8 ohms connected between its terminals. Calculate the internal resistance of the cell.

14. Describe how the errors in the scale of an ammeter may be determined by the use of a potentiometer. What would be a suitable value of the standard resistance employed in this standardising experiment if the ammeter reads up to 50 amperes.

15. Describe how the zero error of a potentiometer may be measured. If in Fig. 831 the length of wire to balance the p.d. over AB is 36·5 cm., over BC 29 4 cm., and over AC 66·3 cm., what is the zero error ? What fault in construction in the instrument could such a zero error be due to ?

CHAPTER LXX

ELECTROLYSIS : CELLS AND BATTERIES

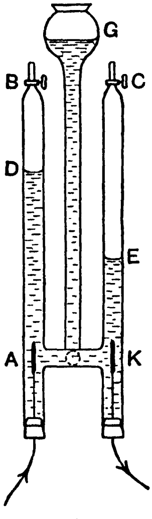


FIG. 836.—Electrolysis of water.

Introduction to electrolysis.—Reference to the fact that the passage of an electric current through certain liquids, mostly solutions, is accompanied by chemical effects has already been made (p. 830). A more complete account of this phenomenon must now be given. It has been known for a very long time that some liquids are capable of carrying a current while others are not. Faraday, who first systematically studied this branch of electrical phenomena, called those liquids which would carry a current **electrolytes.** Water to which a small amount of an acid, or salt, has been added is a moderately good conductor, and is hence an electrolyte. On the other hand, paraffin oil is a good insulator, and is therefore not an electrolyte. Pure water is not an electrolyte.

The conductors which bring the current into and lead it from the electrolyte are called **electrodes.** Thus, if two electrodes consisting of pieces of platinum foil be dipped into an electrolyte consisting of water to which a few drops of sulphuric acid have been added, and the electrodes connected respectively to the poles of a battery of a few cells in series, a current will flow through the water. While the current is flowing it may be noticed that bubbles form on the electrodes. If the experiment be performed with the apparatus shown in Fig. 836, consisting of two vertical graduated tubes AB and KC, united at their bases by the short piece AK, the gases formed on the electrodes may be caught in the tubes and measured.

The electrode at which the current enters the electrolyte is called the **anode**, and that by which it leaves is the **kathode**. It will be found that the gas evolved at the kathode K has twice the volume of that evolved at the anode in the same time. On opening the tap C, the gas in CE escapes, and if a light be applied, the escaping gas burns. It is hydrogen. The gas in BD as it escapes will ignite a glowing taper or splinter of wood, showing that it is oxygen. Thus hydrogen is evolved at the kathode, oxygen at the anode, and the two are in the proportion in which they combine to form water. The significance of this fact will be seen shortly.

In the event of the electrolyte being a solution of a salt, the metal is liberated at the kathode and the acid radicle or element at the anode. These substances are deposited upon the electrode if they can exist in a stable form in water, but in many cases there is some reaction with the water. When the current passes between platinum plates immersed in a solution of copper sulphate, metallic copper is deposited upon the kathode, since there is no reaction between the copper and the water. At the anode, the acid radicle SO_4 is liberated. This cannot exist alone, so with the water forms sulphuric acid, oxygen being liberated in the gaseous form. This reaction follows the equation :

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2.$$

A similar reaction occurs when the current passes through acidulated water. The hydrogen of the sulphuric acid is liberated at the kathode and SO_4 at the anode. This, with the water, forms sulphuric acid and oxygen as above. Faraday called the materials liberated in electrolysis **ions**. This name is still sometimes used, but is also employed in a more particular sense (p. 1034).

EXPT. 211.—**Examples in electrolysis.** (i) Dip two carbon rods (arc-lamp carbons) into water slightly acidulated with sulphuric acid. Bind a piece of bare copper wire round each of the rods at the part not immersed, and join one wire to each pole of a battery of a few secondary cells. Test the direction of the current by means of a compass needle and so determine which carbon rod is the anode and which the kathode. Note that bubbles are formed on both rods, but more copiously at the kathode. Collect some of the gas from each electrode by means of

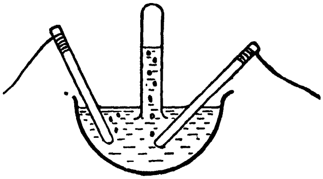


FIG. 837.—Experiment in Electrolysis.

inverted test-tubes (Fig. 837) and test the gas, showing that one is hydrogen and the other oxygen.

(ii) Replace the dilute acid by a solution of copper sulphate and observe that when the current flows, a red deposit of copper is formed on the carbon rod which is acting as kathode and bubbles of gas form on the anode.

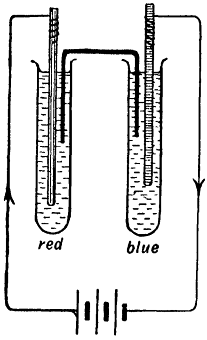


FIG. 838.—Electrolysis of sodium chloride.

(iii) Using two lead plates as electrodes, pass a current through a solution of lead acetate. A beautiful fern-like deposit of metallic lead forms upon the kathode.

(iv) Place each carbon rod in a test-tube containing a solution of sodium chloride to which a few drops of litmus have been added, and unite the tubes by means of a strip of blotting paper. Observe that the litmus in the anode tube (Fig. 838) turns red. This is due to the formation of hydrochloric acid. Chlorine is liberated, which, with the water, forms hydrochloric acid, which turns the litmus red.

$$4Cl + 2H_2O = 4HCl + O_2.$$

At the kathode, sodium is liberated, which dissolves in the water, forming sodium hydroxide, which keeps the litmus blue.

$$2Na + 2H_2O = 2NaOH + H_2.$$

Hydrogen escapes in bubbles at the kathode and oxygen at the anode.

Laws of electrolysis.—The quantitative laws of electrolysis were first given by Faraday and are known by his name. In order to understand them let us consider the case of a current flowing through a number of electrolytic cells A, B, C and D in series (Fig. 839). Let all the electrodes be platinum or carbon, so that there are no secondary actions of the ions with the electrodes to consider. For a moment we will disregard all secondary reactions and concentrate our attention upon the materials liberated by the process of electrolysis, leaving the consideration of their ultimate condition for a time.

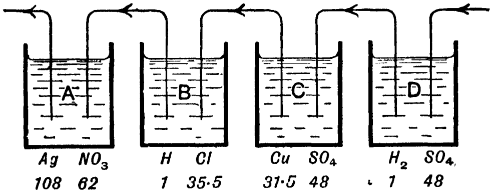


FIG. 839 —Laws of electrolysis.

Suppose that A contains a solution of silver nitrate, B one of hydrochloric acid, C of copper sulphate and D of sulphuric acid. From what we have already seen, there will be liberated at the respective kathodes, silver, hydrogen, copper and hydrogen ; and at the anodes NO_3, Cl, SO_4 and SO_4. It is a matter of experiment to determine how much of each substance is liberated by any current in a given time. The **first law of electrolysis states that the mass of any substance liberated is proportional to the current flowing and to the time for which it flows ; that is, it is proportional to the product of current and time,** or,

$$\text{Mass liberated} \propto \text{current} \times \text{time}.$$

It will be shown later (p. 902) that the product of current and time represents a quantity of electricity, in fact, the first law of electrolysis is usually stated, that **the mass of any substance liberated is proportional to the quantity of electricity passing through the electrolyte.** Thus, when a current of 10 amperes flows for 30 seconds, the same quantity of electricity passes as when a current of 5 amperes flows for 60 seconds, and in each case the amount of electrolysis occurring is the same.

The relation of the mass of substance liberated to the chemical nature of the substance must now be considered. Suppose that the current flows until 1 gram of hydrogen has been liberated in B (Fig. 839), then, since the current and time of flow are the same for all the cells, 1 gram of hydrogen also is liberated in D. Now, in B, 1 gr. of hydrogen is combined with 35·5 gr. of chlorine to form hydrochloric acid. Hence, when 1 gr. of hydrogen is liberated at the kathode, 35·5 gr. of chlorine will be liberated at the anode. Similarly, in D, 48 gr. of SO_4 will be liberated in the same way. Again, 48 gr. of SO_4 is liberated in C, and hence the equivalent amount of copper, that is, 31·5 gr., is liberated. In A there will be 108 gr. of silver and 62 gr. of NO_3 liberated.

For any other current, or time of flow, the whole of these quantities will vary in the same ratio, so that we may state that **the masses of various substances liberated are proportional to their chemical equivalents.** In the case of an acid radicle the chemical equivalent is the mass of the substance that would combine with 1 gr. of hydrogen, and in the case of the base it is the mass that would replace 1 gr. of hydrogen in an acid to form the salt.

Thus, with the monovalent substances the chemical equivalent is the atomic weight ; with divalent substances, half the atomic weight, and so on. The two laws of electrolysis may thus be stated formally as follows :

Faraday's laws of electrolysis:

(i) **The mass of any substance liberated in electrolysis is proportional to the quantity of electricity passing through the electrolyte (or to the current and the time for which the current flows).**

(ii) **The masses of different substances liberated by a given current in a given time are proportional to the chemical equivalents of the substances.**

The coulomb.—When a current of one absolute unit flows for one second, one absolute unit of quantity of electricity passes along the conductor. Since, however, the current is usually measured in amperes, it is convenient to give a name to the quantity of electricity which passes along the conductor when 1 ampere flows for 1 second. It is called one **coulomb**, and is the practical unit of quantity of electricity, upon the same system as that upon which the ampere, volt and ohm were devised.

The ampere being one-tenth of the absolute unit of current we see that the coulomb is one-tenth of the absolute unit of quantity of electricity.

Electro-chemical equivalent.—From Faraday's laws it is evident that if the mass of some one substance liberated by a given current in a known time can be determined experimentally, then the corresponding mass of any other substance for any current and time may be calculated. It does not follow that every substance when liberated can be collected and weighed with accuracy. Hence, for the purpose of exact experimental work it is necessary to choose the substance with care. The choice falls upon silver, as this has a very high chemical equivalent, and, when deposited from silver nitrate solution by means of a moderate current, forms a hard metallic deposit of great purity.

The mass of any substance liberated by unit current in unit time is called its electro-chemical equivalent. The electro-chemical equivalent of silver has been found to be 0·0011183. That is, a current of 1 ampere flowing for 1 second, or the passage of 1 coulomb, liberates 0·0011183 gram of silver. From this, the electro-chemical equivalents of the other substances may be calculated, using the knowledge of their chemical equivalents. Thus, the atomic weight of silver being 107·88, it follows that the electro-chemical equivalent of hydrogen is

$$z_H = \frac{0{\cdot}0011183 \times 1{\cdot}008}{107{\cdot}88} = 0{\cdot}00001045;$$

also the electro-chemical equivalent of copper is

$$z_{\mathrm{Cu}} = 0{\cdot}0011183 \times \frac{63{\cdot}57}{2 \times 107{\cdot}88} = 0{\cdot}0003295.$$

In this case the atomic weight is 63·57, but since copper is usually divalent, its chemical equivalent is ½ (63·57).

TABLE OF SOME ATOMIC WEIGHTS AND ELECTRO-CHEMICAL EQUIVALENTS.

	Atomic weight O=16.	Valency.	Electro-chemical equivalent.
Aluminium - - (Al)	27·1	3	0·0000936
Bromine - - (Br)	79·92	1	0·0008285
Chlorine - - (Cl)	35·46	1	0·0003676
Copper - - - (Cu)	63·57	1 or 2	0·0003295
Gold - - - (Au)	197·2	3	0·0006814
Hydrogen - - (H)	1·008	1	0·00001045
Iron - - - (Fe)	55·84	2 or 3	—
Lead - - - (Pb)	207·10	2	0·001073
Oxygen - - - (O)	16·0	2	0·00008293
Platinum - - (Pt)	195·2	4	0·0005059
Potassium - - (K)	39·10	1	0·0004053
Silver - - - (Ag)	107·88	1	0·0011183
Sodium - - - (Na)	23·00	1	0·0002384
Zinc - - - (Zn)	65·37	2	0·0003388

Voltameters.—Owing to the exactness of Faraday's laws of electrolysis and hence of our knowledge of certain electro-chemical equivalents, it follows that electrolytic methods may be employed for the accurate measurement of current. Knowing the electro-chemical equivalent of any substance, a suitable solution of it is used as an electrolyte, and the current to be measured is passed through the electrolyte for a known time. Since the amount of deposit is proportional to the time for which the current passes, a small current may be employed, the time being made large enough to produce a fair amount of deposit. In fact, large currents must not be employed, since with large currents, in many cases, the deposit, if metallic, is friable and easily detached from the electrode.

If z be the electro-chemical equivalent of the material employed, this is the mass liberated by 1 ampere in 1 second, and it follows from Faraday's laws (p. 902) that the total mass of the deposit is given by the equation,

$$\mathrm{W} = \mathrm{I}zt,$$

where I is the current in amperes and t the time in seconds for which the current flows.

An instrument designed to make use of this principle for the measurement of currents is known as a **voltameter.** There are many forms of voltameter, but the variation in type is only such as is necessary for the accurate determination of the mass of the ion liberated.

Copper voltameter.—In the case of the **copper voltameter,** a strong solution of copper sulphate, with a few drops of sulphuric acid added, is used as the electrolyte, the electrodes being copper plates. This gives a very cheap and useful form of the instrument, by means of which currents of the order of one or two amperes may be measured with a fair degree of accuracy. From the simplicity of its form it is readily made in any laboratory. The measurement of any given current requires considerable time, and this voltameter is therefore chiefly used in the standardisation and calibration of ammeters and tangent galvanometers.

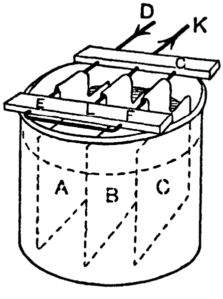


FIG. 840.—Copper voltameter.

The anode is in the form of two thin copper sheets A and C (Fig. 840) hanging one on either side of a similar copper sheet B which serves as kathode. With this arrangement the deposit occurs on both sides of the kathode and thus the maximum area is utilised. A bent brass or copper rod DEFG serves to support the plates A and C, and a similar rod KL supports the kathode B.

In order to measure the current, the circuit is arranged as in Fig. 841. The voltameter V is joined in series with the ammeter A, the battery B, a rheostat R and a key K, the kathode having previously been well cleaned with emery paper. On closing the key, the rheostat must be adjusted until the current has a suitable value, say, 1 ampere. No more than $\frac{1}{50}$ ampere per available square centimetre of kathode surface should be used, or the deposited copper will not be hard and compact, and is liable to be washed off the plate.

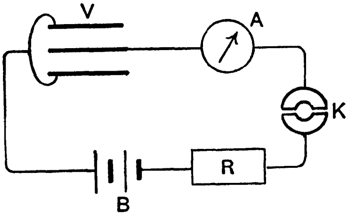


FIG. 841.—Calibration of an ammeter by electrolysis.

The circuit is then broken at K and the kathode removed, washed in clean water, dried and carefully weighed. The drying may be performed by waving the plate about **some distance above** the flame of a spirit lamp. The plate must not be heated too much, or it is liable to oxidise. The kathode is then replaced and the key closed, the instant of performing this being carefully noted on a watch or clock and recorded. All the time that the current is flowing it should be kept constant by means of the rheostat. If considerable variations occur, the observations so far are wasted and must be repeated. After an interval of about half an hour, or longer for smaller currents, the circuit is broken at a carefully noted time, and the kathode again removed, washed, dried and weighed. The observations may be recorded as follows :

Reading of ammeter - - = amperes.
Time of starting current - - = h. m. s.
,, stopping ,, - - = h. m. s.
Interval for which current flowed = m. s.
$\therefore t =$ seconds
Weight of kathode at start - = grams.
,, ,, stop - = grs.
Gain in weight of kathode - = grs. = W.

Since $W = Izt$,

$$\text{Current} = \frac{W}{0{\cdot}0003295 \times t}$$

Error of ammeter = .

Expt. 212.—**Calibration of an ammeter by a copper voltameter.** Arrange a circuit with ammeter and voltmeter included as in Fig. 841. Find the error of the instrument for one of the low readings as directed above. Repeat for larger currents, the last determination of the error being made with the largest ammeter scale reading. Plot a graph having ammeter readings as abscissae and errors as ordinates.

Silver voltameter.—For ordinary purposes the copper voltameter is sufficiently accurate, but for the highest order of accuracy the **silver voltameter** must be employed.

This usually takes the form of a platinum basin (Fig. 842) which has been previously cleaned with nitric acid, dried and weighed. It contains the electrolyte consisting of a solution of silver nitrate (15 gr. of $AgNO_3$ to 100 c.c. of water). Suspended in the solution, by means of platinum wire, is a plate of silver which constitutes the anode. From the reactions described on p. 901 it will be seen that silver is deposited upon the platinum

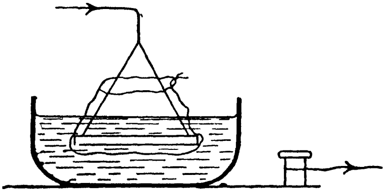


Fig. 842.—Silver voltameter.

basin, and also silver goes into solution at the anode, so that the strength of the solution remains constant. The silver anode is wrapped in a piece of blotting paper, which serves to catch any impurities liberated as the silver dissolves away, and prevents them falling on the platinum basin. The principle of the use of the silver voltameter is identical with that for the copper voltameter, the mass of silver deposited in a known time being found and the current calculated from the relation,

$$\text{Current} = \frac{W}{0{\cdot}0011183 \times t}.$$

Water voltameter.—It is possible to use the graduated tubes in Fig. 836 as a voltameter, by collecting the hydrogen liberated by the current in a given time. The gas collected is not at the atmospheric pressure, since the level of the water at G differs from that at E. On correcting for this, the volume must be reduced to that for standard pressure and temperature, taking into account the saturated aqueous vapour present (p. 460). Then on multiplying by the density of hydrogen (0·0000899 gr. per c.c.) the mass of hydrogen is known and the equation

$$\text{Current} = \frac{W}{0{\cdot}00001045 \times t},$$

may be used to find the current.

Owing to the difficulty of determining the volume of the gas with accuracy, and the number of corrections to be applied, the method is inferior to those in which the amount of ion liberated is found by direct weighing.

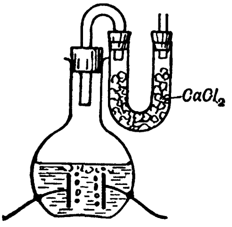


FIG. 843.—Water voltameter.

The method may be modified, in order to make it depend upon direct weighings, by electrolysing the dilute acid solution in a flask provided with sealed-in platinum terminals (Fig. 843). The oxygen and hydrogen escape together, and since they are insoluble they pass away into the air. On weighing the flask before and after passing the current for a known time, the weight of water decomposed is known. Any moisture carried off by the escaping gases is caught by a drying tube through which they pass before leaving, which tube must, of course, be weighed with the flask. Since the oxygen and hydrogen escaping are both weighed, the electro-chemical equivalent used in calculating the current is the sum of those of hydrogen and oxygen, *i.e.*

$$Z_{H_2O} = 0{\cdot}00001045 + 0{\cdot}00008293 = 0{\cdot}00009338.$$

Thus, $$\text{Current} = \frac{W}{0{\cdot}00009338 \times t}.$$

Expt. 213.—**Water voltameter.** Perform the calibration of an ammeter as in Expt. 212, using the water voltameter (Fig. 843) instead of the copper voltameter.

Theory of electrolysis.—It would be out of place to attempt to give in this volume a comprehensive account of the modern theory of electrolysis, but certain important facts may be considered. The laws of electrolysis certainly suggest that the electricity is carried through the solution by the atoms in it, all monovalent atoms carrying the same amount of electricity, divalent atoms twice that amount, and so on. Faraday considered the primary decomposition occurring, when the current passed, to be that of the water which is acting as solvent, the deposition of the dissolved substances being due to the secondary actions of the oxygen and hydrogen at the electrodes. It is now considered that the primary action is the decomposition of the dissolved substance, and that in cases where the hydrogen and oxygen are liberated, as when a solution of sulphuric acid is electrolysed (p. 898) it is the secondary action of the SO_4 and the water that causes the liberation of the oxygen.

$$2H_2O + 2SO_4 = 2H_2SO_4 + O_2.$$

It is a significant fact that pure water is a non-conductor.

The question then arises as to whether, in the case of a simple substance such as potassium chloride, the passage of the current causes a splitting up of the molecules, or whether these are already dissociated into potassium and chlorine in the solution and are merely directed towards the kathode and anode when the current flows.

The first consideration would lead us to think that Ohm's law would not be true for electrolytes, but the second is consistent with the truth of Ohm's law. Since Ohm's law is true for electrolytes this strengthens our belief in the dissociation theory.

If the first alternative were true, then no current would flow until the electromotive force had a sufficient value to disrupt the molecules, and any excess of electromotive force would freely cause current. Since, however, the current is proportional to the p.d. in the electrolyte, probably the e.m.f. does not cause the splitting up of the molecules, but merely drives the constituents of the already dissociated molecules in one direction or the other.

Evidence from widely differing sources leads to the belief that in any electrolyte, some of the molecules of the dissolved substance are dissociated, the atoms then having positive or negative electric charges, according to their nature. Such atoms, possessing electric charges, are called **ions** and have very different properties to the neutral atoms of the same substance ordinarily met with. Thus, in a given solution of potassium chloride a certain percentage of the molecules will be dissociated into ions. These may be designated by $\overset{+}{K}$ and $\overset{-}{Cl}$. When the external source of electromotive force, such as the battery, is connected to the electrodes, one is raised to a higher potential than the other. There is thus an electric field between them, and we shall see on p. 925 that in such an electric field positive charges are urged in one direction and negative charges in the other. It will thus be seen from Fig. 844 that the $\overset{+}{K}$ ions having positive charges are driven towards the kathode. On reaching it they give up their positive charges to the kathode and resume their ordinary qualities. The potassium would then dissolve in the water in the usual way. Similarly the $\overset{-}{Cl}$ ions on arriving at the anode give up their negative charges and resume their ordinary form. These streams of ions in the electrolyte constitute the current. When there is no dissociation the liquid is non-conducting. Metallic ions such as potassium, silver, copper, hydrogen, etc., have positive charges, and hence are driven towards the kathode, while, on the other hand, the acid radicles have negative charges, and are driven towards the anode.

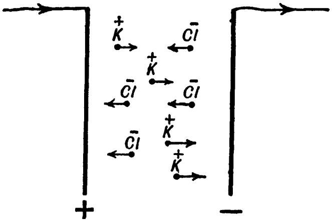


FIG. 844.—Electrolytic conduction.

The distinction between the chemical dissociation that occurs at high temperature and the electrolytic dissociation that occurs in solutions may well be illustrated by the case of ammonium chloride. At high temperature this substance dissociates according to the equation

$$NH_4Cl = NH_3 + HCl.$$

In solution the dissociation is represented by the equation

$$NH_4Cl = \overset{+}{NH_4} + \overset{-}{Cl}.$$

Electroplating.—Electrolytic processes are used for many industrial purposes. The deposition of silver or gold upon the baser and more corrodible metals is well known. For these purposes solutions which give a hard metallic deposit are essential. In the case of silver plating, a solution of the double cyanide of silver and potassium is used as electrolyte. On adding potassium cyanide to a solution of silver nitrate, silver cyanide is precipitated, which however dissolves when excess of potassium cyanide is added. The articles to be silver plated must first be thoroughly cleaned with wet sand or emery, afterwards being dipped in strong caustic soda to remove all traces of grease, and then washed before being placed in the electrolytic bath. The articles are then suspended by conducting holders and the current passed in such a direction that the article is kathode. The current must not be excessive, or the deposit will not be hard and adherent. After sufficient deposition has taken place, the article is washed and finally burnished to give it the familiar bright metallic finish.

For gold plating the process is similar to the above, but the solution is obtained by dissolving gold fulminate, or sometimes gold chloride, in potassium cyanide.

Electrotyping.—In some printing processes a copper reproduction of each page of type is made. A coating of wax is placed upon the set-up type and forced upon it by hydraulic pressure, the type having been previously sprinkled over with powdered graphite to prevent the wax adhering. After removal from the type, the wax mould is coated with powdered graphite to render its surface conducting. To increase this conductivity the surface is washed and a solution of copper sulphate poured upon it. Iron filings are then sprinkled upon it, which displace copper from the solution, so that in a few minutes a very thin layer of copper is deposited upon the surface. The mould is then placed in an electrolytic bath of copper sulphate solution, the electrical contact being so made that the mould is kathode. In the course of an hour or so a layer of copper is formed which is a good copy of the original type. On removing the wax and pouring molten type metal into the copper shell to stiffen it, the block used for the actual printing is obtained.

Simple voltaic cell.—It is well known that a piece of zinc immersed in a dilute solution of sulphuric acid is dissolved with the formation of zinc sulphate and the liberation of hydrogen.

$$H_2SO_4 + Zn = ZnSO_4 + H_2.$$

The hydrogen liberated forms bubbles on the surface of the zinc. If a rod of copper be now immersed by the side of the zinc and the two connected externally through a galvanometer, as in Fig. 845, it will be observed that a current will flow in the external circuit from the copper to the zinc, and at the same time the bubbles of hydrogen will appear upon the copper instead of the zinc. The zinc still dissolves, with formation of zinc sulphate, and it is the

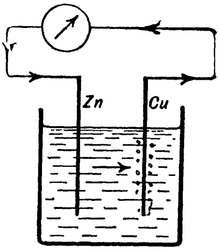


FIG. 845.—Simple cell.

energy liberated from this reaction which is available for the maintenance of the current.

Remembering that hydrogen appears at the kathode and SO_4 at the anode, we see at once that the current leaves the cell at the copper electrode. The copper is called the **positive pole** of the cell and the zinc the **negative pole.** The copper may be replaced by carbon or platinum with similar results. The original voltaic cell due to Volta was of this type, the arrangement alone being different. Alternate discs of zinc and copper are laid upon each other in a column with pieces of cloth moistened with dilute acid in the following order. Beginning at the bottom (Fig. 846) is a copper disc, then cloth, then zinc. Again comes copper, cloth, zinc and so on. This forms a pile consisting of simple cells of copper-acid-zinc in series. It was by means of such a pile that Volta first demonstrated many of the effects of an electric current.

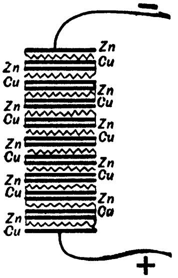


FIG 846.—Voltaic pile.

Polarisation.—A simple cell of the above type (Fig. 845) is of little practical utility, since in use the current drops rapidly. The cause of this is the layer of hydrogen bubbles which collects upon the copper. There are two deleterious effects of this layer of bubbles. In the first place, the effective area of the copper in contact with the solution is diminished, thereby increasing the resistance of the cell. In the second place, the presence of the hydrogen produces an electromotive force which tends to send a current through the cell in the opposite direction to the current actually passing. Hence the effective electromotive force of the cell is diminished by the amount of this e.m.f. which is called a **polarisation e.m.f.**

The presence of this **back e.m.f.** may easily be demonstrated in the following way. Two platinum sheets A and B are immersed in dilute sulphuric acid (Fig. 847). When the key K is depressed a current passes from A to B. A then becomes coated with bubbles of oxygen and B with hydrogen bubbles. On raising the key, the battery becomes disconnected and the galvanometer

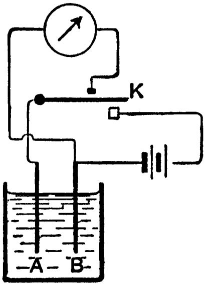


FIG. 847.—Polarisation e.m.f.

connected to the plates A and B. It will then be noticed that a current flows for a short time, in fact as long as the bubbles remain. The electromotive force is due to the presence of these gases; they play a similar part in forming a cell, to the copper and zinc in the simple cell. The plate on which the oxygen is deposited is the positive pole of the cell and that on which the hydrogen is deposited is the negative pole.

Daniell's cell.—Although cells are never used now for the production of large currents, they are frequently used when small currents are required. Zinc is almost universally employed for the negative pole. Pure zinc does not dissolve when immersed in dilute sulphuric acid unless the battery circuit is complete and a current flows. Commercial zinc, however, does dissolve and would consequently waste away when the cell is not in use. The reason for this is that pieces of impurity in the zinc, chiefly iron, form local cells with the bulk of the zinc, which therefore dissolves away. To get over this difficulty the zincs are always rubbed over with mercury, forming an amalgam which presents a surface of uniform composition to the solution and thus prevents local action.

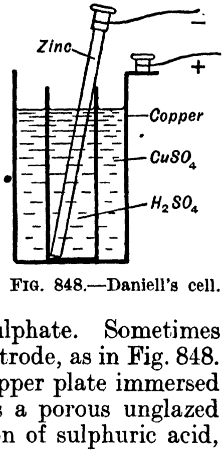


FIG. 848.—Daniell's cell.

To overcome the difficulty of polarisation many devices are used. In the **Daniell's cell** the positive pole is copper and the electrolyte in contact with it is a strong solution of copper sulphate. Sometimes the outer pot is of copper and itself forms the electrode, as in Fig. 848. Sometimes an earthenware jar is used, and a copper plate immersed in the copper sulphate solution. Inside this is a porous unglazed earthenware pot which contains a dilute solution of sulphuric acid, or sometimes zinc sulphate, and in this is immersed the amalgamated zinc rod. The object of the porous pot is to keep the copper sulphate solution and the dilute acid from mixing while allowing contact between them within the pores of the earthenware.

On completing the external circuit, the current flows from zinc to copper within the cell. Thus, at the anode zinc is dissolved,

$$Zn + SO_4 = ZnSO_4.$$

At the same time copper is deposited on the kathode,

$$CuSO_4 = Cu + SO_4.$$

At the junction of the two solutions hydrogen ions from the zinc side and SO_4 ions from the copper side form sulphuric acid:

$$\overset{+}{H}_2 + \overset{-}{SO}_4 = H_2SO_4.$$

There is consequently no hydrogen liberated upon the copper,

and hence no polarisation. The e.m.f. of the Daniell's cell is fairly constant and is about 1·1 volt.

Leclanché cell.—In the case of the Daniell's cell there is no polarisation, because copper from the solution is deposited upon the copper positive plate. Other cells are rendered non-polarising by the addition of some oxidising reagent to remove the hydrogen as it is liberated. Thus, in the bichromate cell, potassium bichromate is added to the sulphuric acid solution and carbon is used as the positive plate. In the Bunsen's and Grove's cells strong nitric acid is the oxidising reagent, and surrounds the positive plate, which is of carbon or platinum. But the only cell of this type used to any extent at the present time is the **Leclanché cell,** because there are no objectionable fumes from it and it gives a fair current for a short time, and recovers its original e.m.f. after a period of rest.

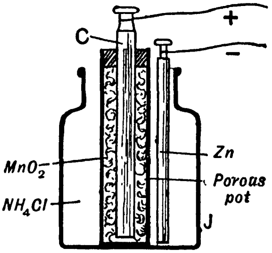


FIG. 849.—Leclanché cell.

The liquid of the Leclanché cell is a saturated solution of ammonium chloride (sal ammoniac) contained in a glass jar J (Fig. 849). In this is immersed the negative pole, consisting of an amalgamated zinc rod. The positive pole is a carbon rod, around which is closely packed a mixture of manganese dioxide and crushed gas carbon, contained in a porous pot which is cemented with pitch at the top. The sal ammoniac solution diffuses through the porous pot and the manganese dioxide mixture and reaches the carbon rod. When the current passes, zinc is dissolved at the negative pole according to the equation

$$2Cl + Zn = ZnCl_2.$$

The ions of NH_4 are liberated at the positive pole and there form ammonia (NH_3) and hydrogen,

$$2(NH_4) = 2NH_3 + H_2.$$

The ammonia escapes and the hydrogen is gradually oxidised by the manganese dioxide,

$$2MnO_2 + H_2 = Mn_2O_3 + H_2O.$$

On the disappearance of the hydrogen the cell regains its original e.m.f. of about 1·5 volt.

On account of its power of recovery after use, the Leclanché cell is largely used for telephones and electric bells.

Dry cells.—For the sake of portability many forms of Leclanché cell have been constructed in which there is no free liquid present. In most of these there is a paste containing manganese dioxide surrounding a carbon rod. This is in contact with a layer of sawdust, or in some cases plaster of paris, saturated with sal-ammoniac. The whole is contained in a zinc case which forms the negative electrode. Sometimes the outer case is of millboard, a zinc rod serving as electrode.

Standard cells.—Cells in which polarisation occurs are useless as standards of electromotive force, since they polarise, and therefore vary in electromotive force, directly a current flows. On referring again to the Daniell's cell (p. 911) it is seen that there is no polarisation, and the electrodes do not change in character when the current flows. The reason for this is that the solution in contact with each electrode contains as active ion the same metal as the electrode itself. Also, if the cell runs for some time, zinc is dissolved, forming zinc sulphate, and copper sulphate is decomposed with deposition of copper. If a current be now sent through the cell in the reverse direction by any external means, copper is redissolved and zinc is deposited. Thus a reverse current can bring the cell back to its original condition. Such a cell is said to be **reversible.** Reversible cells have fairly constant electromotive force. The Daniell's cell may, for rough purposes, be used as a standard of approximately 1·1 volt. More reliable standards must, however, be used for exact measurements. Such reliable standards are the **Latimer Clark cell** and the **Cadmium or Weston cell.**

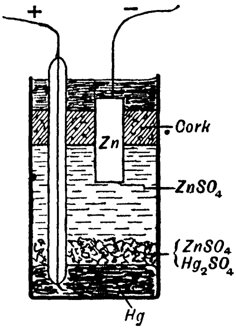


Fig. 850.—Latimer Clark cell.

Latimer Clark cell.—In this cell a pool of mercury forms the positive and a pure zinc rod the negative electrode. Upon the mercury (Fig. 850) rests a layer of paste made of mercurous sulphate and a saturated solution of zinc sulphate. Upon this rests a saturated solution of zinc sulphate, into which dips the zinc electrode. In order to make sure that the solution is saturated, some crystals of zinc sulphate are placed upon the top of the paste. Contact is made with the mercury by means of a platinum wire sealed into a glass tube, so that the tip of the wire projects into the mercury. The glass tube and the zinc electrode are kept in place by a cork, and the vessel is sealed up by a layer of marine glue.

Since the negative electrode, zinc, is in a solution of zinc sulphate, and the positive electrode, mercury, is in contact with mercurous sulphate, it will be seen that the cell is of the reversible type. If made carefully with pure chemicals, its electromotive force at any given temperature is very constant. It is given by the equation

$$\text{e.m.f.} = 1{\cdot}433 - 0{\cdot}0012\,(t - 15) \text{ volt,}$$

where t is the centigrade temperature. Thus, at 15° C. its electromotive force is 1·433 volt, and that at any other temperature may be calculated.

Cadmium or Weston cell.—This is a modification of the Clark cell and is constructed by replacing the zinc by cadmium amalgam and using cadmium sulphate instead of zinc sulphate. The form of the cell adopted by the National Physical Laboratory is given in Fig. 851. The electromotive force of this cell is given by

$$\text{e.m.f.} = 1{\cdot}0183 - 0{\cdot}0000406\ (t - 20)\ \text{volt}$$

at t° C. Thus, at 20° C. the electromotive force is 1·0183 volt. Owing to its very small variation of voltage with temperature this cell has been adopted as the international standard of electromotive force.

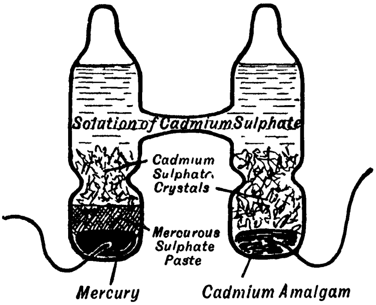


FIG. 851.—Standard cadmium cell.

In using the Latimer Clark or the cadmium cell great care must be taken not to allow more than a very minute current to pass through it. For this reason it is desirable, when using the cell, to keep a resistance of several thousands of ohms permanently in series with it.

Source of energy and e.m.f. of cell.—It has already been indicated (p. 910) that the source of energy of the current maintained by a cell is the chemical action occurring in it. Let us consider the case of the Daniell's cell. When 1 ampere flows for 1 second through the cell, 0·00003388 gr. of zinc is dissolved and 0·0003295 gr. of copper deposited. Now when 1 gr. of zinc is dissolved in sulphuric acid in a calorimeter, energy to the extent of about 1630 calories is liberated. This energy for 1 amp. flowing for 1 sec. is $1630 \times 0{\cdot}0003388 = 0{\cdot}553$ calories $= 0{\cdot}553 \times 4{\cdot}2 \times 10^7$ ergs $= 2{\cdot}32 \times 10^7$ ergs, and this is the energy liberated. On the other hand, 1 gr. of copper dissolved in sulphuric acid liberates 881 calories, so that 1 amp. in 1 sec. requires an amount of energy of

$$881 \times 0{\cdot}0003295 \times 4{\cdot}2 \times 10^7 = 1{\cdot}22 \times 10^7 \text{ ergs}$$

for the liberation of the copper. The balance of energy

$$(2{\cdot}32 \times 10^7) - (1{\cdot}22 \times 10^7) = 1{\cdot}1 \times 10^7 \text{ ergs}$$

is available for driving the current.

Now, the work done in maintaining the current is $E \times I \times 10^7$ ergs per second, which becomes E ergs per second when $I = 1$ amp.

$$\therefore \; E \times 10^7 = 1\cdot1 \times 10^7,$$

$$E = 1\cdot1 \text{ volt.}$$

This reasoning rests entirely upon the assumption that **the energy liberated from the chemical changes is entirely converted into energy of electrical current.** The closeness of the result to the known value of the electromotive force shows that in the case of the Daniell's cell this assumption is very nearly justified. But this is not necessarily always the case. In fact, in some cells some of the energy is liberated directly in the form of heat, and only the remainder is available for maintaining the current. Such cells become hotter when running, and decrease in electromotive force as the temperature rises, as in the case of the Latimer Clark cell (p. 913), and the Cadmium or Weston cell (p. 914). On the other hand, in some cells heat energy is drawn directly from the cell and goes to increase the energy available for maintaining the current. The cell then becomes colder when running, and its electromotive force increases with rise in temperature.

Minimum e.m.f. in electrolysis.—When polarisation occurs, an opposing electromotive force is brought into existence (p. 910). In the case of the electrolysis of water this is of importance, as it follows that any cell whose electromotive force is less than this is unable to maintain the electrolysing current. In the case of water the polarisation e.m.f. may be calculated. When 1 gram of water is formed by the combustion of hydrogen in oxygen there is a liberation of heat to the extent of about 3800 calories (p. 361). We may take this as the amount of energy required to separate the hydrogen from the oxygen for one gram of water. If 1 ampere flows through the electrolytic cell for 1 second 0·00009338 gr. of water is decomposed (p. 906). Hence the energy required for this decomposition is

$$3800 \times 0\cdot00009338 \times 4\cdot2 \times 10^7 \text{ ergs} = 1\cdot49 \times 10^7 \text{ ergs.}$$

Again, the work done in opposition to the polarisation e.m.f. (E) is $E \times I \times 10^7$ ergs per second for I amperes, or $E \times 10^7$ ergs per second for 1 ampere;

$$\therefore \; E = 1\cdot49 \text{ volt.}$$

This is the minimum electromotive force necessary to maintain the current, and it is thus seen why a single Daniell's cell is not sufficient for the electrolysis of water. It would, of course, start a current, but when polarisation begins, the back e.m.f. due to it rises until it equals the e.m.f. of the Daniell, when, of course, the current will cease.

Secondary or storage cells.—In the case of the cells already described, some substance is used up chemically in the production of the current. This is expensive, and moreover these cells cannot be used for the production of large currents. In 1859 Planté succeeded in making a cell in which energy put into the cell as electric current can be stored up and subsequently drawn again from the cell in the form of current. Such cells are called **secondary** or **storage** cells, or sometimes **accumulators.**

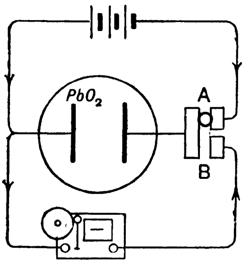


FIG. 852.—Principle of the secondary cell.

Two lead plates are used as electrodes in a solution of sulphuric acid. On the first passage of current through the cell, oxygen is liberated at the anode, which oxidises the surface of the lead plate, forming PbO_2. The hydrogen bubbles away at the kathode.

On stopping the current and joining the lead plates by a conductor, a current will pass for a time from the oxidised plate (+) to the unoxidised plate (–) through the external circuit. If the experiment be performed as shown in Fig. 852 the key must be placed in A for the charging current to flow, and in B for the discharge, which may be used to ring an electric bell.

When the discharge takes place, the oxidised plate is reduced by the hydrogen liberated and forms PbO, which, with the sulphuric acid, forms $PbSO_4$. The current passes until the negative plate is oxidised and forms $PbSO_4$, and ceases when both plates have reached the same condition.

If the charging and discharging be repeated for a number of times it will be found that the storing capacity of the cell becomes greater. This is generally carried out not merely by letting the cell discharge, but by reversing the current in it until the kathode is reduced to lead. This reduced lead forms a soft layer upon the plate, known as **spongy lead**, and with repeated reversals of current this spongy lead forms a deeper and deeper layer, and the storage capacity of the cell increases. This process is called '**forming**' the plate.

When the plates have been properly 'formed' the processes of charging and discharging may be represented approximately by the following equations :

$$\textit{Charging.}\left\{\begin{array}{c}\textit{Positive Plate.}\\ PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4.\\ \textit{Negative Plate.}\\ PbSO_4 + H_2 = Pb + H_2SO_4.\end{array}\right.$$

$$\textit{Discharging.}\left\{\begin{array}{c}\textit{Positive Plate.}\\ PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O.\\ \textit{Negative Plate.}\\ Pb + H_2SO_4 + O = PbSO_4 + H_2O.\end{array}\right.$$

It will be seen that during the process of charging, sulphuric acid is formed, and consequently the density of the electrolyte rises. During discharge, sulphuric acid disappears and the density of the electrolyte falls. Observation of the density of the 'acid' gives the best indication of the condition of the cell. It should not exceed 1·21 when the cell is fully charged, nor fall below 1·15 when it is discharged.

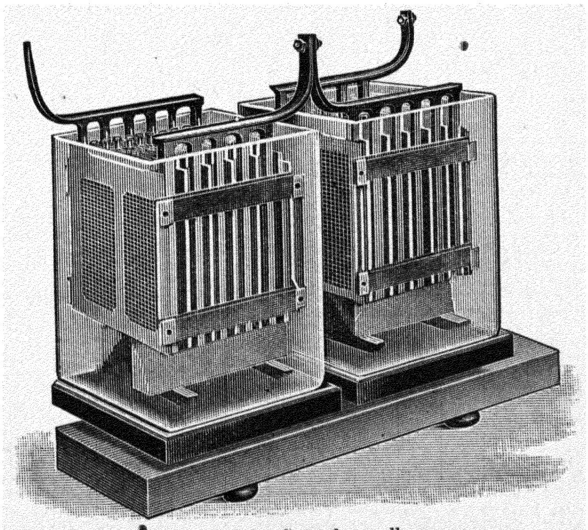


FIG. 853.—Secondary cells.

The secondary cell of this type has a remarkably constant electromotive force of about 2·1 volts throughout the greater period of its discharge. It is usually constructed of a series of parallel lead plates, alternate plates being connected to one of two lead strips at the top which form the leads for the current (Fig. 853). Owing to the high conductivity of the electrolyte, and the large area of the plates and their closeness together, the internal resistance is very small, and hence considerable currents can be obtained from them. For a 100 volt supply circuit a battery of 50 to 55 of such cells would be used in series.

Faure, or paste cell.—The secondary cell of the Planté type is costly to produce on account of the long 'forming' required. In order to cheapen the production, Faure constructed the plates in the form of a network or grid, into the interspaces of which a paste consisting of red lead (Pb_3O_4) and sulphuric acid is pressed. This forms lead oxide and lead sulphate on both plates,

$$Pb_3O_4 + 2H_2SO_4 = PbO_2 + 2PbSO_4 + 2H_2O.$$

On passing the charging current, the $PbSO_4$ on the positive plate is oxidised to PbO_2,

$$PbSO_4 + O + H_2O = PbO_2 + H_2SO_4,$$

while at the negative plate the oxide and sulphate are reduced to spongy lead,

$$PbSO_4 + H_2 = Pb + H_2SO_4,$$

$$PbO_2 + 2H_2 = Pb + 2H_2O.$$

Thus the first charging produces the requisite layer of spongy lead, in this way obviating the lengthy and costly process of 'forming.'

Paste plates, however, are not so durable as 'formed' plates. Some makers use 'formed' plates for the positive and paste plates for the negative. Many varieties of both kinds are on the market.

EXERCISES ON CHAPTER LXX.

1. State the laws of electrolysis, and explain what is meant by the electro-chemical equivalent of an element.

Describe how you would determine the electro-chemical equivalent of copper. Sen. Camb. Loc.

2. Give an account of the laws of electrolysis; and explain what you mean by electropositive and electronegative elements.

It is found that in 1 minute 40 seconds a certain current deposits 0·112 gm. of silver; and in twice the time 0·081 gm. of potassium. Given that the chemical equivalent of silver is 108, find that of potassium. L.U.

3. What is meant by the *polarisation* of a voltaic cell, and how may this effect be exhibited ?

What, also, is *local action* in such a cell, and how may it be diminished ?
L.U.

4. What is *electrolysis* ? Describe an experiment in which occurs what is commonly called the 'electrolysis of water.' Give your view of the appropriateness or otherwise of this way of describing the experiment and its results. L.U.

5. Explain how the electromotive force of a Daniell cell can be calculated from the chemical changes that take place therein when a current flows. How can the electromotive force of two cells be compared by experiment ?
L.U.

6. A water voltameter, a conductor in a calorimeter, and a tangent galvanometer are connected in series. A current causes an evolution of 10 cubic centimetres of hydrogen, a rise of 4° C. in the same time, and a deflection of 10° in the galvanometer. The current is then doubled. Describe the effect in each part of the circuit.

7. State the laws of electrolysis.

A circuit includes a silver voltameter and a tangent galvanometer of 20 turns of 16 cm. diameter. If the galvanometer shows a steady deflection of 45° and if ·115 gm. of silver is deposited in 15 minutes, find the strength of the earth's horizontal magnetic field.

(Electro-chemical equivalent of silver = 001118 gram per coulomb.)
L.U.

8. State Faraday's laws of electrolysis.

Point out the most important differences between electrical conduction in metals and in solutions. L.U.

9. State and explain Faraday's laws of electrolysis. A tangent galvanometer was joined in series with a battery and a silver voltameter. The deflection of the needle was 45°, and in the course of an hour the mass of silver deposited was 0·1052 gr. Given that the electro-chemical equivalent of silver is 0·001118, calculate the constant of the galvanometer.
L.U.

10. Describe the parts of a storage cell or accumulator, and state the changes that occur in them during the process of charging and discharging. Why is it important that the voltage of the cell should not be allowed to fall below 1·9 ? L.U.

11. A copper voltameter and a wire of resistance 2·8 ohms immersed in 350 grams of water are in series and a current passes through them. If 0 86 gr. of copper is deposited in 18 minutes, find the rise in temperature of the water in half-an-hour.

12. The coil of a tangent galvanometer, having 12 turns and radius 15 cm., is placed in series with a copper voltameter. If the deflection is 60° and $H = 0{\cdot}18$, calculate the amount of copper deposited in 15 minutes.

13. Describe the Leclanché cell and give an account of the chemical reactions occurring in it. For what purposes is it chiefly used and why ?

14. Give a description with sketch of either the cadmium cell or the Latimer Clark cell. For what purpose is such a cell used, and what precautions must be taken in using it ?

15. A current is passed for 45 minutes through acidulated water and the hydrogen liberated is dried and collected over mercury. If the volume of hydrogen at 68 cm. pressure and 15° C. is 430 cc. and an ammeter through which the current also passes reads 1·2 amp., what is the error of the ammeter ?

CHAPTER LXXI

STATIC ELECTRICITY: ELECTRIC CHARGES

Charge of electricity.—In the previous chapters electric currents have been treated, the name itself implying something flowing along the conductor, to which the name electricity or electric charge is given. Its properties at rest are quite different from those exhibited when it is flowing in the form of a current Thus, there is no magnetic field due to it, no heat produced, and no electrolysis unless it is moving.

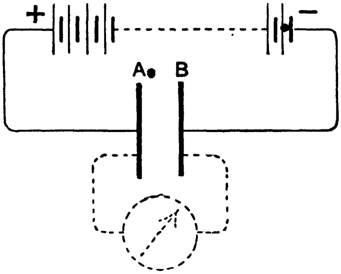


FIG. 854.—Existence of electric charge.

If a great many cells be joined together in series, and the positive pole connected to a conductor A (Fig. 854) and the negative to B, no current flows until A and B are connected together by a conductor. If, however, A and B are joined to the poles of the battery, and the battery connections **broken** and the two **afterwards** connected by a wire, a current will flow through the wire **for a short time**, and may be detected if a delicate galvanometer be in series. For the success of the experiment 100 or more cells are necessary in the battery, and the conductors A and B should be large plates, close together but not touching. Otherwise the current will be too small to be detected.

The explanation is that the battery tries to produce a current, that is, to cause electricity to circulate in the circuit. Since there is no complete circuit, electricity, or electric charge accumulates upon A and B. That on A is said to be positive electricity (+) and that on B negative electricity (−). On breaking the battery connections these charges remain, and on connecting A and B the charges flow through the connecting conductor until they are used up, their flow constituting the current.

Forces acting on charges.—In the above case, the current flows from A to B. This means that the positive charge on A flows through the conductor to B, or the negative charge flows from B to A, or both processes go on. On connecting A and B to the battery, the electromotive force of the battery causes a difference of potential (p. 852) between A and B. Also, we know that the positive charge upon A is acted on by a force driving it towards B, and will travel if a conducting path is provided for it. Similarly, the negative charge on B is urged towards A, and the question arises, can these forces be discovered without joining the two by a conductor ?

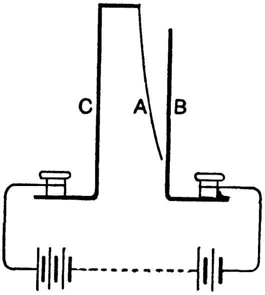


FIG. 855.—Attraction between opposite charges.

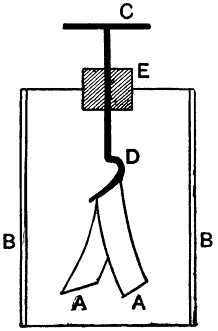


FIG. 856.—Gold-leaf electroscope.

Let B be a fixed piece of metal (Fig. 855) and A a strip of gold leaf suspended from a metal carrier C. Then, on connecting C and B to the battery, it will be noticed that the gold leaf A is displaced towards the fixed conductor. The same thing happens if the poles of the battery are interchanged. Thus A experiences a force urging it towards B, and B a force urging it towards A.

We may thus say that **positive and negative charges attract each other, or that a positive charge experiences a force driving it from points of higher to points of lower potential, and a negative charge experiences a force driving it from points of lower to points of higher potential.** Both statements amount to the same thing, as we shall see later. It is advisable to place a piece of paper or a piece of mica between A and B, so that if they happen to touch no harm will be done.

Electroscope.—There is another convenient form of apparatus for exhibiting the forces on charges, which is commonly used for their detection. Two strips of gold leaf AA (Fig. 856) hang from a wire support CD, which passes through a stopper E in a box. E is made of sulphur or paraffin wax, which is non-conducting. BB are two strips of tinfoil pasted on to the sides of the box, which should have glass windows at the back and front to allow observation of the gold leaves. On connecting C to one pole of the battery and B to the other, a slight divergence of the leaves may be detected, since each of the leaves A is urged towards the nearest strip B, as explained on p. 922. The leaves are driven apart, and we may say if we choose, that the like charges on the leaves repel each other. This apparatus is called a **gold-leaf electroscope,** although the leaves may be made of any thin metallic foil.

Production of electric charges by friction.—Electric charges are nearly always produced when two substances are rubbed together, but if the substances are conducting, the charges escape, and therefore cannot be detected. If, however, a rod of ebonite be rubbed with a piece of fur, both substances being non-conductors, the ebonite will be found to have a negative charge and the fur a positive charge. Both the fur and the ebonite must be thoroughly dry; if they are damp the charges escape. The charges may be detected by bringing the bodies in turn into contact with the cap of an electroscope, when they impart part of their charge to the electroscope and the leaves diverge. Glass rubbed with silk exhibits a similar phenomenon, the glass acquiring a positive and the silk a negative charge. The method of deciding the sign of the charge is given later (p. 934).

Conductors and insulators.—It was seen in Chap. LXVII that substances differ very much in electrical conductivity. Thus the metals are excellent conductors, while wood, cotton, etc., are such poor conductors that it might almost be said that they will not carry a current of electricity. This is not strictly true, but the current they will carry is so minute that it can hardly be detected by any of the means used for detecting a current. Nevertheless, if an electroscope be charged (p. 934) and the cap touched with one end of a wooden rod held in the hand it will be found that the leaves soon collapse, showing that the rod has conducted the charge away. The same result follows if a cotton thread be used instead of the wooden rod.

If, however, a stick of sulphur or paraffin wax be used, the charge will not sensibly leak away through it. Substances which will not allow a charge to pass through them at all are called **insulators.** Thus wood, cotton, the human body, etc., are very feeble conductors, but sulphur, paraffin wax, sealing wax, silk, lead-glass, fused quartz, amber, etc., are insulators, provided that their surfaces are kept clean and dry.

The electricity circulating in a conductor, and constituting the electric current produced by a battery, is identical with that collected on the surface of a body by friction, but whereas the battery can produce larger quantities of charge at low potential, only very small quantities can be produced by friction, but these are at relatively enormous potentials. Evidence of this fact may be found by bringing the hand near the surface of an ebonite rod which has been rubbed with fur. A spark will take place between the hand and the ebonite. This means that the difference of potential between the two is sufficient to cause a current to flow, for an instant, through the air between them, and from our experience of batteries we know that the potential difference required to make electric charge jump an air gap is very great.

Law of force between charges.—We saw on p. 922 that charges of opposite kinds attract each other. Now, in considering the electroscope (p. 922) it was observed that the gold leaves move apart when they have charges of the same sign, and hence we may consider that they repel each other. Thus we may describe the behaviour of charges to each other by saying that **charges of the same kind repel each other, and charges of opposite kinds attract each other.** This is closely analogous to the behaviour of magnetic poles (p. 770) and may be exhibited in a similar manner. Rub a piece of ebonite rod with fur and suspend it in a stirrup by means of a silk fibre (Fig. 857). Bring near it a glass rod rubbed with silk. It will be observed that the glass attracts the ebonite. If the silk be thoroughly dry, it will, after rubbing the glass, be found to repel the ebonite. Thus the charges of like sign repel each other and unlike charges attract each other.

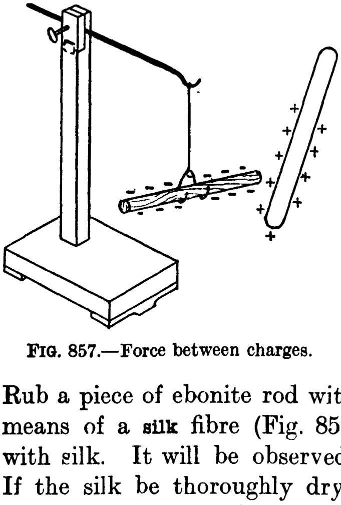


FIG. 857.—Force between charges.

The law of force between charges is also similar to that between magnetic poles. That is, **for two small charges, the force between them varies inversely as the square of their distance apart.**

There is no means of proving the law of force directly by a single experiment. Coulomb, using a torsion balance, proved it very roughly, but this proof is far from satisfactory. The best evidence of the truth of the law is that innumerable calculations based upon it are found to be in accordance with experimentally ascertained facts.

The force between two charges also depends upon their magnitudes, and the treatment of the force between charges may begin with the relation

$$\text{Force} \propto \frac{q_1 \times q_2}{d^2},$$

where q_1 and q_2 are the magnitudes of the charges, and d is their distance apart.

Unit of electric charge.—The above law holds whatever the units in which the charges are measured, but in order to obtain the force in absolute measure, the charges q_1 and q_2 must be given in terms of some definite unit. It is usual to choose the unit of charge so that when the distance between unit charges is 1 centimetre, the force between them is 1 dyne. Hence **the unit charge is such that when placed one centimetre from an equal charge the force between them is 1 dyne.** Thus the force in dynes between any two charges q_1 and q_2 is now given by the relation.

$$F = \frac{q_1 q_2}{d^2} \text{ dynes.}$$

EXAMPLE.—Find the force upon a positive charge of 16 units when placed midway between a positive charge of 10 units and a negative charge of 20 units situated 8 cm. apart.

Force on $+16$ due to $+10 = \frac{16 \times 10}{4^2} = 10$ dynes.

Force on $+16$ due to $-20 = \frac{16 \times 20}{4^2} = 20$ dynes;

$\therefore$ **Total force on charge $+16 = 10 + 20 = 30$ dynes.**

The above definition of unit charge assumes that the charges are situated in a vacuum, and is sufficiently accurate for all ordinary purposes if the changes are situated in air. When situated in other media the force will not be the same (p. 951).

Electric field.—As in the case of magnetism, it is of the greatest convenience to be able to map out the **electric field** at any point (p. 778). The direction of the field at any point is, of course, the direction in which a positive charge situated at that point would be urged. In fact, **the field at any point may be completely defined in magnitude and direction by the force upon unit positive charge situated at that point.** Hence, in order to calculate the strength of field at any point, due to a number of charges, we must imagine a unit positive charge placed at that point, and find the resultant of the forces on it due to the separate charges. The term **electric intensity** is also used to denote this quantity, namely, the force on unit positive charge.

EXAMPLE.—Calculate the strength of field, or electric intensity, at a point situated at equal distances of 5 cm. from two equal charges of 50 units, one of which is positive and the other negative, their distance apart being 8 cm.

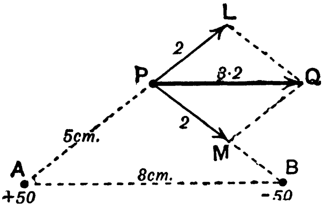


FIG. 858.—Problem on electric fields.

Let P (Fig. 858) be the point at which the electric intensity is required. Imagine a unit positive charge to be situated at P. Then the electric intensity at P, due to +50 units $=50/5^2=2$, and may be represented by PL. Also, that due to -50 is also 2, and may be represented by PM. The resultant PQ is then the required electrical intensity and is obviously parallel to AB. From the similar triangles PLQ and APB,

$$\frac{PQ}{PL}=\frac{AB}{AP},$$

or,

$$\frac{PQ}{2}=\frac{8}{5};$$

$$\therefore\ PQ=\frac{2\times 8}{5}=\underline{3\cdot 2}\text{ dynes.}$$

Lines of force.—Since a positive charge situated in an electric field is subject to a force in the direction of the field, it will, if free to move, travel along some path in the field. The **line along which a free positive charge would move is called an electric line of force.** The whole electric field in the neighbourhood of electric charges may be considered to be mapped out by lines of force, exactly as in the magnetic case (p. 779). For two small equally and oppositely charged spheres the lines of force are as shown in Fig. 859. All the lines of force arise upon the positive charge and end on the

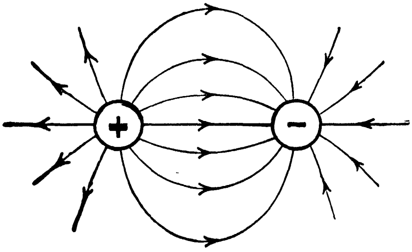


FIG. 859.—Lines of force ; two unlike charges.

negative charge, for a free positive charge would be repelled by the positively charged sphere and attracted by the other. When the spheres are both charged positively the lines of force are as shown in Fig. 860. The lines then all arise upon the spheres. They must, however, end somewhere, and in this case will end upon the walls of the room. The case of a single charged conducting sphere situated in a cubical space is indicated in Fig. 861. It should be noticed that the lines of force always meet the conducting surfaces upon which the charges are situated so that the angles made are right angles.

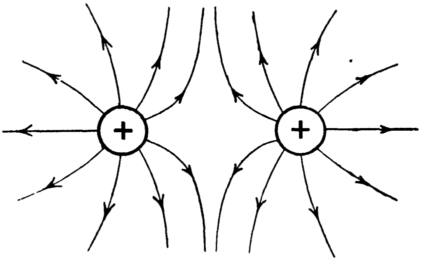


FIG. 860.—Lines of force ; two positive charges.

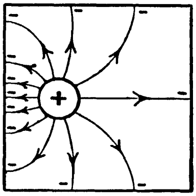


FIG. 861.—Lines of force, positive charge in a conducting enclosure

It will also be seen that where the electric intensity is greatest, that is, near the charges, the lines of force are more densely packed. In fact, the convention of representing the electric intensity by the number of lines of force per square centimetre may be adopted here, as in the case of the magnetic field (p. 811).

EXERCISES ON CHAPTER LXXI.

1. Describe the gold-leaf electroscope, and the method of using it to detect, and to ascertain the nature of, an electric charge.

2. State and explain the law of electric forces between two point charges. Calculate the position of the point in the neighbourhood of two point charges of $+50$ and -18 units situated 40 cm. apart, where a third charge would experience no resultant force. L.U.

3. Two small spheres 30 cm. apart have charges $5q$ and $-3q$. Show the distribution of lines of force, before and after the spheres are connected by a wire ; and calculate the force exerted between them in each case. L.U.

4. Define unit quantity of electricity.

Small spheres carrying charges $+5$, $+10$, $+5$, and -5 units are placed in order at the angular points of a square ABCD of 10 cm. side. Calculate the force on a charge of $+2$ units placed at the intersection of the diagonals of the square. L.U.

5. Equal and opposite charges of electricity are placed at two points l cm. apart. Compare the electric intensity midway between the points with that at a distance r cm. from the centre, (*a*) on the production of the line joining the points, (*b*) on a line through the centre at right angles to that joining the points. L.U.

6. Describe any two experiments to illustrate the essential identity of frictional and current electricity. What are the differences in the two under ordinary conditions ? L.U.

7. Define unit quantity of electricity.

A small conductor carries a charge of 10 units. Show, by means of a graph, the variation of the force which it exerts on a unit charge, as the latter is moved from 1 cm. distance to 10 cm. distance. L.U.

8. Define unit charge of electricity. Two small charged and insulated conducting spheres repel each other with a force of F dynes. Find the repulsion if the charge on one sphere is trebled and the distance between them is also trebled. Allahabad University.

CHAPTER LXXII

POTENTIAL: CAPACITY

Work done in moving a charge.—Whenever a charge of electricity is situated in an electric field it experiences a force, just as a piece of matter in a gravitational field experiences a force. In order to move the charge in opposition to this force, work must be expended upon it, exactly as to lift a body against gravitational force, work is necessary. On the other hand, when the charge is allowed to move in the direction of the force, work is done upon it; similarly, when gravity causes a body to move from higher to lower level, it does work upon it. The analogy between the two cases, gravitational and electrical, is mathematically very close. The law of inverse squares holds in both cases.

Potential.—In the gravitational case, a body when free to move will always move from higher to lower level; in the same way, a positive charge of electricity will, if free to move, travel from a point of higher to a point of lower potential. Potential, then, is analogous to level, and determines the direction in which a charge will travel when free to move. It thus determines the direction of the force upon the charge. Hence, **a positive charge experiences a force tending to drive it from points of higher to points of lower potential, that is, down the gradient of potential.** On the other hand, a negative charge is urged up the gradient of potential, that is, from points of lower to points of higher potential. There is nothing in the gravitational case corresponding to negative electricity.

We define the difference of potential between two points as **the work done in moving a unit positive charge from one point to the other.** Thus, difference of potential is measured in ergs per unit charge. Suppose that a positive charge of 10 units is moved from one place to another where the potential is 20 units higher, work to the amount of $10 \times 20 = 200$ ergs must be done upon it. If the field moves the charge in the reverse direction it will do work to the extent of 200 ergs upon the charge.

Potential due to electric charge.—In most cases it is very difficult to calculate the electrical potential due to a system of charges, but in some cases it is easy. When there is only one charge to consider, we can imagine a unit positive charge carried from one point to another in its neighbourhood, and since we know the force on it at each point, we can find the work done, and therefore the difference of potential between the two points.

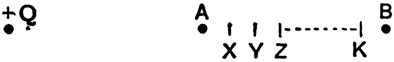


FIG. 862.—Potential due to a charge.

We will find the difference of potential between A and B (Fig. 862) due to the positive charge Q. Call the distance of A from Q, a; from B to Q, b; and so on; then,

$$\text{Force on unit positive charge at A} = \frac{Q}{a^2}.$$

$$\text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad B = \frac{Q}{b^2}.$$

The path from A to B must now be imagined to consist of a number of very short steps, AX, XY, YZ, ... , and KB.

Now for the step AX the force on unit charge at A is Q/a^2 and that at X is Q/x^2. Since these forces are very nearly equal, the step being small, we may, without sensible error, take the average force over the path AX to be Q/ax.

∴ Work done in moving unit charge from A to X

$$= \text{force} \times \text{distance}$$

$$= \frac{Q}{ax}(x-a) = \frac{Q}{a} - \frac{Q}{x}.$$

Similarly, for path XY, work done $= \dfrac{Q}{x} - \dfrac{Q}{y}$.

$$\text{"} \quad YZ \quad \text{"} \qquad = \frac{Q}{y} - \frac{Q}{z}$$

$$\text{etc.,} \qquad \text{etc.,}$$

$$\text{"} \quad KB \quad \text{"} \qquad = \frac{Q}{k} - \frac{Q}{b}.$$

Then, in going over the whole path AB,

$$\text{Work done} = \frac{Q}{a} - \frac{Q}{x} + \frac{Q}{x} - \frac{Q}{y} + \frac{Q}{y} - \frac{Q}{z} + \frac{Q}{z} - \ldots. - \frac{Q}{k} + \frac{Q}{k} - \frac{Q}{b}$$

$$= \frac{Q}{a} - \frac{Q}{b}.$$

However many steps there are in the path from A to B, the only terms remaining in the expression for the total work done in going from A to B are Q/a and Q/b. Hence, if we make the steps infinitely great in number, each step being then infinitesimal, we are justified in writing Q/ax for the average between Q/a^2 and Q/x^2.

Hence **the difference of potential between A and B due to the charge Q is $\frac{Q}{a} - \frac{Q}{b}$.**

There is no means of determining absolute potential, any more than there is of fixing absolute level. For convenience levels are reckoned from mean sea-level. In a similar way we may choose any convenient place to measure potential from. At an infinite distance from the charge q, the force on unit charge due to it is zero, and the charge could be moved about at an infinite distance without doing any work upon it; hence all points at infinity are at the same potential. It is therefore convenient to choose the zero of potential to be that at infinity. The difference of potential between the point A and one at infinity is therefore obtained by putting $b = \infty$, so that

$$\frac{Q}{a} - \frac{Q}{b} = \frac{Q}{a} - \frac{Q}{\infty} = \frac{Q}{a} - 0 = \frac{Q}{a}.$$

Thus, reckoning from the potential at infinity as zero, we should say that the absolute potential at A due to the charge $+Q$ is $+Q/a$.

For some purposes it is convenient to consider the potential of the earth to be zero.

Example.—Find the potential at a point 5 cm. distant from each of two charges of +50 and −50 units respectively. Also find the potential at a point 10 cm. from the charge +50 and 20 cm. from the charge −50.

Referring to Fig. 858.

Potential at P due to $+50 = +\frac{50}{5} = +10.$

" P $-50 = -\frac{50}{5} = -10;$

$\therefore$ potential at $P = 50 - 50 = \underline{0}.$

Again, potential at point 10 cm. from charge $+50 = +\frac{50}{10} = +5.$

" " " 20 " " $-50 = -\frac{50}{20} = -2{\cdot}5;$

$\therefore$ resultant potential $= +5 - 2{\cdot}5$

$= \underline{2{\cdot}5}$ units.

Equipotential surfaces.—For any given distribution of charge there will be a number of points at which the potential is the same. **A surface drawn through points having the same potential is called an**

equipotential surface. The meaning of the term indicates that all points of an equipotential surface are at the same potential.

It follows from the nature of such a surface that a charge may be moved about upon it without the performance of work, since there is no difference of potential between points on the surface. Further, as regards static electricity, **the surface of a conductor must be an equipotential surface.** For if this were not so, there would be a difference of potential between certain points on the surface, and then a current would flow from the point of higher to that of lower potential, and the conditions would thus not be statical.

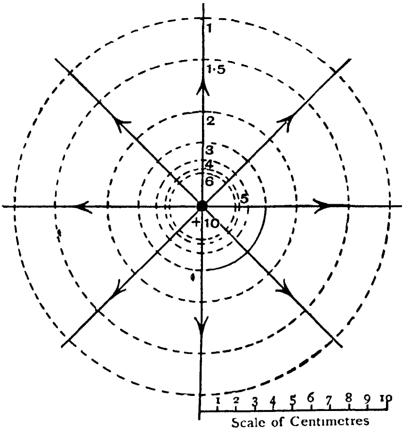


Fig. 863.—Equipotential surfaces and lines of force.

It is easy to find the equipotential surfaces near a single charge. For all points at the same distance from it are at the same potential $(v = Q/r)$, and thus the equipotential surfaces are spheres, having the charge as centre. The equipotential surfaces and lines of force for a charge of +10 units are shown in Fig. 863.

Since no work is done in moving a charge along an equipotential surface, it follows that there is no force on the charge in the direction of the surface. Therefore, **the direction of the electrical field must always be at right angles to every equipotential surface.** If this were not the case there would be a component of the field parallel to the surface. It follows that lines of force always meet conducting surfaces at right angles, as may be seen on examining Fig. 863.

Charging a conductor by influence.—In the earlier experiments with the electroscope (p. 923) this was charged by touching it with the rubbed ebonite rod. There is a better way than this. Let a positively charged body Q (Fig. 864) be brought near to a conductor AB supported upon a glass rod in order to insulate it. The presence of Q raises the potential everywhere in its neighbourhood ; hence if AB were originally at the potential of earth it will now have a positive potential. But the nearer parts A will be at a higher potential than the more distant parts B$(V = Q/r)$. Hence a current will flow from A to B, which means that the end B will acquire an excess of positive charge and A of negative charge. This flow will go on until all parts of AB are brought to the same potential, since when the charges are at rest, the surface of a conductor must be an equipotential surface. Let the conductor be now connected to earth. A current will flow to earth, since the conductor is at a higher potential than earth, owing to the presence of Q. The flow will continue until AB is reduced to the potential of earth (zero) and AB will then retain just sufficient negative charge everywhere to produce a negative potential equal to the positive potential Q, so that the actual potential, being the sum of these, is zero. The distribution of charge will be somewhat as indicated in Fig. 865 (*a*).

Now let the earth connection be broken, and then let Q be removed. The negative charge on AB cannot get away, since AB is now insulated. The charge will distribute itself over the body AB (Fig. 865 (*b*)), which will now have a negative potential due to this negative charge.

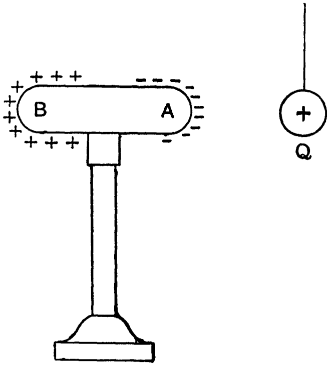


FIG. 864.—Effect of a charged body near a conductor.

Attraction of light bodies.—It is now easy to explain the earliest known electrical phenomenon, namely that a piece of amber rubbed with cloth will attract light bodies, such as small pieces of paper or pith. The amber represents the charged body Q in Fig. 864 and the conductor AB the light body. The presence of the charged body causes a redistribution of charge upon the light body, the charge nearer to Q being opposite in kind to that of Q. The attraction between the unlike charges is greater than the repulsion between the like charges, since these are at greater distance from each other. If the body is then light enough, and free to move, it will, as a whole, be attracted, and move towards Q. The experiment may easily be performed with a rod of ebonite or sealing-wax. The Greek name for amber is **elektron**, and it is from this that the name ‘electricity’ arises.

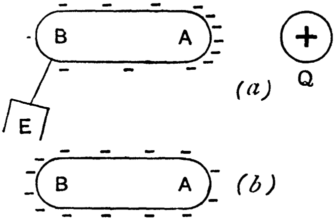


FIG. 865.—Charging by influence.

If Q be a charged conductor, the light bodies after striking it will be repelled from it, since they will then be charged by conduction from Q, and having then the same kind of charge, repulsion results.

Expt. 214.—**The electroscope.** See that the electroscope (p. 922) is perfectly dry. Rub an ebonite rod with fur and bring it near the electroscope. Observe that the leaves diverge. While the rod is still near, momentarily touch the cap of the electroscope with the finger. The leaves collapse because the potential is reduced to zero. Now take away the rod. The leaves diverge because the potential due to the charge upon them is unbalanced by that due to the charge upon the rod. Since the rubbed ebonite has a negative charge, that upon the electroscope is positive.

Thoroughly dry the pieces of fur, silk, and cloth. Also rods of ebonite, sealing wax, glass, shellac, and a brass tube having a glass handle. Take each rod in turn and rub it with each rubber, and in each case test the charge upon the rod and the rubber by bringing each in turn near the electroscope. Note that since the electroscope has a positive charge, its potential is positive, and on bringing a positive charge near it, its potential is raised still further and the leaves diverge more. Bringing a negative charge near it lowers its potential and the leaves collapse.

Tabulate the results as follows:

Rod.	Sign of charge.	Rubbed with	Sign of charge.
Ebonite - -	−	Fur - - -	+
Glass - -		Silk - - -	
Brass - -		Fur - - -	
....	....	....	....
....	....	....	....
Etc.	Etc.	Etc.	Etc.

Faraday's ice-pail experiments.—Several very important facts concerning electrified conductors were established by Faraday, hollow conducting cans (such as ice-pails) being used.

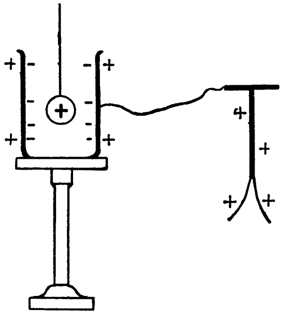


Fig. 866.—**Faraday's ice-pail experiment. Distribution of charge.**

I. The can, or ice-pail, is placed upon an insulating stand and connected to an electroscope. A charged conductor suspended by a silk thread is then lowered into the can. Assuming that the charge is positive, the can and electroscope will now have a positive potential, and the leaves diverge. The distribution of charge is indicated by signs, and follows that indicated on p. 933. When the body is well inside the can, it may be moved about without altering the divergence of the leaves of the electroscope. On removing the body the leaves collapse, as the potential then returns to zero.

If, however, while inside, the body be allowed to touch the inside of the can, no alteration of the leaves is observed; and on removing the body there is still no alteration, the leaves remain diverged and the body, if tested, will be found to have entirely lost its charge. Thus the charge on the body has gone entirely to the can, its own positive charge and the negative charge on the inside of the can having neutralised each other, being, therefore, equal in amount.

Also, since the body, on being removed, after touching the inside of the can, is uncharged, it shows that **no charge resides upon the inside of a hollow conductor.**

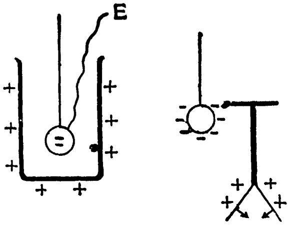


FIG. 867.—Faraday's ice-pail experiment. Distribution of potential.

II. Let the electroscope be disconnected from the can and positively charged, and let a positive charge be also given to the can, as in the last experiment.

On lowering the suspended uncharged body into the can and then connecting it to earth for a moment by touching it by a wire held in the hand, its potential is lowered, from that of the space inside the can, to zero, a current flowing to earth, thus leaving the body negatively charged. This negative charge may be demonstrated by bringing the body near the positively charged electroscope, when the leaves will wholly or partially collapse (Fig. 867).

It can be shown that the charge produced by earthing the body inside the can is always the same, wherever the body may be situated within the can, provided that it is not near the opening. This shows that the potential is the same throughout the whole of the space in the interior of the can, except near the opening. In fact, **the whole of the space within a closed conductor is at the same potential, and this is the same as the potential of the conductor.**

Proof plane.—It has already been seen that when a conductor is hollow, the charge resides entirely upon the outside. This fact may be demonstrated much more satisfactorily by means of **a proof plane.** This is a small metal plate carried upon an insulating handle. It may be made to coincide approximately with the surface, the charge upon which it is required to examine. Two forms of proof plane A and B are shown in Fig. 868. On placing the proof plane upon the surface, it becomes part of the surface and the charge is shared with it. On taking the proof plane away it carries the charge upon it, and on bringing it near a charged electroscope the charge carried may be examined. When using the hollow conductor in Fig. 868 no charge can be taken from A, but charge can be taken from B; thus showing that, except in the neighbourhood of the opening, no charge resides upon the inside of a charged conductor.

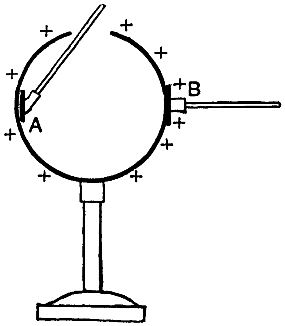


FIG. 868.—Proof plane.

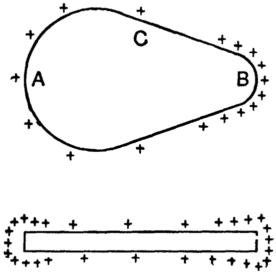


FIG. 869.—Distribution of charge on a conductor.

Distribution of charge on a conductor.—It is only upon a spherical conductor within a concentric conducting space that charge is ever distributed uniformly. In the case of a conductor such as AB (Fig. 869) the distribution of charge is somewhat as indicated. Where the surface has different curvatures at different parts, the accumulation of charge is greater where the curvature of the surface is greater. This may be demonstrated by means of the proof plane, by touching various parts of the surface with it, and examining the charge by means of the electroscope.

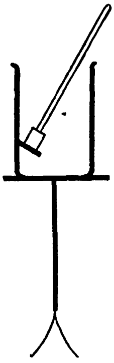


FIG. 870.—Faraday cylinder.

To obtain some idea of the relative magnitudes of the charges from different parts of the surface, it does not suffice merely to bring the charged proof plane near the electroscope. *Its charge must be given up* completely to the electroscope and the divergence of the leaves noted. For this purpose a can, or hollow vessel, is placed upon the cap of the electroscope

(Fig. 870) and the proof plane with its charge put inside the can, contact with the can being made. It is then certain that the whole charge is given up to the can and electroscope (p. 935). This vessel is sometimes known as the **Faraday cylinder**. If the experiment be performed after touching the conductor (Fig. 869), first at A, then at B, and then at C, it will be found that the surface density of the charge is greatest at B and least at C.

Discharge from points.—Since the density of the charge on a conductor is greater the greater the curvature, it will be seen that at a point the density should become infinite. An absolute point cannot, of course, be obtained, but at a fairly sharp point the density of the charge does become very great. In the neighbourhood of a great density of charge the electric field becomes very great (p. 955). Now air is, as a rule, non-conducting, but for a very strong electric field this ceases to be the case, and the air becomes a conductor (Chap. LXXX). Hence the charge in this strong field passes away from the point, the carrier of the charge being the air itself, which streams away from the point. This stream is sometimes called an '**electric wind.**' It may be exhibited by connecting a pointed wire to the conductor of an electrical machine (p. 960), which produces a quantity of charge rapidly. The air now streams from the point and may be detected by a candle flame near the point (Fig. 871). A further illustration of this may be given by making four pointed wires into a little wheel (Fig. 872) with the points directed all the same way. On balancing the wheel on a needle point and charging it strongly by an electrical machine, the air streams from the points. The momentum given to the air has its counterpart in momentum given to the wheel, which thus rotates as shown.

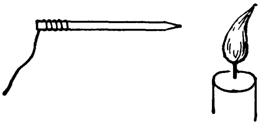


FIG. 871.—Discharge from a point.

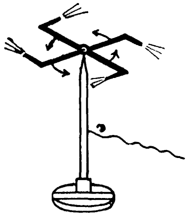


FIG. 872.—Demonstration of electric wind.

Capacity.—Since the presence of an electric charge changes the potential at all points in its neighbourhood, it is obvious that, when a positive charge is placed upon a conductor, its potential is raised. Similarly, a negative charge placed upon a conductor lowers its potential. Unless otherwise stated, it will be assumed that the charge placed upon a conductor is positive.

In certain cases it is possible to calculate the change of potential of a conductor produced by placing a given charge upon it. But when the conductor has not a simple geometrical form the potential due to a given charge can only be determined by experiment. However, in all cases there is a definite relation between the charge and the potential produced by that charge, and we shall call the ratio of the charge placed upon a conductor to the change in the potential produced by that charge, the **capacity** of the conductor.

Thus, if C is the capacity, Q the charge, and V the potential of the conductor due to that charge, then

$$C=\frac{Q}{V} \quad \text{.................................} \quad (1)$$

Capacity of a sphere.—Imagine a sphere, situated at a considerable distance from all other conductors. A charge placed upon it will be uniformly distributed over it. In this case the strength of field at any point outside may be calculated on the assumption that the charge is all concentrated at the centre of the sphere (p. 813). Thus, in Fig. 873, if the charge Q be uniformly distributed over the sphere whose centre is at O, the electric intensity at P is Q/OP^2; exactly as though the charge Q were all concentrated at O.

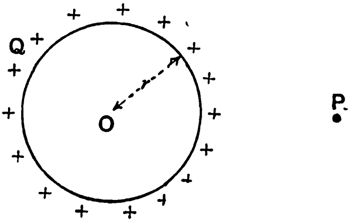


FIG. 873.—Charged sphere.

It follows from the reasoning on p. 930 that the potential at P is Q/OP. Also the distance of the surface of the sphere from the centre is r, the radius of the sphere. Hence the potential at the surface is Q/r. The capacity of the sphere is therefore given by

$$\text{Capacity}=\frac{\text{charge}}{\text{potential}}=\frac{Q}{\frac{Q}{r}}=r. \quad \text{..........................} \quad (2)$$

Thus **the capacity of a sphere is numerically equal to its radius,** and since the centimetre is the unit of length for all scientific purposes, the capacity of a sphere is numerically equal to its radius in centimetres.

Concentric spheres.—It was seen on p. 935 that when a charged conductor is situated within a hollow conductor, an equal charge of opposite kind is situated upon the inner surface of the hollow conductor. Thus, if a charge $+Q$ be situated upon the sphere A (Fig. 874) of radius a cm. there will be produced a charge $-Q$ upon the inner surface of the concentric sphere B of radius b. This sphere being earthed, its potential is, of course, zero. In fact, the charge upon B must be such that the potential is zero. Now the potential of B due to the charge $+Q$ upon A is $+Q/b$, and the potential due to its own charge is $-Q/b$, the resultant potential of B being zero.

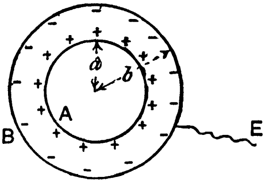


FIG. 874.—Concentric spheres.

Again, the potential of A due to the charge $+Q$ upon it is $+Q/a$. Remembering that the potential within a closed conductor due to the charge upon it is uniform and equal to that of the conductor (p. 935) we see that the potential inside B due to the charge $-Q$ upon it is $-Q/b$.

Hence, resultant potential of $\mathrm{A}=\dfrac{Q}{a}-\dfrac{Q}{b}$

$$=Q\left(\frac{b-a}{ab}\right).$$

Now charge upon A is Q; hence, by definition

$$\text{Capacity of A}=\frac{Q}{Q\left(\dfrac{b-a}{ab}\right)}=\frac{ab}{b-a}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Thus the effect of surrounding a sphere by an earthed concentric sphere is to increase its capacity from a to $\dfrac{ab}{b-a}$. This may be written $\dfrac{a}{1-a/b}$, which is very nearly equal to a if b is very great; but as b becomes smaller the capacity increases until, when b is very nearly equal to a, the capacity becomes very great.

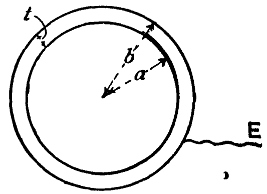


FIG. 875.—Nearly equal spheres.

Another way of expressing the capacity when the spheres are of nearly equal radius is of importance. The thickness of the air space is $b-a=t$ (Fig. 875).

Then, $\qquad \text{Capacity of sphere}=\dfrac{ab}{b-a}=\dfrac{ab}{t}.$

When b is very nearly equal to a, a^2 may be written for ab.

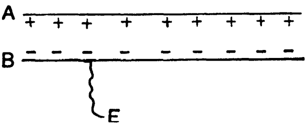
$$\text{Capacity} = \frac{a^2}{t}.$$

But the whole area of the sphere is $4\pi a^2$;

$$\therefore \text{ Capacity per square centimetre} = \frac{a^2}{t} \cdot \frac{1}{4\pi a^2}$$

$$= \frac{1}{4\pi t}. \quad \text{.................................(4)}$$

Parallel plates.—Equation (4) only becomes strictly valid when the radius of the sphere is infinite. But in this case the surface is plane. Hence the capacity per square centimetre of a plate A, at a distance t from an earthed parallel plate B, is $1/4\pi t$, and for an area of A square centimetres,



FI . 876.—Parallel plates.

$$\text{Capacity} = \frac{A}{4\pi t}. \quad \text{........................(5)}$$

Of course the plates must be of such a great extent that the area considered is not near the edges. Near the edge of any actual pair of parallel plates the charge is not distributed uniformly and the capacity is no longer given by the above simple expression.

It will be noticed that the capacity of the insulated plate increases, the nearer the earthed plate is brought to it. This may be demonstrated by connecting the insulated plate A (Fig. 877) to the electroscope and charging it. The leaves diverge to an extent corresponding to the potential of A. On bringing the earthed plate B nearer to A the divergence of the leaves will decrease, showing that the charge upon A no longer raises it to such a high potential as before. Thus the capacity of A must have increased. On removing B to its original distance the leaves diverge again to their first position, showing that the potential has again risen and the capacity therefore decreased. The arrangement is called a **condenser**.

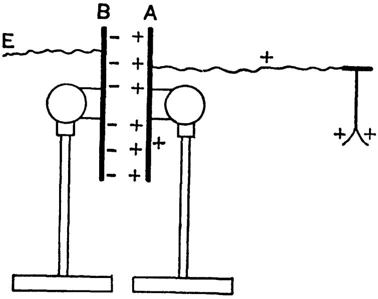


FIG. 877.—Plate condenser.

Leyden jar.—A convenient form of conductor having large capacity may be made by coating a glass jar A (Fig. 878) with a layer of tinfoil C inside, and another layer B outside. These layers form approximately parallel sheets, of which C is insulated and B is earthed by standing on the table or being held in the hand. D is a wire conductor which makes contact with C, and is provided with a knob for making contact with external bodies. The capacity of C is much greater than if there were no earthed parallel conductor B. On touching D to the conductor of an electrical machine (p. 961) C will acquire a considerable charge, and on short circuiting D and B by a pair of discharging tongs held by a glass handle, a considerable spark will occur when the air gap becomes small enough. Both the glass jar A and the glass handle of the discharging tongs should be varnished with shellac varnish in order to render them good insulators. The apparatus is called a **Leyden jar.**

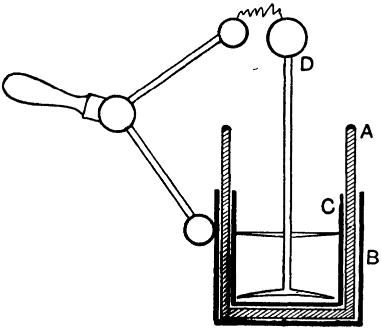


FIG. 878.—Leyden jar.

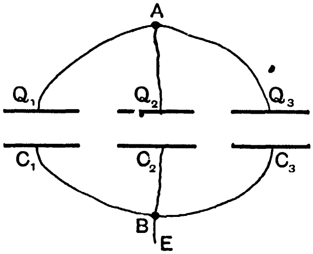


FIG. 879.—Condensers in parallel.

Condensers in parallel.—Let a number of condensers, C_1, C_2, C_3, etc., be joined in parallel between two points A and B (Fig. 879). To find the resulting capacity, imagine a charge Q_1 situated upon the insulated plate of C_1, Q_2 upon that of C_2, etc. Then the potential of A is $Q_1/C_1 = Q_2/C_2 = Q_3/C_3 = V$, these three potentials being the same, since the insulated conductors are joined together;

$$\therefore \quad Q_1 = C_1V, \quad Q_2 = C_2V, \quad \text{and} \quad Q_3 = C_3V.$$

Now the total charge Q is

$$Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V = Q,$$

$$\therefore \quad \frac{Q}{V} = C_1 + C_2 + C_3.$$

But $\frac{Q}{V}$ is the resultant capacity C;

$$\therefore \quad C = C_1 + C_2 + C_3. \quad \text{..............................(1)}$$

Thus, **when condensers are joined in parallel the resulting capacity is the sum of the separate capacities.**

Condensers in series.—Let the condensers be connected in series, as in Fig. 880, the last plate of the last condenser being earthed. Since the opposite plates of any condenser have equal and opposite charges, when charge $+Q$ is given to A, B will have charge $-Q$. But B and C together are insulated from other conductors, and their total amount of charge, if zero at first, must remain zero. Therefore, charge on $C = +Q$. Similarly, that on D or M is $-Q$, and that on E, $+Q$. Now,

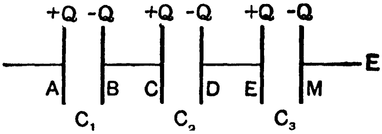


FIG. 880.—Condensers in series.

Difference of potential between A and B $= \dfrac{Q}{C_1}$.

" " " C and D $= \dfrac{Q}{C_2}$.

" " " E and M $= \dfrac{Q}{C_3}$;

∴ Total difference of potential between A and M

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = V.$$

But if C is the resultant capacity between A and M,

$$V = \frac{Q}{C};$$

$$\therefore \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3},$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{.................................(2)}$$

Therefore, **for condensers in series (sometimes said to be in cascade) the reciprocal of the resultant capacity is the sum of the reciprocals of the separate capacities.**

Energy of charge.—From the occurrence of a spark when the Leyden jar is discharged, it may be inferred that the jar has energy stored in it when charged. In fact, whenever a conductor is discharged, work is being done, a strict measure of which is the energy dissipated in the form of heat, or otherwise, when the current flows. It is possible to calculate the energy associated with any charge when we know the potential at the place occupied by the charge.

Let us imagine that a conductor is at first at zero potential and without charge. From the definition of potential (p. 929) it will then be seen that to bring up a very small charge from infinity and put it on the conductor requires an amount of work depending upon the potential of the conductor. It is zero if the potential is at first zero. But after placing the charge upon the conductor its potential will no longer be zero. If the charge upon the body and the potential be plotted in the form of a graph (Fig. 881), then to bring an additional small charge q, when the potential is v, the work is $q \times v$, and is represented by the rectangular strip of height v and width q. Continuing this process for the whole increase of charge from zero up to Q, the work done is represented by the sum of all such strips as qv, and is the area OAB when the strips are made sufficiently narrow. Since the potential is proportional to the charge, OB is a straight line, and therefore the total work done is $\frac{1}{2}$OA × AB. But if OA is the final charge Q, and AB the final potential V,

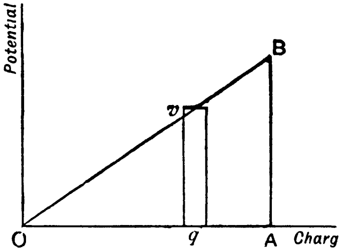


FIG. 881.—Energy diagram.

$$\text{Work done, or energy of charge} = \tfrac{1}{2}\mathrm{QV} \text{ ergs.}$$

Remembering that $\mathrm{Q} = \mathrm{CV}$, we have

$$\text{Energy of charge} = \tfrac{1}{2}\mathrm{QV} = \tfrac{1}{2}\mathrm{CV}^2 = \tfrac{1}{2}\frac{\mathrm{Q}^2}{\mathrm{C}} \text{ ergs.}$$

This energy is available to produce the current when the conductor is discharged. In most cases it appears entirely in the form of heat in the wire carrying the current, or in the case of a spark discharge, owing to the high temperature, light and also sound may be produced.

Loss of energy on sharing charge between two conductors.—Let the charge Q be situated upon a conductor A of capacity C_1. Then on connecting A to a second conductor B of capacity C_2, the charge becomes shared between A and B by some of it flowing from A to B. At first the potential of A was Q/C_1, but when connected to B the potential is $\mathrm{Q}/(\mathrm{C}_1+\mathrm{C}_2)$, which is less than Q/C_1. Hence the same charge Q is situated upon a conductor of less potential than previously, and the energy must therefore be less. The energy has been used up in driving the charge through the conductor connecting A and B, which is, of course, heated in the process.

Practical unit of capacity (the farad).—In practice it is useful to have a unit of capacity founded upon the volt and the coulomb.

Thus, a capacity such that a charge of 1 coulomb raises the potential to 1 volt is said to be **1 farad**. The farad is rather large for ordinary purposes, so that the standards usually constructed for laboratory purposes are one-millionth of a farad. This is called a micro-farad ;

$$\therefore \ 1 \text{ farad} = 10^6 \text{ micro-farads.}$$

The farad is founded upon the electro-magnetic system of units, being derived from the volt and ampere. There is, however, a relation between this unit and the electrostatic unit of capacity, which cannot be discussed here. Suffice it to say that

$$1 \text{ farad} = 9 \times 10^{11} \text{ electrostatic units of capacity.}$$

EXERCISES ON CHAPTER LXXII.

1. Describe a Leyden jar and explain its action.

If you were given two Leyden jars, a means of charging them at a constant potential, and a gold-leaf electroscope, how would you determine which jar has the greater capacity ? Sen. Camb. Loc.

2. An insulated sphere of radius 26 cm. is surrounded by a concentric earthed sphere of radius 30 cm. Find the charge required to raise the potential of the insulated sphere to 30 units, and also the energy of the charge.

3. How would you show that there is no electric charge on the inner surface of a charged insulated hollow conductor ?

A conductor A has a capacity of 10 and a potential of 50 ; another conductor B is of capacity 6 and potential 65. Calculate the charges on A and B after they have been connected by a long thin wire.

4. Describe how it may be shown experimentally that there is no free charge on the inner surface of a hollow charged conductor.

Two equal spheres of water, having equal and similar charges, coalesce to form a larger sphere. If no charge is lost, how will the surface density of electrification change ? L.U.

5. What is meant by the electrical capacity of a condenser, and on what does it depend.

Two Leyden jars of capacity C_1 and C_2 are respectively connected (*a*) in parallel, (*b*) in series ; find the resultant capacity in each case. L.U.

6. Two concentric metal spheres are insulated from earth and from one another and a charge of $+e$ is given to the inner sphere. What will be the electrical condition of the outer sphere ? How will it be changed (*a*) by connecting the outer sphere to earth momentarily, and (*b*) by afterwards connecting the inner sphere to earth ? L.U.

7. Explain how the distribution of a charge of electricity over the surface of a conductor depends on its shape, and describe how you would verify your statements.

Explain the action of the pointed rods used as collectors on electrical influence machines. L.U.

8. Show how the energy of a charged condenser depends on its charge and the potential difference between the plates.

What is the energy of a sphere of radius r when charged with Q units of electricity? L.U.

9. Define electrostatic potential.

Obtain an expression for the potential at a point due to a spherical conductor charged with q units of electricity.

Two small conductors 30 cm. apart have positive charges of 10 and 20 units respectively. Calculate (*a*) the force, (*b*) the potential, at a point midway between them. L.U.

10. Define the term "capacity of a conductor."

Calculate the capacity of an insulated conducting sphere 24 cm. in diameter, surrounded by a concentric earthed sphere 26 cm. in diameter.

What is the energy of a charge of 5 electrostatic units communicated to the insulated sphere? L.U.

11. Define the electrostatic and the electromagnetic units of capacity.

Three equal condensers joined in parallel and connected to a cell of e.m.f. 2 volts produce 1 8 microcoulombs on discharge through a galvanometer. What would they produce if joined in series or cascade? What is the capacity of each condenser? L.U.

12. A condenser whose capacity is 3020 units has a potential of 35 units. Calculate the potential when this condenser is connected to another of capacity 4530 units.

13. The insulated conductor of a spherical condenser A has a charge of +40 units and that of a similar condenser B has a charge of +25 units. The spheres of A have radii 18 cm. and 20 cm. and those of B 45 cm. and 50 cm. Find the direction of flow of the current on connecting the insulated sphere of A to that of B.

14. In Question 13, calculate the total energies of the charges before and after connecting the insulated spheres together.

15. A parallel plate condenser has area 35 sq. cm., a charge of 50 units, and the distance between its plates is 3 mm. Calculate the energy of the charge.

CHAPTER LXXIII

THE ELECTROMETER: DIELECTRICS

Gold-leaf electroscope used as an electrometer.—An **electrometer**, as its name implies, is an instrument for measuring electrical potential, whereas the ordinary gold-leaf electroscope as usually employed will only indicate a rise or a fall of potential, without measuring it. The electroscope, however, has been made into an instrument capable of fair accuracy for measuring potentials by Mr. C. T. R. Wilson.

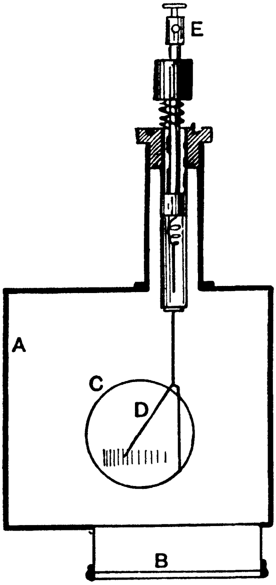


FIG. 882.—The Wilson electroscope.

The leaf of gold, or aluminium D, is supported by a wire (Fig. 882) and connection can be made between it and the conductor E by means of a piece of wire carried by a spring. On depressing E, the wire comes into contact with the wire supporting the leaf, making metallic connection. The tube carrying the conductors is insulated by an ebonite plug. The leaves are surrounded by the brass box A, having a circular window through which they are observed by a short-focus telescope having a finely divided scale in the eyepiece. The position of the image of the leaf on the eyepiece scale indicates the potential of the leaf. It is necessary, however, to calibrate the scale by applying known potentials and noting the positions of the leaf, before any readings of potential can be made by the instrument.

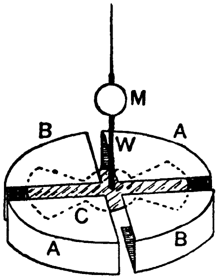


FIG. 883.—The quadrant of the electrometer.

Quadrant electrometer.—For accurate measurement of small potentials the **quadrant electrometer** is much superior to the gold-leaf electroscope. This consists of four hollow quadrants AABB (Fig. 883) with a paddle-shaped conductor C (sometimes called the 'needle') hanging within them. This paddle is carried by a wire support W to which a small concave mirror is attached, the whole being suspended by a quartz fibre. This arrangement enables the deflection of the paddle to be observed, exactly as in the case of the reflecting galvanometer (p. 864). In Fig. 884 the arrangement of a quadrant electrometer is shown. The quadrants Q are supported upon amber blocks A, and two of the quadrants are drawn back so that the paddle inside them can be seen. F is the quartz fibre and M the mirror. K is a conductor through which the paddle can be charged before any measurements are made.

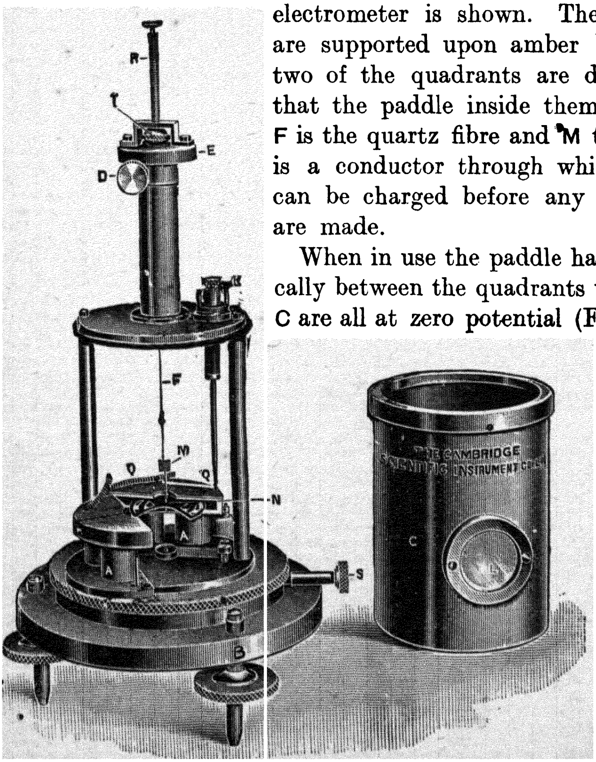


FIG. 884.—The quadrant electrometer.

When in use the paddle hangs symmetrically between the quadrants when A, B and C are all at zero potential (Fig. 883). The quadrants AA are connected together by a wire so that they must always have the same potential, which we shall call V_A; similarly, BB are connected together, and their potential will be called

V_B. It can then be shown that with these potentials the needle will be deflected from its zero position by an amount θ, where

$$\theta \propto (V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right),$$

V_C being the potential of the paddle. In practice V_C is always a much higher potential than V_A or V_B, so that when these are small the term $\left(V_C - \frac{V_A + V_B}{2}\right)$ is practically constant, and we may write

$$\theta = K\ (V_A - V_B),$$

where K is now a constant, to be determined by experiment. It is thus seen that **the deflection is proportional to the difference of potential between the pairs of quadrants.**

That the needle will undergo a deflection may be seen from Fig. 885. Suppose the paddle to be positively charged and the pair of quadrants AA positively and BB negatively charged. It will be noticed that the upper half of the paddle in the figure will experience a force driving it towards the left, and the lower half a force driving it towards the right. This is equivalent to a couple, and the paddle will rotate until the opposite couple due to the twist in the suspension fibre brings it to rest.

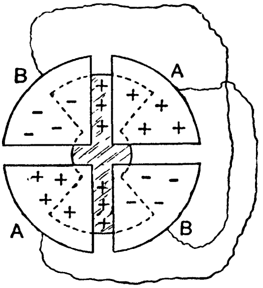


FIG. 885.—Charges on the quadrants.

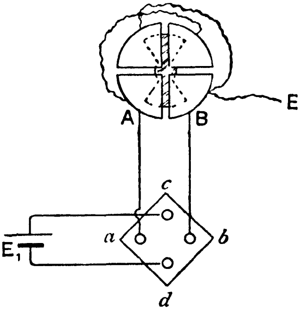


FIG. 886.—Use of the electrometer.

Note that if the quadrants B are connected to earth they may be taken to be at zero potential, and then the deflection is proportional to the potential of A, thus $\theta = KV_A$.

EXPT. 215.—**Comparison of e.m.f.'s of cells.** Connect the case of the electrometer to earth by means of a piece of copper wire bound round one of the supply water-pipes in the laboratory or outside. Connect the A quadrant to (*a*) one of four holes (*a*, *b*, *c* and *d*) (Fig. 886) drilled in a piece of paraffin wax and filled with mercury. Connect B to *b*, taking care that in each case the connecting wires are well insulated. Connect the paddle of the electrometer for a moment to one terminal of a 100 volt supply, the other terminal being earthed. These connections should only be made by the student under supervision. Now let B be also connected to earth. To adjust the instrument, join *a* and *b* by a short wire connection. All the quadrants are now earthed. The instrument should now be levelled and the spot of light adjusted to zero by turning the suspension head. When all is ready, join one of the cells, E_1, whose e.m.f.'s are to be compared, to *c* and *d*. Place connectors to join *a* and *c* and also *b* and *d*, and observe the deflection. Now remove the connectors to join *a* and *d*, *b* and *c*. The deflection is reversed and should be again noted.

Replace the cell E_1 by another E_2 and repeat the observations. Tabulate the results as follows :

Name of cell.	Deflection to right.	Deflection to left.	Mean deflection (θ).	E.m.f. of cell.

Then, $E_1/E_2 = \theta_1/\theta_2$. If one of the cells be a standard, the e.m.f. of the other can be calculated.

Electrostatic voltmeter.—For certain purposes the needle of the electrometer is connected to one pair of quadrants, and has then the potential of that pair. Let it be connected to the A quadrants. Then $V_C = V_A$, and the expression for the deflection,

$$\theta \propto (V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right)$$

becomes

$$\theta \propto (V_A - V_B)^2.$$

The points between which it is required to find the difference of potential are then connected to A and B and the deflection is proportional to the square of the difference of potential. Moreover, since the deflection is proportional to the square, it is in the same direction whether $V_A - V_B$ is positive or negative. Hence, since the deflection is always in one direction, the instrument may be used on alternating-current circuits. If the scale be calibrated to read volts, then on an alternating-current circuit it reads **virtual volts.**

A quadrant electrometer is not mechanically strong enough to be used as a voltmeter. It is therefore generally made with a stout suspension, and to obtain sufficient sensitiveness, the axle (Fig. 887) has a number of paddles attached, which hang in the spaces between a number of quadrants. A pointer is attached, which moves over a scale calibrated in volts. Such an instrument is usually called a **multicellular electrostatic voltmeter.**

One great advantage of the electrostatic voltmeter lies in the fact that on a continuous-current circuit no current flows in the instrument. In this respect it is an ideal voltmeter (p. 875).

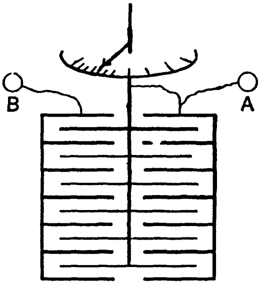


FIG. 887.—Electrostatic voltmeter.

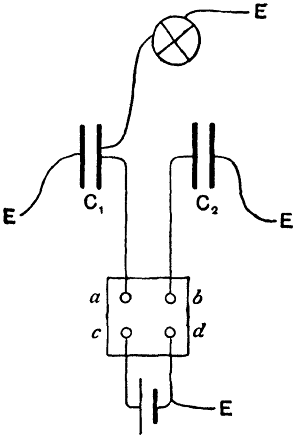


FIG. 888.—Comparison of capacities.

Comparison of capacities.—There are many ways of comparing capacities, but the following is one of the simplest. On giving a charge $+Q$ to the condenser of capacity C_1 (Fig. 888) its potential V_1 is Q/C_1. Now let the second condenser of capacity C_2 be connected to the first, so that the charge is shared between them. Their potentials must now become the same, a charge $+q$ passing from the first to the second condenser. This lowers the potential of the first from V_1 to V_2, the final common potential;

$$\therefore\ V_1 - V_2 = \frac{q}{C_1}.$$

At the same time the potential of the second condenser is raised from zero to V_2;

$$\therefore\ V_2 = \frac{q}{C_2};$$

$$\therefore\ C_1(V_1 - V_2) = C_2V_2,$$

or,

$$\frac{C_1}{C_2} = \frac{V_2}{V_1 - V_2}.$$

If the potentials V_1 and V_2 are measured by means of the quadrant electrometer, the ratio of the capacities can thus be found.

EXPT. 216.—**Comparison of capacities.** Set up the electrometer as in Expt. 215. Connect the insulated plate of each condenser to one terminal *a*, *b* of the key, the other two terminals, *c* and *d*, being joined to a cell. The earth connections are shown in Fig. 888. Connect *a* and *c*, thus charging the condenser C_1 to potential V_1, and observe the electrometer deflection θ_1, while terminal *b* is earthed. Insulate *b* and remove the connector from *c* to *b*, so that the charge in C_1 is shared with C_2, and again note the deflection θ_2. Then, since the deflections are proportional to the potentials,

$$\frac{C_1}{C_2}=\frac{V_2}{V_1-V_2}=\frac{\theta_2}{\theta_1-\theta_2}.$$

Dielectrics.—In all the discussions so far, it has been considered that the charged conductors have been surrounded by air. The air, however, might be replaced by any non-conducting medium without altering the distribution of the charges. If the medium between the charges were conducting, the charges would at once move in the direction of the electrical field at each point, and would continue to move until the potential throughout the whole of the medium became uniform. Thus there would no longer be any electrical field in the medium. Hence there cannot be any statical condition involving an electrical field in a conductor. But when the charges are separated by a non-conductor the charges cannot move and the electrostatic field may therefore persist permanently in the medium. For this reason non-conductors are frequently called **dielectrics**, or media in which there can be an electric field, without the charges being caused to move.

Dielectric constant.—The definition of unit charge and the law of force between charges (p. 925), are, strictly speaking, only valid when the charges are situated in a vacuum. But for most practical purposes, the replacing of vacuum by air would not produce an appreciable difference. While true for most gases besides air, this is certainly not true for liquid or solid dielectrics.

If two charges, q_1 and q_2, be immersed in a dielectric, such as, say, paraffin oil, the force between them will be less than in air, and must be represented by the modified equation :

$$\text{Force}=\frac{q_1q_2}{kd^2}\text{ dynes},$$

where k is some number depending upon the nature of the medium.

This quantity (k) is called the **dielectric constant,** or **specific inductive capacity** of the medium. For solid paraffin the dielectric constant is about 2·3, and therefore, for charges immersed in it, the force between them is $q_1q_2/2{\cdot}3d^2$ dynes. The following is a table of the dielectric constants of several important substances :

Substance.	Dielectric constant (k).
Ordinary glass - -	8·45
Plate glass - - -	4·67
Ebonite - -	3·15
Sulphur - - -	3·84
Mica - - - - -	6·64
Paraffin (solid) - -	2·3
Petroleum - - -	2·0
Water - - - - -	80
Air (at 76 cm. pressure) -	1·0006
Hydrogen (76 cm. press.)	1·0003

Effect of dielectric upon potential.—On referring to p. 930 it will be seen that the potential at any point due to a charge + Q is obtained by finding the work done in transferring a unit charge from infinity to the point. In order to find the work done in any small step of this transference, the average force is multiplied by the distance. When the dielectric is other than air or vacuum, the distances are, of course, all unchanged, but every force is diminished in the ratio $1/k$; that is, it becomes $1/k$ of what it was for air. Hence the average force for the step Ax (Fig. 862) becomes Q/kax instead of Q/ax, and the work becomes

$$\frac{Q}{kax}(x-a)=\frac{Q}{ka}-\frac{Q}{kx}.$$

This factor $1/k$ enters into every term in the series, the sum of which gives the work done. Therefore

$$\text{Work done}=\frac{Q}{ka}-\frac{Q}{kb}.$$

Considering B to be at infinity, the potential at A is Q/ka. Hence **the effect of changing air for a dielectric of constant k, is to reduce the potential to 1/k of its previous value.**

Effect of dielectric upon capacity.—It is now easy to see that the dielectric surrounding a conductor affects its capacity. For on putting a charge + Q upon the conductor, the potential at any point with air as dielectric would be, say, V, but with the dielectric of constant k instead of air the potential is V/k. Thus the capacity will, by definition (p. 938), be $Q \div V/k = kQ/V$. That is, **it is k times the capacity with air as dielectric.** It follows that

$$\textbf{Capacity of sphere} = kr;$$

$$\textbf{Capacity of concentric spheres} = \frac{kab}{b-a};$$

$$\textbf{Capacity of parallel plates} = \frac{kA}{4\pi t}.$$

In the case of the sphere, the dielectric must extend uniformly to a great distance, practically infinity, all round the sphere. On the other hand, for the concentric spheres and the parallel plates, the dielectric need only occupy the space between the earthed plate and the insulated plate; because this is the only region occupied by the electric field under consideration. It is only the region occupied by the electric field that need be filled with the dielectric.

EXPT. 217.—**Effect of the dielectric upon capacity.** Connect one of two insulated parallel plates A (Fig. 889) to the gold-leaf electroscope and charge it. Earth the other plate B. The divergence of the leaves now corresponds to the potential due to the charge upon A. Now introduce a slab of glass or ebonite between the plates. Note that the leaves partially collapse. This shows that the potential of A has fallen; the charge upon it does not now raise it to the original potential, so that its capacity must have increased. Remove the slab and note that the leaves return to their original divergence. The effect of introducing the slab of dielectric is therefore to increase the capacity of the condenser.

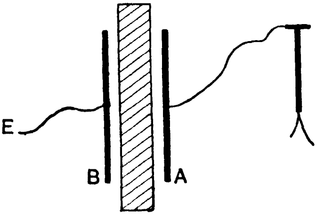


FIG. 889.—Condenser with dielectric slab.

Capacity of Leyden jar.—If the glass of the Leyden jar (Fig. 878) be not very thick, the inner and outer coatings may be considered to be parallel plates. Let the total area of the inner tinfoil be A sq. cm. and the thickness of the glass be t cm., then the capacity of the jar is $kA/4\pi t$. Taking the dielectric constant of the glass as 8, the capacity will be $8A/4\pi t$. Thus, with an area of inner tinfoil of 750 sq. cm. and a thickness of glass 2 mm., the capacity would be $\frac{8 \times 750}{4\pi \times 0{\cdot}2} = 2380$ units approximately. Thus the capacity would be equal to that of a sphere of radius 2380 cm., or 47·6 metres diameter.

EXAMPLE.—Find the capacity of a condenser consisting of two rectangular sheets of tinfoil 30 cm. × 20 cm. separated by a sheet of mica 0·2 mm. thick, taking the dielectric constant of mica as 6·6.

$$\text{Capacity} = \frac{k\text{A}}{4\pi t} = \frac{6{\cdot}6 \times 30 \times 20}{4\pi \times 0{\cdot}02} = \underline{15750} \text{ units.}$$

Measurement of dielectric constant.—From the effect of the dielectric upon the capacity of a condenser (p. 953) it is possible to derive a method for measuring dielectric constants. If the capacity C of a condenser, having air as dielectric, can be measured, and then the capacity C_1 with some other medium as dielectric, we have

$$C_1 = kC, \quad \text{or} \quad k = \frac{C_1}{C},$$

where k is the dielectric constant of the medium. It is not necessary to measure the capacities in absolute measure; the ratio of the capacities with the medium and with air as dielectric only need be found. Faraday was the first to employ this method, and by means of it he investigated the properties of several insulators. Two spherical condensers, AB, A′B′ (Fig. 890), as nearly as possible alike, are employed. The two insulated conductors A and A′ are first joined together and charged. Since they must now be at the same potential, they will have equal charges if the capacities are the same. On insulating B and B′ and connecting A′ to B and A to B′ the respective positive and negative charges neutralise each other if they are equal, so that no charge remains. If the remaining charges are zero then the two capacities must be equal. The space inside A′B′ is now filled with the dielectric shellac, or sulphur, etc., which is poured into the space in a molten condition. The ratio of the capacities C of AB and C′ of A′B′ is now measured as on p. 950. The value C_1/C is the dielectric constant k.

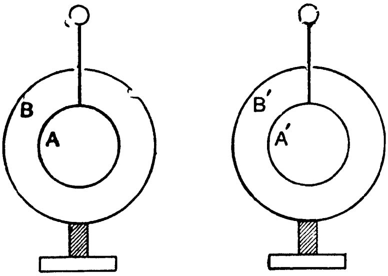


FIG. 890.—Faraday's method of measuring k

Faraday only half filled A′B′ with the dielectric, in which case

$$C' = \frac{C}{2} + \frac{kC}{2} = \frac{C}{2}(1+k); \qquad \therefore\ k = 2\frac{C'}{C} - 1.$$

The quadrant electrometer was not invented until after Faraday's time. He employed a calibrated electroscope similar to the gold-leaf electroscope, but having two light suspended pith-balls instead of the gold leaves. In this way he found, for shellac, $C_1 = 1{\cdot}5C$;

$$\therefore\ k = (2 \times 1{\cdot}5) - 1 = 2.$$

Parallel-plate method for measuring k.—For solid substances Faraday's method is not applicable unless the substance has a low melting point. For glass, ebonite, etc., the parallel-plate method is necessary. The principle of Expt. 217 is employed. Before this can be followed, it is necessary to find the capacity of a parallel-plate condenser partially filled with dielectric. Let A be the insulated and B the earthed plate (Fig. 891). Let t be the distance apart of the plates, and h the thickness of the slab of dielectric introduced between them. On p. 940 it was seen that the capacity of the condenser without the slab is $A/4\pi t$; and if $+\sigma$ and $-\sigma$ are the amounts of charge per square centimetre of the plates, $A\sigma$ is the charge upon A;

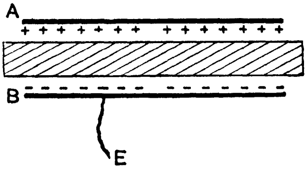


FIG. 891.—Measurement of k.

$$\text{Difference of potential between A and B} = \frac{Q}{C} = \frac{A\sigma}{\frac{A}{4\pi t}}$$

$$= 4\pi\sigma t.$$

But the difference of potential between the plates is equal to the force on unit charge × distance;

$$\therefore\ \text{Force on unit charge} \times t = 4\pi\sigma t,$$

or, $$\text{Force on unit charge} = 4\pi\sigma. \quad\ldots\ldots\ldots\ldots(1)$$

This is the force on unit charge in air or vacuum, due to the charges $+\sigma$ and $-\sigma$.

Inside the slab, the force on unit charge will be $4\pi\sigma/k$, outside it it is still $4\pi\sigma$;

$\therefore$ Work done in carrying unit charge through the slab

$$= \frac{4\pi\sigma}{k} \cdot h,$$

and, Work done in carrying unit charge through the air space $(t-h)$

$$= 4\pi\sigma\,(t-h);$$

∴ Work done in carrying unit charge from B to A

$$=\frac{4\pi\sigma}{k}h+4\pi\sigma(t-h)$$

$$=4\pi\sigma\left\{t-h+\frac{h}{k}\right\}.$$

But this is also the difference of potential between the plates due to the charge Aσ upon A ;

$$\therefore \text{ Capacity}=\frac{\text{A}\sigma}{4\pi\sigma\left\{t-h+\frac{h}{k}\right\}}=\frac{\text{A}}{4\pi\left\{t-\left(h-\frac{h}{k}\right)\right\}}. \quad\ldots\ldots\ldots\ldots(2)$$

Thus the capacity has changed from A$/4\pi t$ to the value given in (2) by introduction of the slab. The same change in capacity might have been produced by bringing B nearer to A by the amount $(h-h/k)$ instead of introducing the slab.

This change in value of t gives us a method of measuring k. Let A be connected to the electrometer or electroscope with the slab in, and the deflection or divergence noted. If the slab be then withdrawn the deflection will increase. Move B nearer until the original deflection is restored, so that the capacity regains its original value. This travel of B must be measured ; let it be l.

Then $$l=h-\frac{h}{k},$$

or, $$\frac{l}{h}=1-\frac{1}{k},$$

and $$k=\frac{h}{h-l}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

h and l being known, k may be found.

EXERCISES ON CHAPTER LXXIII.

1. What is meant by the electric capacity of an insulated conductor ? Describe an experiment you would make to show that the capacity of an insulated conductor is increased when a second conductor connected to earth is brought near it. Sen. Camb. Loc.

2. Distinguish between the electric potential and the energy of a charged conductor. An isolated sphere of radius 8 cm. receives a charge of 720 E.S. units. Show how to calculate the potential and the energy when the dielectric is air, and also when the medium has a dielectric coefficient 2·5. L.U.

3. Define dielectric constant and describe how it may be measured in the case of a liquid such as paraffin oil.

4. Describe one form of electrometer and describe its action.
L.U.

5. How does the energy of a charged condenser depend on the substance between the plates?

What conclusions do you draw regarding the source of the energy of electrical charges? L.U.

6. Calculate the capacity of a spherical condenser of which the radii of the spheres are 15·6 and 16·8 cm. respectively. What is the energy of the charge if the insulated sphere be raised to a potential of 80 units?

7. A Leyden jar has a diameter of 15 cm., a depth of tinfoil of 18 cm. and thickness of glass 2·5 mm. If the value of *k* for the glass is 6·4, find the capacity of the jar.

8. Explain why the capacity of a condenser is changed when the dielectric is changed from air to some other substance.

9. A condenser consists of eleven rectangular pieces of tinfoil each measuring 15 cm. × 20 cm. all joined together, with ten similar pieces of tinfoil joined together and alternating with the first set. If the tinfoils are separated by sheets of mica of thickness 0·2 mm. whose specific inductive capacity is 6·28, what is the capacity of the condenser? Also find the amount of work necessary to put a charge of 100 electrostatic units upon it. L.U.

10. An air condenser with plates 10 centimetres square and half a centimetre apart is charged with 100 electrostatic units of electricity. Find the loss of electric energy when it is plunged under oil of specific inductive capacity 2. L.U.

11. Two parallel plate condensers, A and B, have the following dimensions: (A) area of plate 35 sq. cm., thickness of dielectric 3 mm.; (B) area 75 sq. cm., thickness of dielectric 5 mm. The dielectric of A has a constant 6·2 and that of B 7·5. If A receives a charge of +80 units and B a charge of +70 units, find the direction of flow of the current on connecting A to B.

CHAPTER LXXIV

ELECTRICAL MACHINES

The electrophorus.—Any apparatus for the production of an unlimited supply of electrical charge may be called an electrical machine. The **electrophorus** can hardly be given the name of 'machine,' but it represents a type from which more efficient machines have been developed. It consists of a sheet of ebonite B (Fig. 892) which can be rubbed with a piece of fur, giving it a negative charge. Upon this, a brass plate A, carried by an insulating handle, can be placed. On earthing A by touching it with the finger or a wire, it becomes positively charged, the negative charge upon it escaping to earth (p. 933). On lifting A by its insulating handle, the positive charge is carried with it and is available for use.

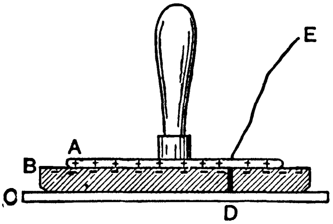


FIG. 892.—The electrophorus.

The ebonite sheet B may be mounted upon a metal sole-plate C, through which passes a metal pin D that makes contact with A automatically when this is placed upon the ebonite and so earths it. This, however, is not essential; the earthing may be done by hand.

The plates A and B are only in actual contact at a few points, so that the charge upon B diminishes with extreme slowness. Thus the amount of positive charge produced by the electrophorus is almost unlimited. Since this charge has considerable energy it is of interest to note the source of this energy. After earthing the plate A it is, of course, at zero potential, but has a positive charge. The positive potential due to this charge is, of course, equal to the negative potential due to the negative charge upon B. When A is lifted off B work must be done in opposition to the force between the opposite charges. This work is the source of the energy of the positive charge, for as A is removed from the neighbourhood of the negative charge its resultant potential will rise, and eventually, when A is far from B, become that due to its own positive charge alone.

The water dropper.—Two tin cans, A and B (Fig. 893), with the bottoms removed, are placed vertically above two others, C and D, in which funnels or pieces of gauze are fixed. A water supply ends in two jets, E and F, one situated within each of the cans, A and B. The jets must be so regulated that they break up into drops while still situated within the cans A and B. These drops in falling must make contact with the interiors of the cans C and D. A and C are connected together, as are also B and D.

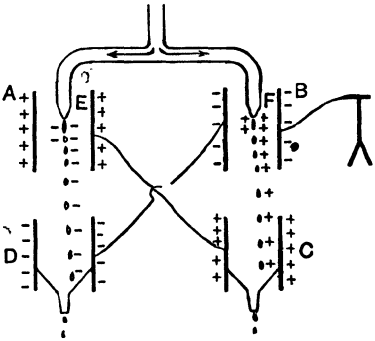


FIG. 893.—The water dropper.

To begin with, a small charge is given to one pair of cans; let a positive charge be given to A. This makes the potential of A positive, and since the interior of a hollow conductor acquires the potential of the conductor (p. 935) the jet is in a region of positive potential. But since the jet is earthed, through the water pipes, a current flows to earth, leaving the jet negatively charged. As it breaks up into drops, the negative charge is imprisoned upon the drops and is carried to D, where, on the drop making contact with the interior, the charge passes to the outside, and is, of course, shared with B. Upon the other side, B being now negatively charged, a similar process goes on, positive charge being carried to C and A. The higher the charges the more rapid the process, so that once started, the accumulation of charge goes on more and more rapidly.

The source of energy of these accumulated charges is the work done by gravity in pulling the negatively charged drops at E out of the neighbourhood of the positive charge upon A, and the positively charged drops at F out of the neighbourhood of B's negative charge.

In setting up the apparatus, the cans may be fixed to a wooden framework by means of sealing wax, and the length of the jet must be adjusted until the accumulation of charges begins. There is generally enough charge upon the apparatus to start the process without the necessity for putting any upon the cans.

Kelvin replenisher.—This type of influence machine is almost the same, electrically, as the water dropper, but it is mechanically constructed so that the carriers, which are the counterpart of the drops, are carried round an axis continually. The conductors A and B (Fig. 894) act both as inductors, in this respect resembling A and B in the water dropper, and also as collectors like the cans C and D. The carriers E and F are fixed upon an insulating arm which is so mounted that it can be made to rotate freely about the axis O.

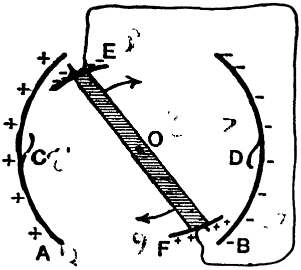


FIG. 894.—Kelvin replenisher.

Imagine A to be given a small positive charge to begin with. As the carriers rotate in the direction of the arrows they come simultaneously into contact with two light strips of metal connected by a wire, at the position shown in the diagram. This connects E and F electrically. But E being nearer than F to the positive charge upon A, is at a higher potential than F, so that a current flows from E to F, leaving E negatively charged and F positively. As the motion continues, E and F leave the contact pieces and carry away the charges. Soon E arrives in contact with the metal tongue D and gives up its negative charge to the conductor B. At the same time F comes in contact with C and gives up its positive charge to A. The process is then repeated during the next half turn. Owing to the collection of negative charge upon B and positive charge upon A the charges produced upon E and F become greater and greater as time goes on, and the rate of accumulation of charge becomes more rapid.

Wimshurst influence machine.—If we could imagine the conductors A and B of the Kelvin replenisher to rotate in the opposite direction to the carriers E and F, we should get twice the number of operations in a given time. This alteration is accomplished in the Wimshurst machine, together with a further multiplication produced by increasing the number of carriers and inductors. These parts, in this machine, are exactly like each other in form. They consist of metal sectors, fixed to glass or ebonite discs which rotate in opposite directions. As the electrical arrangements cannot very well be shown in a picture of these discs, they are represented by two concentric circles in Fig. 895. The plates, with their sectors, are supposed to rotate in the directions shown by the arrows.

As in the previous two cases, some charge must be given to one set of sectors in order to start the action, but we will describe the state of affairs after the discs have made a few turns. At A and B are two wire brushes, connected by a conducting rod, which touch simultaneously two diametrically opposite sectors upon one disc. At this moment charges of opposite sign upon the sectors on the other disc are passing, so that the sector at A will receive a negative charge and that at B a positive charge. These charges are eventually given up, the negative to the collector at E and the positive to the collector F. Before arriving at the collectors they play a similar part to the sectors passing the brushes C and D, giving those in contact with C a positive charge and those at D a negative charge. It will thus be seen whence the charges passing A and B were derived.

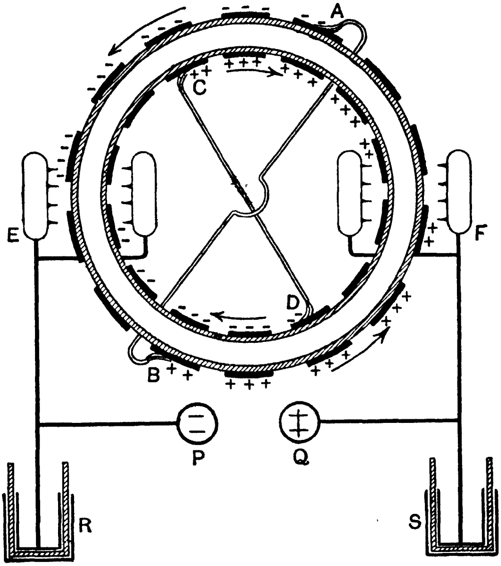


FIG. 895.—Diagram of the Wimshurst machine

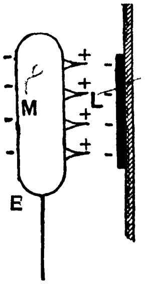


FIG. 896.—Collection of the charges.

The collection of the charges at E and F requires notice. Consider Fig. 896. Several points fixed upon the collector are directed towards the sector, which is supposed to have a negative charge. This causes the points L to be at lower potential than the further parts M of the conductor, the consequence being that positive charge flows to the points and negative charge to M. The effect of the points is to cause this positive charge to flow from them, as described on p. 937, and the stream of positive charge falling upon the sector neutralises its negative charge. Hence the sector passes away uncharged, while a corresponding negative charge remains upon the collector.

Two discharge knobs, P and Q (Fig. 895), are connected to the collectors, and on the difference of potential between them rising to a sufficient amount, owing to the accumulation of the opposite charges, a spark discharge will take place. Most Wimshurst machines are provided with Leyden jars, R and S, one connected to each collector. Their function is to increase the capacity so that a greater accumulation of charge takes place before the difference of potential between P and Q is sufficient to cause a discharge. The discharge will then be much more violent than when only a small amount of charge has collected.

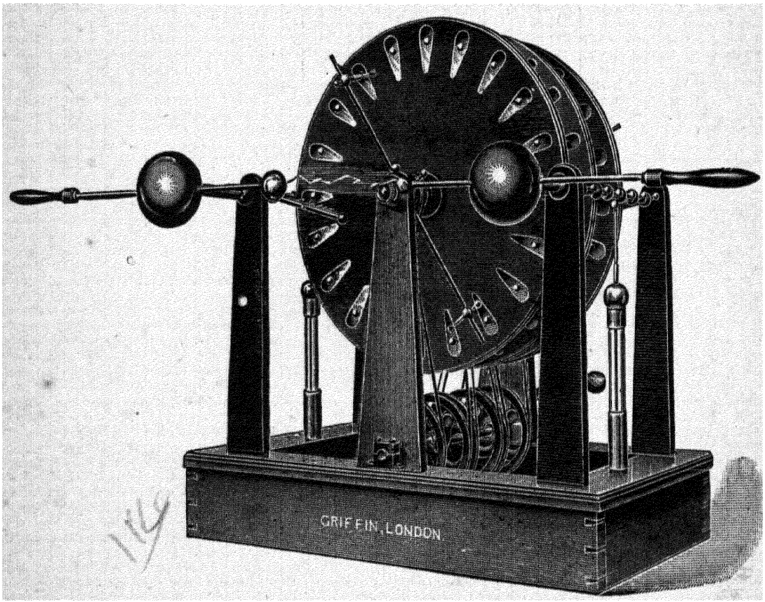


FIG. 897.—The Wimshurst influence machine.

In Fig. 897 the general appearance of the Wimshurst machine is shown. It has six plates, alternate plates being connected together so that adjacent plates rotate in opposite directions.

Some makers enclose the machine in an outer case which is filled with coal gas, as it is thought that the machine then works more efficiently than in air.

Influence machines are sometimes used for working X-ray tubes, but they are not so convenient or reliable as the induction coil. There are several other forms of influence machine, but those described above will serve as types.

Exercises on Chapter LXXIV.

1. A charged ebonite rod is brought near to a pin, which is fixed to the knob of an electroscope, and the rod is then removed. State and explain what may be observed.

Explain what practical use is made of the effect observed.

Sen. Camb. Loc.

2. Explain the action of some machine for producing electrostatic charge, indicating clearly the source of the energy of the charge obtained.

L.U.

3. Explain the action of a Wimshurst machine.

Show how the potential and charge of a sector vary during a revolution.

4. Describe the action of the electrophorus, and explain why the amount of charge that can be given to any insulated conductor by means of it is limited.

5. Describe the action of (*a*) the water dropper, or (*b*) the Kelvin replenisher.

CHAPTER LXXV

ELECTROMAGNETICS

Work done in carrying a magnet pole round an electric current.—It was seen in Chapter LXV that the system of units developed in connection with electric currents is founded upon the relation between the current and its magnetic field. The field in a certain case (p. 833) is taken as a measure of the current, and we may thus say that the strength of magnetic field at any point is proportional to the current to which it is due. This leads, in a manner which cannot here be stated, to the fact that the work done in carrying a magnetic pole once round a conductor in which a current is flowing is proportional to the current. Further, if the current is measured in absolute units (p. 833) **the work done in carrying a unit magnetic pole once round the current is 4π times the strength of the current.**

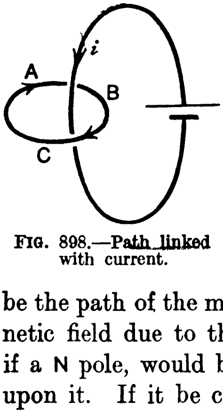


FIG. 898.—Path linked with current.

In taking the magnetic pole round the current in this way, its path must be linked with the current circuit. In Fig. 898 if ABC be the path of the magnetic pole, we know from p. 831 that the magnetic field due to the current is in such a direction that the pole, if a N pole, would be urged along the path, and work is thus done upon it. If it be carried round in the direction CBA, the current's magnetic field opposes the motion, and energy must be expended by some outside agency to effect this motion. In either case, however,

$$\text{Work done} = 4\pi i \text{ ergs.}$$

Magnetic field due to a long straight current.—We will now apply this relation to calculate the magnetic field due to a current in several important cases. The first case is that of a current flowing in a straight wire, of sufficient length to be considered infinitely long. The magnetic lines of force due to such a current are circles whose centres lie upon the wire. Hence, from symmetry, it is obvious that the strength of magnetic field is the same all round any given circle having its centre upon, and its plane perpendicular to, the wire.

To find the strength of magnetic field H at a point P (Fig. 899) at distance r from the wire, remember that the force on a unit magnetic pole at P is H dynes, and is tangential to the circle (p. 830). As the pole is carried round this circle,

$$\text{Work done} = \text{force} \times \text{distance}$$
$$= \mathrm{H} \times 2\pi r \text{ ergs.}$$

But from the law given above,

$$\text{Work done} = 4\pi i \text{ ergs ;}$$
$$\therefore\ 2\pi \mathrm{H} r = 4\pi i\,;$$
$$\therefore\ \mathrm{H} = \frac{2i}{r} \text{ gauss.} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

It is thus seen that the strength of magnetic field due to a long straight current is inversely proportional to the distance from the current. The unit of magnetic field is defined on p. 781 as the force on unit pole, and is sometimes called the **gauss**.

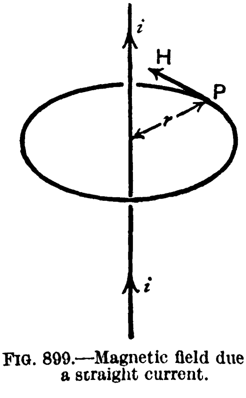


FIG. 899.—Magnetic field due to a straight current.

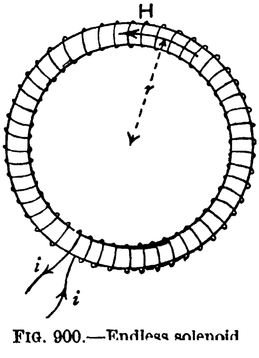


FIG. 900.—Endless solenoid

Magnetic field inside an endless solenoid.—A circular ring upon which a wire is uniformly wound, so that the magnetic field is everywhere in the direction of the circumference of the ring, is called an **endless solenoid**. If there are n turns per centimetre length of the solenoid, measured circumferentially, the total number of turns is $2\pi rn$, where r is the radius (Fig. 900). Thus, for a current i in the wire, a unit magnetic pole carried round the curved axis of the solenoid passes $2\pi rn$ times round the current, and the work done is therefore $4\pi(2\pi rni)$ ergs. But if H is the strength of magnetic field, this is the force on the unit pole, and the work done upon the pole in making a complete circuit along this path is $2\pi r\mathrm{H}$;

$$\therefore \quad 2\pi r\mathrm{H} = 4\pi(2\pi r n i),$$

or,

$$\mathrm{H} = 4\pi n i \text{ gauss.} \quad \ldots\ldots(2)$$

If the current be given in amperes, the strength of field is then

$$\mathrm{H} = \frac{4\pi n\mathrm{I}}{10} \text{ gauss.} \quad \ldots\ldots(3)$$

Unless the circular path of the magnetic pole has the mean radius of the solenoid, the expression for the field is not so simple as the above, for the field is not quite uniform over the cross-section of the coil. If, however, the section of the solenoid is small compared with the radius r, this want of uniformity is negligible, and when r is very great, it may be entirely ignored.

Magnetic field inside a long straight solenoid.—It will be noticed that the radius r does not enter into the expression $4\pi ni$ for the magnetic field inside the endless solenoid. If then r be made very great without altering n the number of turns per centimetre, the magnetic field remains the same. On continuing to increase the radius, the solenoid eventually becomes straight. Hence the strength of magnetic field inside an infinitely long straight solenoid is $4\pi ni$.

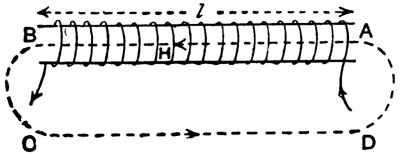


FIG. 901.—Long straight solenoid.

This may be proved independently by considering a long straight solenoid of length l, having n turns per centimetre. For a path ABCD (Fig. 901), the work done in carrying a unit pole from A to B is $\mathrm{H}l$ ergs. For the rest of the closed path BCDA, the field is nearly zero if the solenoid is very long, so that the work for this part of the path is zero ;

$$\therefore \text{ Work done for the closed path} = \mathrm{H}l = 4\pi n l i,$$

or,

$$\mathrm{H} = 4\pi n i \text{ gauss.} \quad \ldots\ldots(4)$$

The only difficulty in the above reasoning arises from the indefiniteness near the ends, but it must be remembered that, strictly speaking, the field is only $4\pi ni$ for a very long solenoid, in which case the path near the ends is an insignificant part of the whole path.

Magnetic field due to a short solenoid.—The above law cannot be applied to the case of a short solenoid, as it is impossible to find a complete path, enclosing the current, along which the field has constant strength. Fig. 902 illustrates the form of the magnetic field for a short solenoid. The lines of force spread out from the ends, and have even begun to spread out within the coil. On comparison with the lines of force due to a bar magnet (p. 778) it will be seen that the external magnetic field due to the solenoid is very similar to that of the magnet. There is the equivalent of a N pole at one end and a S pole at the other.

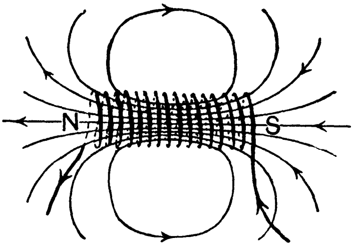


FIG. 902.—Magnetic field of a short solenoid.

EXPT. 218.—**Magnetic effect of a short solenoid.** Wind some cotton-covered copper wire upon a piece of brass tube, 10 cm. long and about 1 cm. diameter. Pass a current of 1 or 2 amperes through the coil. By means of a suspended magnet, as in Expt. 162, find the N and S poles of the solenoid, and show that this agrees with the law for relation of direction of current to that of magnetic field given on p. 831.

EXPT. 219.—**Force between solenoids.** Wind another solenoid as in the last experiment, but bring the free ends of wire to the middle of the solenoid and tie them firmly with cotton, taking care to keep them insulated from each other. Attach to each free end a piece of copper wire A and B, and suspend the coil as in Fig. 903, so that the current may enter at A and leave by B. Bring the pole of a bar magnet near each end of the solenoid in turn, and show by the attractions and repulsions that each end of the solenoid is a magnetic pole, one end N and the other S. Verify the statement that **on looking at the end of the coil at which the current travels in an anti-clockwise direction that end is a N pole.** The end at which the current flows clockwise is a S pole (Fig. 902).

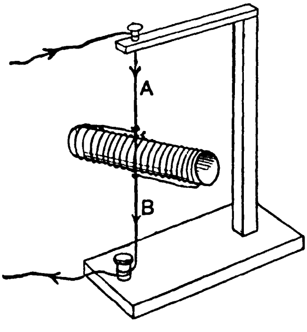


FIG. 903.—Magnetic effect of a solenoid.

Repeat the experiment, using the coil of Expt. 218 in place of the bar magnet.

Place a piece of iron rod in each coil and show that the poles are now very much stronger, but of the same kind as before.

Force on a straight current in a magnetic field.—The results of Expts. 218 and 219 show that electric-current circuits behave as magnets. It is therefore evident that a conductor carrying a current will experience a force when situated in a magnetic field. The laws governing this force are as follows :

(i) The force on the conductor is at right angles to the plane containing the direction of the current and that of the magnetic field. It is in the direction shown in **Fig. 904,** which may be remembered by the following **Left-Hand Rule.** Place the thumb, fore-finger and middle finger of the **left hand** mutually at right angles to each other. Let the mIddle finger point in the direction of the current I and the Fore-finger in the direction of the magnetic Field ; then the thuMb points in the direction of Motion in which the circuit is urged.

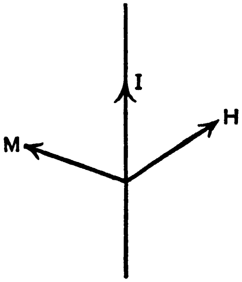


FIG. 904.—Force on electric current in a magnetic field.

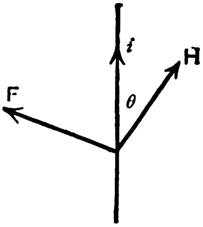


FIG. 905.—Force on current inclined to magnetic field.

(ii) When the magnetic field and the current are at right angles to each other, the force acting on every centimetre of the conductor is Hi dynes. Or, if the current is measured in amperes, the force per centimetre of conductor is $HI/10$ dynes.

If the current and magnetic field are inclined to each other at an angle θ, the force per centimetre of the conductor is then $Hi \sin \theta$ dynes. When $\theta = 0$, that is, when the current and field have the same direction, there is no force on the conductor since $\sin \theta = 0$. When, however, $\theta = 90°$, $\sin \theta = 1$, and the force is Hi dynes, as given above.

Couple acting on a rectangular coil in a magnetic field.—Let ABCD (Fig. 906 (*a*)) be a rectangular coil, situated in a magnetic field. The forces on the sides AB and CD are equal, opposite, and in the same straight line, and therefore cancel each other out. The force on the side AD, however, is $F = liH$ dynes, where l is the length of AD, i the current and H the strength of magnetic field. There is an equal and opposite force on BC, and the two give rise to a couple, whose moment $= li\mathrm{H} \times \mathrm{EB}$, where EB is the perpendicular distance between the forces. Fig. 906 (*b*) is a plan of the coil, in which it will be seen that if the normal to the coil makes an angle θ' with the direction of the magnetic field, then the angle $\mathrm{EAB} = \theta'$ and

$$\mathrm{EB} = \mathrm{AB} \sin \theta'.$$

Calling AB, the breadth of the coil, b, we have

$$\text{Couple} = \mathrm{H}ilb \sin \theta'.$$

But $\quad l \times b = \text{area of coil} = \mathrm{A},$

Then $\quad \text{Couple} = \mathrm{H} \,.\, \mathrm{A}i \,.\, \sin \theta'.$

Comparing this with the expression for the couple on a magnet situated in a magnetic field (p. 781), that is, $\mathrm{HM} \,.\, \sin \theta'$, we see that as regards the magnetic field, the coil may be considered to have a magnetic moment equal to $\mathrm{A}i$. This result is quite general for plane circuits, whatever the shape of the coil. Thus, for a circular coil of radius r cm., in which a current i flows, the magnetic moment is $\pi r^2 i$.

It will be seen that magnetic lines of force due to the coil (Fig. 906) emerge at the front face and enter at the back. Thus, the front face is a N polar face, which will correspond to the direction of rotation shown.

The following is a convenient rule for finding the direction of rotation of a current circuit situated in a magnetic field: **the forces acting in the circuit are in such a direction that the circuit moves to embrace the greatest number of magnetic lines of force.** This rule is quite general and applies to flexible as well as to rigid current circuits.

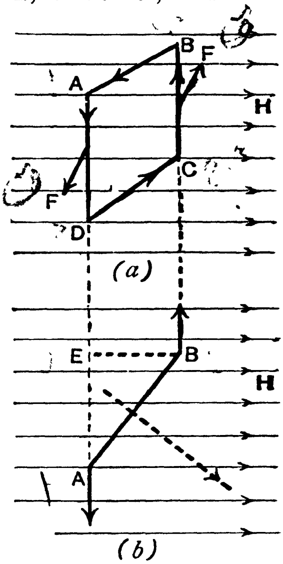


FIG. 906.—Coil in a magnetic field.

Suspended coil galvanometer.—One of the most important uses to which the above fact is put is in the construction of the modern type of galvanometer (p. 866).

If the magnetic field between the poles (Fig. 797) be uniform, and of strength H, the couple acting on the coil is $n\mathrm{A}i \,.\, \mathrm{H}$, where n is the number of turns in it, provided that the plane of the coil is parallel to the field. In other positions the couple is $n\mathrm{A}i\mathrm{H} \cos \theta$, where θ is the angle between the magnetic field and the plane of the coil. The coil then rotates until the twist in the suspension produces an equal and opposite couple. Since the latter couple is proportional to the angle of twist, it may be written $c\theta$, where c is a constant depending on the length, thickness and material of the strip. Hence, for equilibrium,

$$nAiH\cos\theta = c\theta\ ;$$

$$\therefore\ i = \frac{c}{nAH}\cdot\frac{\theta}{\cos\theta}. \qquad (1)$$

For very small deflections $\cos\theta \doteq 1$, and the current is proportional to the deflection, but for large deflections the current is proportional to $\theta/\cos\theta$, which is so complicated an expression that it is not of much use.

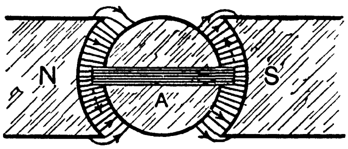


FIG. 907.—Magnetic field of the suspended coil galvanometer.

To get over this difficulty the soft iron cylinder A (Fig. 907) is placed between the pole pieces, so that the sides of the coil move in the cylindrical space between the pole pieces and the soft iron cylinder. This ensures that the permanent magnetic field shall be radial, and therefore the sides of the coil, even when deflected, are still in a field of the same strength. Also, the force on the sides is still at right angles to the plane of the coil. Hence the couple is always $nAiH$, unless, of course, the deflection be so great that the sides of the coil come near the edges of the pole pieces.

Thus, with the soft iron cylinder present,

$$nAiH = c\theta\ ;$$

$$\therefore\ i = \frac{c}{nAH}\theta. \qquad (2)$$

The current is therefore proportional to the deflection. This is of great convenience in practice, and renders this arrangement particularly serviceable for the construction of ammeters (p. 870).

Force between currents.—Since a current has a magnetic field in its neighbourhood, and a current in a magnetic field experiences a force, it follows that two current circuits exert forces upon each other. Indeed, we have seen in Expt. 219, p. 967, that this is the case

Let A and B (Fig. 908) be two

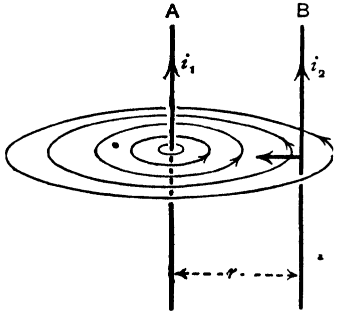


FIG. 908.—Force between parallel currents.

long straight parallel wires carrying currents i_1 and i_2 in the same direction. If the distance between them be r centimetres, the strength of magnetic field at B due to A is $2i_1/r$ (p. 965). B being situated in this magnetic field experiences a force

$$F = \frac{2i_1}{r} \times i_2 = \frac{2i_1 i_2}{r},$$

per centimetre of its length. By applying the left-hand rule (p. 968) it will be seen that the direction of this force is such that B is urged towards A.

By a similar reasoning it may be shown that A is urged towards B with a force $2i_2 i_1/r$ per centimetre of its length.

Further, we see that **currents in the same direction attract each other, and currents in opposite directions repel each other.**

When the circuits are not of such a simple form, it may not be easy to calculate the force between them, but **for any given position the force is always proportional to the product of the current strengths.** Many current measuring instruments are based upon this principle.

Kelvin current balance.—The current to be measured passes through six coils CLMABD in series (Fig. 909). A, B, C and D are fixed, while L and M are attached to an arm which can rotate about an axis GH. A scale is also attached to the arm and carries a weight W which can be caused to slide along this scale. The arm and scale must be so balanced that when the weight W is on the zero mark the beam is horizontal, as indicated by a pointer, when no current is flowing. The coils are so connected that when the current flows, the forces between A and L, C and L are in such a direction that L is pushed downwards. Similarly M is pushed upwards. The arm thus rotates. It is brought back to its equilibrium position by sliding the weight W to the right, which introduces a couple equal and opposite to that due to the current. The scale is so graduated that the movement of the weight upon it gives a direct reading of the current. This scale, however, is not equally spaced, for the couple is proportional to the product of the current strengths in adjacent coils. Since, however, the current in these is the same, the couple is proportional

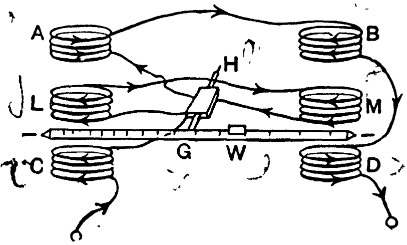


FIG. 909.—Diagram of the Kelvin current balance.

to i^2. But the couple is also proportional to the linear displacement of the weight ; $\therefore$ Displacement $\propto i^2$.

Before being sent out by the instrument maker, the marks upon the scale are so placed that the scale reads directly in current.

By changing the weight, the range of current to be measured may be varied.

It should be noted that if the direction of the current be reversed, **it is reversed in all the coils**, so that the direction of every force is the same as before. Hence the direction of the current is immaterial, which is essential if the instrument is intended to measure alternating currents.

EXPT. 220.—**Calibration of an ammeter by the Kelvin current balance.** Place the ammeter to be calibrated in series with the current balance and an adjustable rheostat. Set the movable weight W upon one of the fixed marks on the scale of the balance, and adjust the rheostat until the current is such that the movable arm returns to its zero position. Observe the readings of both ammeter and current balance. Repeat for other currents and record the results in a table as below :

Ammeter reading.	Position of weight on movable arm.	Current.

The current for each position of the movable weight is given by the instrument maker for each weight, in a table supplied.

Kelvin watt balance.—A similarly designed instrument has been made for the direct measurement of the power absorbed in any circuit. In this case the coils are differently connected, but the method of measuring the couple by displacing the sliding weight is the same as before.

Suppose that it is desired to measure the power in watts absorbed in the lamp PQ (Fig. 910). A, B, C and D, the fixed coils, are then placed in series with PQ, so that the current I in the lamp also flows through these coils. In this case the movable coils LM consist of a great many turns of fine wire, so that they have considerable resistance. They are joined in parallel with the lamp, so that the current in them is proportional to the p.d. E between the lamp terminals. Hence, the force between A and L is proportional to E × I ; that is, it is proportional to the product of the strengths of the currents in A and L. Similarly, all the other forces between the coils are proportional to

EI; therefore the couple acting on the movable arm is also proportional to EI. Thus, the displacement of the sliding weight necessary to restore equilibrium is proportional to EI. But EI is the power in watts used in driving the current through the lamp. Therefore, the arm on which the movable weight slides may be calibrated so that the instrument reads directly in watts.

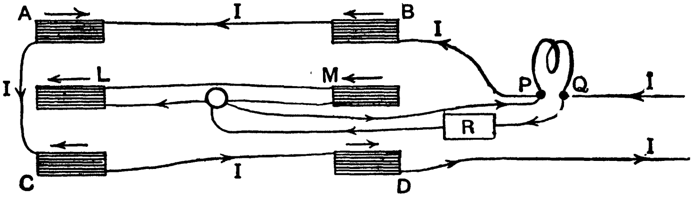


FIG. 910.—Diagram of the Kelvin watt balance.

Owing to the fact that the movable coils LM are placed in parallel with the lamp, their resistance must be high, for the same reason that the resistance of a voltmeter must be high (p. 875). For this reason it is customary to introduce an extra fixed resistance R (Fig. 910) into the movable coil circuit, and the wattmeter is usually calibrated with this high resistance included in the shunt circuit.

EXPT. 221.—**Measurement of watts absorbed by an electric lamp.** Set up the wattmeter and connect it to the lamp as shown in Fig. 910. Move the sliding weight until equilibrium is attained and record the power in watts. If possible, measure the candle-power of the lamp at the same time (p. 547) and obtain the watts per candle-power of the lamp. Repeat for several other lamps.

Siemens' electrodynamometer. — The principle of the **electrodynamometer** is the same as that of the Kelvin current balance, but the design of the apparatus is different. The couple is here measured by the twist in a spring instead of by means of a movable weight. Two coils ABCD and PQRS (Fig. 911), approximately rectangular in form, are connected in series. PQRS is fixed, but ABCD is suspended so that it can rotate about a vertical axis. In rotating it twists the spring whose upper end is attached to a pointer at the torsion head T. When the current flows, the forces

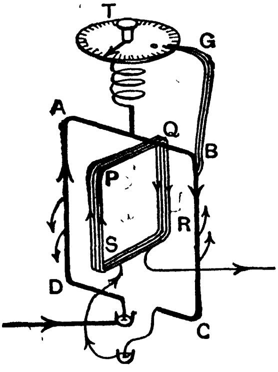


FIG. 911.—Diagram of Siemens' electrodynamometer.

between the coils cause rotation of the suspended coil in the direction of the arrows. It is then brought back to its original position by rotating the pointer to which the upper end of the spring is attached. The number of degrees twist thus given to the spring is a measure of the couple required to maintain the suspended coil in its original position. This position is indicated by the pointer G, attached to the movable coil.

Each force acting between the adjacent sides of the two coils is proportional to the product of the currents in them. But since these currents are both the same, namely, the current I to be measured, each force is proportional to I^2, and the couple is proportional to I^2. If the twist in the spring, as measured by the torsion head, is θ, the couple is also proportional to this :

$$\therefore\ \theta = kI^2,$$

or,

$$I = K\sqrt{\theta},$$

where k and K are constants whose values must be found if the current is to be measured in amperes. If the direction of the current is reversed, that in **both** coils is reversed, so that the forces are in the same direction as before. Hence the reading does not depend upon the direction of the current, and the instrument is applicable for use with alternating currents.

Some types of this instrument are designed for use as wattmeters, when one coil is placed in series and the other in parallel with the apparatus in which the power is to be measured. The arrangement is similar to that of Fig. 910, but it should be remembered that the coil used as shunt must have a very high resistance.

EXPT. 222.—**To calibrate the Siemens' electrodynamometer.** Connect the instrument in series with a standard ammeter and an adjustable rheostat. Turn the pointer T (Fig. 911) until the indicator G is at the zero of the scale when no current is flowing. If T is not at the zero of the scale, its position must be recorded, and this zero error applied to all readings taken with the instrument. Pass the current. Then G moves from its zero position, and must be brought back to it by rotating T. Note the new position of T and also the ammeter reading for the current. Repeat for other currents, until the whole scale has been traversed. Record the results in a table.

Current.	θ, uncorrected for zero error.	θ, corrected for zero.	$\sqrt{\theta}$.

Plot a graph connecting current and θ. This will be a parabola. Plot another graph connecting current and $\sqrt{\theta}$. This should be a straight line. From the latter graph find the constant K of the instrument from the relation

$$I = K\sqrt{\theta}.$$

EXERCISES ON CHAPTER LXXV.

1. State the law for the work done when a magnetic pole is carried round a closed path linked once with a current circuit, and apply the law to calculate the strength of magnetic field at distance *r* from a long straight wire carrying a current.

2. Calculate the strength of magnetic field inside a long straight solenoid, and state why the result is inapplicable to the case of a short solenoid.

What is the strength of magnetic field inside a solenoid of length 60 cm. having 300 turns in which a current of 1·2 amperes is flowing ?

3. A straight solenoid of length 75 cm. consists of 600 turns in which a current of 0·2 ampere flows. What is the magnetic flux through the solenoid, if its radius is 1·8 cm. ?

4. An iron ring forms a closed magnetic circuit, having a mean length of 30 centimetres, and a section of 1·5 sq. cm. On the ring are wound 100 turns of insulated wire, and 1 ampere in the wire gives a total flux of 12,000 lines in the ring. Find the permeability of the iron. C.G.

5. Describe, giving sketches, two forms of current-measuring instruments, one depending on the interaction of currents and magnets, the other of current and currents. Explain the action of each instrument. L.U.

6. A coil of a single turn of wire in the form of a rectangle of height 15 cm. and width 8 cm. is suspended in a horizontal magnetic field of strength 2·5. Draw a diagram indicating the forces acting on each side of the rectangle, and calculate the couple acting upon it when its plane makes an angle of 45° with the magnetic field and a current of 20 c.g.s. units flow in the wire. L.U.

7. Find an expression for the current flowing in a suspended coil galvanometer (*a*) when no soft iron cylinder is used, (*b*) when a soft iron cylinder is situated between the pole pieces.

8. Describe some form of wattmeter and the method of using it to measure the watts per candle power for an incandescent lamp.

9. State in as general a form as possible the quantitative laws relating to the magnetic forces due to electric currents.

A current of one ampere flows round a wire bent into a circle of 20 cm. in diameter. An equal current flows round a circle of 2 cm. in diameter suspended at the centre of the larger coil. What couple is required to hold the small circuit with its plane at right angles to that of the large one ? L.U.

10. Two long straight parallel wires carry currents of 10 and 15 amperes respectively, and are situated 12 cm. apart. Find the force on 5 cm. length of each wire.

11. Describe some form of suspended coil galvanometer and obtain an expression for the deflection in terms of the current and the constants of the instrument. Why is a soft iron cylinder usually placed between the pole pieces?

12. A circular coil of 18 turns of radius 12 cm. carries a current of 3·5 amperes. Calculate the value of the couple required to maintain it with its plane parallel to a magnetic field of strength 25 gauss.

CHAPTER LXXVI

ELECTROMAGNETICS (*CONTINUED*): MUTUAL AND SELF-INDUCTION

Electromotive force due to cutting across a magnetic field.—Whenever there is relative motion between a conductor and a magnetic field, there is an electromotive force in the conductor. This electromotive force is of very great importance, its existence being the foundation for an enormous number of applications of electricity, the electric dynamo, for example, depending upon it. A few simple experiments demonstrate at once the presence of this electromotive force. If a straight conductor (Fig. 912) be moved through the field of an electromagnet in a direction indicated by the arrow M, and a galvanometer be placed in series with the conductor, the galvanometer will indicate a current flowing, **so long as the conductor is moving across the magnetic field.** If the direction of motion of the conductor be reversed, the direction of the current is also reversed. When the conductor is part of a closed circuit, a current will flow, but the primary effect of the motion across the magnetic field is to produce an electromotive force. The resulting current depends, of course, upon the resistance of the circuit.

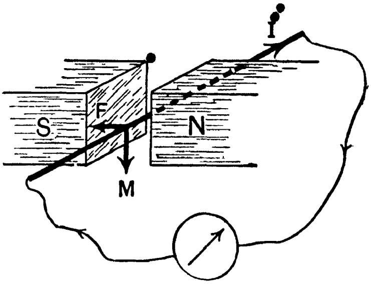


FIG. 912.—Conductor cutting across a magnetic field.

The direction of the electromotive force and current is given by the **Right-Hand Rule,** similar in form to the left-hand rule given on p. 968. Place the thumb, fore-finger and middle finger of the **right hand** mutually at right angles to each other. Let the Fore-finger point in the direction of the magnetic Field, and the thuMb in the direction of Motion ; then the middle finger will point in the direction of the current I, or the electromotive force.

A further experiment illustrates this production of electromotive force. Let a magnet NS (Fig. 913) be brought up to a coil of wire connected in series with a galvanometer. While the magnet approaches the coil there will be a deflection of the galvanometer, which ceases directly the magnet comes to rest. A reverse current is produced on withdrawing the magnet, or on using the S pole of the magnet instead of the N pole.

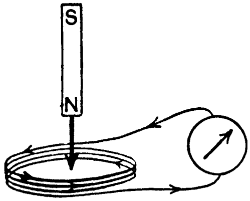


FIG. 913.—Induced e.m.f.

If, instead of moving the magnet, the coil be advanced towards it, the effects are exactly the same as though the coil were at rest and the magnet moved. Thus the electromotive force depends only upon the **relative motion** of the magnet and the coil.

EXPT. 223.—**Induced electromotive forces.** Connect a coil or solenoid of about 100 turns of wire in series with a galvanometer. Push a magnet into the coil, N pole first, and observe the direction of the kick of the galvanometer. After letting the magnet remain in the coil until the galvanometer needle comes to rest, withdraw the magnet quickly from the coil and again note the direction of the kick. Repeat, using the S pole of the magnet. Again repeat, keeping the magnet at rest and moving the coil on to the magnet. Tabulate the results as follows :

Magnet advancing to coil.	Direction of kick.	Magnet withdrawn from coil.	Direction of kick.
N pole S „		N pole S „	
Coil advancing to magnet			
N pole S „		N pole S „	

Value of e.m.f. due to cutting magnetic flux.—In any case in which magnetic flux is cutting across a conductor, **the value of the electromotive force is the rate of cutting of magnetic flux, or the amount of**

magnetic flux cut per second. Magnetic flux being usually measured in terms of magnetic lines of induction, we may say that **the electromotive force in a conductor is equal to the number of magnetic lines of induction cut per second.**

Thus, if a conductor cuts a number of magnetic lines of induction in t seconds,

$$\text{e.m.f.} = \frac{\text{number of lines of induction cut}}{t} \text{ c.g.s. units.}$$

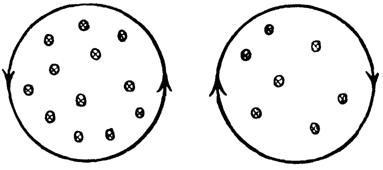
$$= \frac{\text{number of lines of induction cut}}{10^8 \times t} \text{ volts.}$$

Since in air the magnetic induction is identical with the magnetic field, lines of induction are lines of force, and hence,

$$\text{e.m.f.} = \frac{\text{number of lines of force cut}}{10^8 \times t} \text{ volts.}$$

In the cases with which we shall deal, in which the conductor is situated in air, or at any rate in a medium of unit permeability, we shall always refer to lines of force rather than lines of induction, but if the permeability should differ from unity the above distinction must be observed.

Application to a closed circuit.—On applying the above rule to every part of a closed circuit, a simple rule for the whole circuit may be found. **The electromotive force in a closed circuit is equal to the rate of change of the magnetic flux (number of lines of magnetic induction) threaded through the circuit.** To find the direction of the electromotive force, look along the lines of induction at the circuit (Fig. 914), then **if the number of lines threaded through the circuit is increasing the direction of the current is anticlockwise, if the number is decreasing the current is clockwise.**



Increasing Decreasing

FIG. 914.—Rule for direction of e.m.f.

Again, if the circuit consist of a number of turns, the total electromotive force is that given by the above relation multiplied by the number of turns, since the above relation applies to each turn.

EXAMPLE.—If the number of lines of induction threaded with a coil of 1500 turns is 10,000 and if this induction is removed at a constant rate in one-tenth of a second; find the momentary electromotive force in the coil.

$$\text{e.m.f. for each turn} = \frac{10000}{10^8 \times 0{\cdot}1} \text{ volts};$$

$$\therefore \text{ e.m.f. for 1500 turns} = \frac{1500 \times 10000}{10^8 \times 0{\cdot}1} \text{ volts}$$

$$= \underline{1{\cdot}5} \text{ volt.}$$

Mutual induction.—It has now been seen that the electromotive force in a closed circuit depends upon the fact that there is a change in the amount of magnetic flux passing through it. The electromotive force depends only upon this flux and not at all upon the origin of the flux. Thus it may be due to a magnet, or it may, on the other hand, be due to a current in its neighbourhood. If the current in a coil A (Fig. 915) be started, magnetic lines of force due to it become established, and in doing so, some of them cut into the coil B. With the direction of current and field as shown, the momentary current in B, as seen from A, will be anticlockwise (p. 979). The current in B only lasts while the current in A is growing. Directly the current in A ceases to grow, the current in B stops. On stopping the current in A, the magnetic lines due to it disappear, and there is a momentary current in B in a clockwise direction. It is usual in an arrangement like this to call A the **primary circuit** and B the **secondary circuit**. It will then be seen that for two neighbouring circuits, while the current in the primary circuit is growing there is an induced current and e.m.f. in the secondary circuit, **opposite in direction to that in the primary.** While the current in the primary is **diminishing** the induced current and e.m.f. are in **the same direction** as the current in the primary.

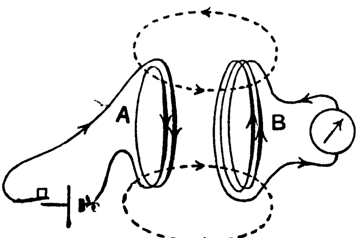


FIG. 915.—Mutual induction.

This effect of one circuit upon another is called **mutual induction.** **The coefficient of mutual induction, or the mutual inductance of two circuits, is the magnetic flux linked with the secondary, when unit current flows in the primary.**

If this magnetic flux be removed by the current in the primary ceasing to flow, and if this change takes place uniformly in one second, the electromotive force in the secondary is numerically equal to the amount of flux which has disappeared; that is, **the mutual inductance of two coils is the e.m.f. in the secondary, when the current in the primary changes at the rate of one unit per second.** It does not matter which of the two coils is primary and which secondary, the mutual inductance is the same in either case.

If the e.m.f. in the secondary is 1 volt when the current changes at the rate of 1 ampere per second the mutual inductance is 1 henry.

Since the volt is 10^8 absolute c.g.s. units and the ampere is 10^{-1} absolute c.g.s. unit, it follows that

$$1 \text{ henry} = \frac{10^8}{10^{-1}} = 10^9 \text{ absolute c.g.s. units of inductance.}$$

Foucault or eddy currents.—When a mass of metal is situated in the neighbourhood of a changing current, it acts as a secondary circuit, electromotive forces and currents being developed in it. The electromotive force is great when the rate of change of magnetic flux is considerable, and the current further depends upon the conductivity of the material. As a good example, take the iron core of an electromagnet, represented diagrammatically in Fig. 916. A few of the magnetic lines of induction are shown. These, in becoming established, cut the iron core and produce considerable momentary currents which circulate as shown by the arrow. These currents are considerable, for the conductivity of the mass of iron is great. Such currents are called **eddy currents**, or **Foucault currents.**

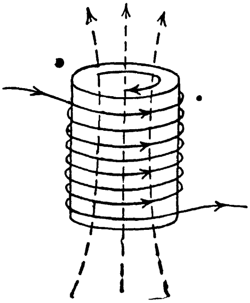


FIG. 916.—Eddy currents.

Part of the energy of these currents becomes dissipated into heat in the iron; for when the primary current ceases to grow, these eddy currents stop. In fact, the iron core acts as a very low resistance secondary circuit. This heat means so much energy wasted. It is not of great consequence in starting or stopping the current once only, but in some cases, such as the core of an armature of a dynamo (p. 992) or of a transformer (p. 984), the magnetic flux is reversed many times per second, and the waste of energy and heating of the iron are highly objectionable.

To diminish the amount of eddy current in such cases the iron core is built up of laminae, or of wires, each lamina or wire having a thin coat of iron oxide upon it which insulates it from its neighbours.

The direction of lamination is so chosen that a maximum of resistance is opposed to the eddy currents. The core in such a case as Fig. 916 would therefore be built up of sheets (or wires) parallel to the axis of the magnetising coil. Such lamination enormously reduces the eddy currents and the consequent energy losses and heating.

Lenz's law.—Since a current in a magnetic field experiences a force, it follows that whenever induced currents arise, forces come into play between these and the field which causes them, which may be a moving magnet or a changing current. By applying the laws already given, the direction of these forces can be found, but there is a simple general expression of them known as **Lenz's law** which enables us to give the direction of such forces at once.

When a circuit and a magnetic field move with respect to each other, the induced currents bring about forces which always tend to oppose the motion.

Let us apply this law to the case of a magnet NS (Fig. 917) approaching a solenoid AB. The law on p. 979 tells us that the induced current, as seen from the magnet, is anticlockwise. Thus there is a N pole of the solenoid at A (p. 967). This repels the N pole of the magnet, and thus **opposes the motion** of the magnet.

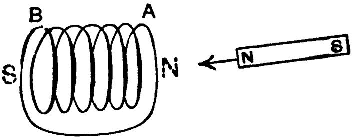


FIG. 917.—Application of Lenz's law.

Whatever may be the nature of the motion of the magnet or coil, the same opposition to the motion will be brought about.

This effect has many important applications. For example, if it is desired to render the motion of the suspended coil of a galvanometer dead-beat, the coil is wound upon a copper frame. If the coil oscillates, this copper frame cuts the magnetic field of the permanent magnets, and according to Lenz's law, forces are brought into play which quickly bring the coil to rest. A useful method of bringing the coil of a galvanometer to rest when it is not provided with a copper frame, is to short circuit the terminals of the galvanometer. As the coil oscillates, induced currents then flow in it and it is rapidly brought to rest. In fact, the coil itself acts as the copper frame just described.

Many modern forms of electro-motor also depend upon this phenomenon. A mass of metal mounted upon an axle is subjected to a rotating magnetic field. By Lenz's law the currents induced in the metal, by their magnetic action, tend to oppose the relative motion of the conductor and the rotating magnetic field. The conductor is thus dragged round after the field.

Induction coil.—It is clear from the definition of mutual inductance (p. 981) that for a given rate of change of current in the primary circuit, the greater the mutual inductance of the two coils, the greater will be the electromotive force produced in the secondary. In the case of the **induction coil**, the mutual inductance is made so great that the electromotive force in the secondary amounts to hundreds of thousands of volts. The mutual inductance is made large by providing an iron core for the coils, and by making the number of turns in the secondary coil very great.

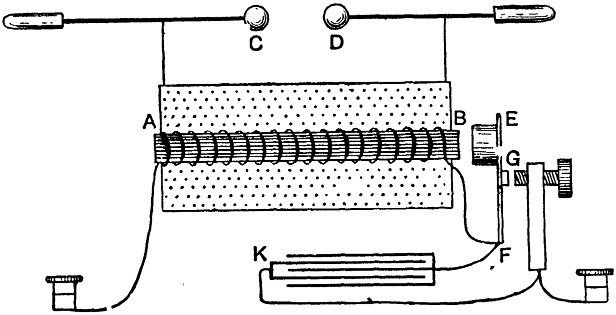


FIG. 918.—Induction coil.

The arrangement of the circuits in the induction coil is shown diagrammatically in Fig. 918. Upon the soft iron core AB, consisting of a bundle of soft iron wires, the primary coil, consisting of a few layers of thick wire, is wound. The secondary coil, consisting of many thousands of turns of fine wire, is shown in section, the ends being connected to the sparking terminals CD.

In a circuit such as the primary, the current does not become established or die away instantaneously. It dies away, however, more quickly than it is established, and consequently the rate of change of magnetic flux through the secondary is greater at the 'break' of the primary circuit than at the 'make.' A higher electromotive force is therefore produced at the break of the primary than at the make. In practice the electromotive force at 'break' is the only one high enough to cause the secondary current to jump the air gap at CD.

In order to render the making and breaking automatic, a spring EF is usually provided. The soft iron head E is pulled towards the iron core AB when this is magnetised by the primary current. This breaks the contact at G, where both contact surfaces are faced with platinum, to diminish the sparking that occurs whenever the circuit is broken. When the break occurs, the iron core becomes demagnetised and the pull on E ceases. The spring then flies back, making contact again at G, and so the process is repeated. The number of makes and breaks occurring per second depends upon the stiffness of the spring and the mass of the iron head E.

Another form of interrupter sometimes employed consists of a rapidly rotating toothed wheel, against which a mercury jet is directed. The primary current is broken every time a tooth leaves the jet.

A commutator for reversing the direction of the primary current is usually provided, but this is not shown in Fig. 918.

For the purpose of increasing the efficiency of the induction coil, a tinfoil condenser is usually connected across the spark gap. The effect of this is to increase considerably the length of spark obtainable from the coil. The function of this condenser is complicated, but its effect is best understood by considering that on the break occurring at G, the current does not merely fall to zero, but proceeds to flow in the opposite direction for a time, the current flowing into the condenser. Thus, instead of a mere stoppage of the current at the break there is an actual reversal of it. Hence the electromotive force in the secondary lasts for a longer time. The drop of current also occurs more quickly, and both effects increase the efficiency of the coil.

The transformer.—It will have been noticed in the case of the induction coil, that the primary current is supplied at low voltage from probably a few secondary cells, while the current in the secondary circuit is at very high voltage. Of course the secondary current is correspondingly smaller than the primary current. In the supply of alternating currents it is often desirable to be able to convert from one voltage to another. This is done by means of the **transformer.** A 'step-up' transformer is one in which the voltage is changed from low to high. In a 'step-down' transformer the reverse is the case. Let the alternating current enter at the terminals AB (Fig. 919) of the coil wound upon a laminated soft iron core, which in this case has the form of a ring. Considering this as the primary, the secondary consists of a great number of turns of fine wire having terminals CD.

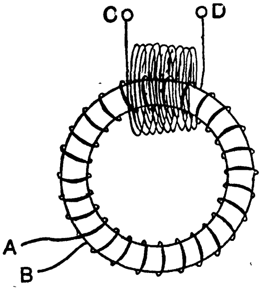


FIG. 919.—Circuits of the transformer.

An alternating current may be represented by a curve, such as the thick line curve in Fig. 920. At points such as A, C and E the current is changing most rapidly, and the magnetic flux through the secondary is also changing rapidly. Hence the secondary e.m.f. is great. It is positive at A and E and negative at C. At B, D and F

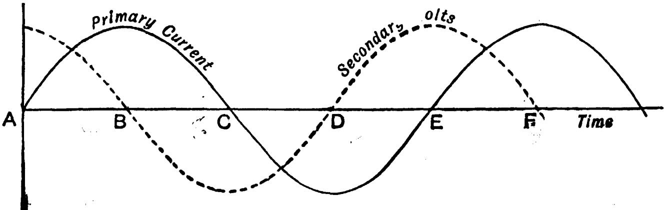


FIG. 920.—Current and e.m.f. curves for the transformer.

the primary current is for a moment constant and the secondary e.m.f. is zero. The dotted curve represents the secondary e.m.f. It should be noticed that the curves are not drawn to scale.

For a step-down transformer CD (Fig. 919) would be the primary and AB the secondary. In this case small current at high voltage would be supplied and large current at small voltage obtained from the transformer.

The ratio of the primary and secondary voltages is approximately the same as that of the number of turns in the primary and secondary coils.

Thus,

$$\frac{\text{primary voltage}}{\text{secondary voltage}} = \frac{\text{number of primary turns}}{\text{number of secondary turns}}.$$

The use of the transformer is to cheapen the transmission of electric current over long distance; the small current at high voltage only requires thin copper wires to transmit it, while for the same rate of transmission of energy at low voltage, the current must be great and stout copper leads are then necessary. Hence it is cheaper to transmit at high voltage and transform down to low voltage at the place where the current is being used. An illustration of a transformer is given in Fig. 921.

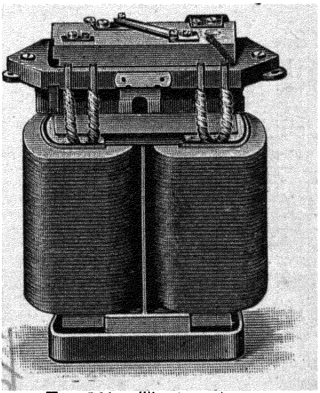


FIG. 921.—The transformer.

Self-induction.—The law that the changing magnetic flux threaded through a circuit causes an electromotive force, applies to the circuit in which the current is changing as well as to neighbouring circuits. Consider the circuit ABC (Fig. 922) in which the current is supposed to be growing. The magnetic flux through the circuit is increasing, and by applying the rule on p. 979 we see that the e.m.f. due to this acts in the direction CBA. That is, it is in the opposite direction to the current. The current, therefore, is less at any given instant than it would be if this induced e.m.f. were absent.

On the other hand, if the current is decreasing, then the magnetic flux is also decreasing, and the induced e.m.f. acts in the same direction as the current, and therefore tends to prevent it from dying away. The current, therefore, decays more slowly than would be the case if no such e.m.f. were present.

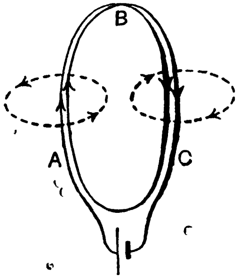


FIG. 922.—Self-induction.

This effect is called **self-induction**, and its general effect is to oppose any change in the current, and hence to make the current change more slowly than it would if there were no self-induction.

The electromotive force, being equal to the rate of change of magnetic flux, is greater, the greater the rate at which the current dies away. Hence on breaking a circuit the rate of change is very great, and is generally sufficient to produce an e.m.f. that will cause the current to jump the gap for an instant. This is the cause of the spark that occurs when a current circuit is broken. With a large magnetic flux, as in the case of an electromagnet, the e.m.f. due to self-induction may be so great that the spark is very violent.

The measurement of self-induction is similar to that of mutual induction (p. 981), being the e.m.f. in the circuit due to unit rate of change of current. It is therefore measured in henrys. The value is called the **coefficient of self-induction**, or the **self-inductance.**

The effect of self-induction on Wheatstone's bridge measurements may be important if precautions are not taken. Thus, on closing the battery circuit the currents in the branches may not grow at the same rate. There will then be a momentary kick of the galvanometer, although the balance may be perfect for steady currents. It is for this reason that the battery key is always closed before the galvanometer key (p. 887). This ensures that the currents will become steady before the galvanometer is joined to the circuit.

In order to render the disturbing effect as small as possible, resistance coils are usually wound **non-inductively.** That is, the wire is first doubled and then wound upon the bobbin (p. 880). In this way the current going and returning is so nearly in the same position that the magnetic field due to it is extremely small, and hence the self-inductance is small.

EXERCISES ON CHAPTER LXXVI.

1. Write an account of the theory and construction of a moving coil galvanometer. Why is the coil sometimes enclosed in a thin silver tube? In what circumstances would the presence of the silver tube be disadvantageous? L.U.

2. Describe two forms of interrupter that may be used with an induction coil.

Discuss the action that takes place in the induction coil, and the characteristics of the currents in the two coils.

What is the function of the condenser sometimes employed? L.U.

3. Explain the terms mutual inductance and self-inductance. How may a resistance coil be wound so that it shall have extremely small self-inductance?

4. Explain the construction and action of a transformer. L.U.

5. Give the laws of production of an electromotive force in a circuit when this is cutting across a magnetic field.

A closed coil of wire rotates slowly about a vertical axis, and a magnetic needle is suspended at its centre. As the coil rotates, it cuts the earth's magnetic field. Represent by a curve, or other diagram, the deflecting couple acting on the needle during one rotation of the coil. L.U.

6. A copper disc of radius 18 cm. rotates with its axle parallel to a magnetic field of strength 20 c.g.s. units. If the disc makes 250 revolutions per minute, find what e.m.f. acting along a radius will be developed.

7. A solenoid of length 50 cm. consisting of 1000 turns of radius 3 cm. carries a current of 0·6 amp. A secondary coil of 500 turns is wound upon the middle part of the solenoid. Calculate the average e.m.f. in the secondary coil if the primary current falls to zero in 0·001 sec.

8. Explain why the resistance coils used in connection with a Wheatstone's bridge should have very small self-inductance. If the coils are known to have considerable self-inductance, what precaution must be taken in making the test.

CHAPTER LXXVII

THE DYNAMO AND MOTOR

Coil rotating in magnetic field.—Consider a plane coil of wire mounted so that it can rotate about an axle O (Fig. 923) at right angles to a uniform magnetic field of strength H. As the coil rotates it cuts across the field and an electromotive force is produced in it. When the plane of the coil is at right angles to the field as at CD, the e.m.f. is zero, since the edges of the coil are not in this position cutting across the field. At EF, where the plane of the coil is parallel to the field, the e.m.f. is a maximum, for the edges of the coil are now cutting perpendicularly across the lines of force.

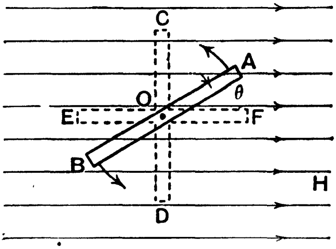


FIG. 923.—Rotating coil.

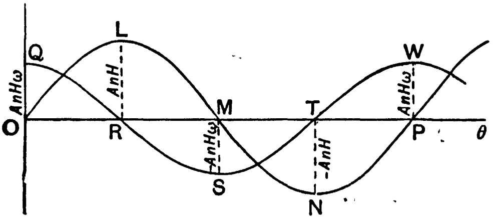


FIG. 924.—Curves of magnetic flux and e.m.f. for a rotating coil.

For a position AB, where the coil makes an angle θ with the magnetic field H, only the component of the field $H \sin\theta$ at right angles to the coil passes through it. Hence the number of lines passing through the coil, if it consists of n turns of area A sq. cm. each, is $AnH \sin\theta$. If this be plotted in the form of a graph for different values of θ as the coil rotates, we get the curve OLMNP (Fig. 924). O corresponds to the position EF (Fig. 923) and L to position CD. As the coil rotates with uniform angular velocity, the e.m.f. is proportional to the rate of change of the flux, which is given by the slope of the curve OLMNP. This is greatest at O, M and P, and zero at L and N. It is well known that the rate of change of a sine curve OLMNP is a cosine curve QRSTW. Hence the curve QRSTW represents the e.m.f. in the coil rotating with uniform angular velocity.

In the neighbourhood of O, M and P, θ is very small, and we may write θ for $\sin\theta$. If the time in which θ is described is t, then the rate of change of $AnH\theta$ is $AnH\theta/t = AnH\omega$, where $\omega = \theta/t$, the angular velocity of rotation of the coil. Hence $AnH\omega$ is the maximum e.m.f. in the coil. The equation for the e.m.f. is therefore

$$\text{e.m.f.} = AnH\omega \cos\theta.$$

It will be seen that the direction of the e.m.f. in the coil is reversed twice in every revolution. If the ends of the coil are connected to metallic rings A and B (Fig. 925), with metal brushes C and D touching them, there will be an alternating e.m.f. acting in the circuit joined to C and D. This arrangement of slip-rings with brushes touching them permits continuous rotation of the coil. The alternating-current (A.C.) dynamo is built upon this principle.

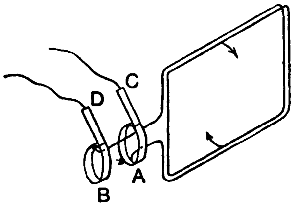


FIG. 925.—Coil producing an alternating e.m.f.

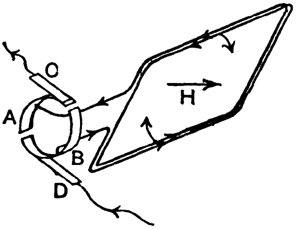


FIG. 926.—Rotating coil with commutator.

Principle of the direct-current dynamo.—When the current in the external circuit is required to be always in the same direction, that is, to be **direct current** (D.C.), the slip-rings (Fig. 925) must be modified in such a way that the connection is reversed every half revolution. This reversal is obtained by using one ring, split into two parts diametrically, the two halves, A and B (Fig. 926), being mounted on a non-conducting axle. One end of the coil **is connected to** A and the other to B. The brushes C and D, connected to the external circuit, bear against the split ring. This arrangement is called a **commutator.** The current is now always in the same direction in the external circuit. An inspection of Fig. 926 shows that the reversal of the direction of the e.m.f. in the coil takes place when the coil is vertical. It will be seen that the descending side of the coil is always in connection with the brush C and the ascending branch with the brush D. Hence the e.m.f. is always directed the same way round the external circuit.

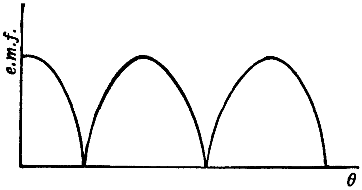


FIG. 927.—e.m.f. curve when a commutator is used.

The electromotive force produced by a single coil of this type is always in the same direction. It is not, however, constant in value. The commutator has merely changed the direction of the reverse halves of the curve of e.m.f. so that for the external circuit the e.m.f. curve will be of the form shown in Fig. 927.

Gramme ring armature.—In a dynamo for actual use, the e.m.f. must be as nearly steady as possible. Hence the unevennesses of the e.m.f. curve for a single coil must be smoothed out in some way. The rotating coil or coils of the dynamo is called the **armature**, and the **Gramme ring** form of armature produces a much more nearly constant e.m.f. than is possible with a single coil. A number of coils are wound continuously upon an iron ring. In Fig. 928 there are eight such coils, *a*, *b*, *c*, *d*, *e*, *f*, *g* and *h*. The commutator is also in eight sections, and each bar of the commutator is connected to the junction of a pair of the coils in the ring. The brushes A and B bear upon the commutator as shown. A better view of the commutator and brushes is given in Fig. 929. The commutator segments are made of hard drawn copper, and are insulated from each other by means of sheet mica, which wears down, by the rubbing of the brushes, at the same

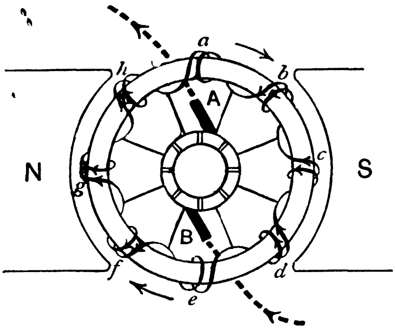


FIG. 928.—Gramme ring armature.

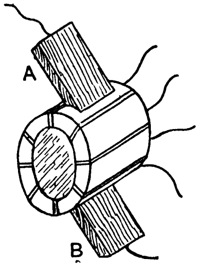


FIG. 929.—Commutator and brushes.

rate as the copper. The segments are also insulated from the shaft by means of mica, and are held in position by two nuts (not shown), from which also they are insulated. The brushes are generally made of copper or brass gauze compressed into a block; but they are sometimes of hard carbon.

N and S are the poles of the field magnet (Fig. 928).

When the armature rotates in the direction shown (Fig. 928) it can be seen, by applying the rule on p. 979, that the e.m.f. in every coil is as indicated by the arrows. The coils *c* and *g*, at the instant shown, have maximum e.m.f., while at this same instant there is no e.m.f. in *a* and *b*. It should be noticed that the currents in *b*, *c* and *d* are all flowing towards the commutator bar in contact with the brush A, and that those in *f*, *g* and *h* are also flowing towards A, and the current in the **external** circuit will therefore flow from A to B.

At the instant given, the coils *a* and *b* are on the point of being short circuited; but since there is no e.m.f. in them this does no harm. Further, since *b*, *c* and *d* are in series, the resulting e.m.f. will be the sum of the separate e.m.f.'s, and the same applies to *f*, *g* and *h*. As the armature rotates, the e.m.f. in each coil will vary as shown in Fig. 927. Thus the e.m.f. in coil *a* will correspond to the thick line in Fig. 930. The e.m.f.'s for *b*, *c* and *d* are drawn dotted, to avoid confusion. At any instant the resultant e.m.f. between A and B is the sum of the e.m.f.'s for the four coils in one side of the armature. These e.m.f.'s are added up in Fig. 930 and the resultant curve LM is obtained. It is then seen that with eight coils in the armature the e.m.f. is very nearly steady. With a greater number of coils the e.m.f. would be still more nearly constant.

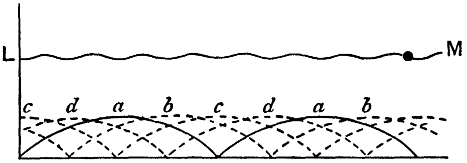


FIG. 930.—Resultant e.m.f. in the gramme ring.

Lead of the brushes.—It will be remembered that each coil in the armature is short circuited twice in a revolution, as the two commutator segments to which its ends are attached come under a brush. Hence it is necessary that this should take place when there is no e.m.f. in the coil, or else the short circuiting will mean a big current in it, with consequent heating of the coil and sparking when the circuit is broken an instant later. If the magnetic field in which the coil rotates were symmetrical about a vertical plane, as it is when the armature is at rest (Fig. 931) the brushes would have to be placed at the ends of a vertical diameter. When the current flows in the armature, however, it produces a component of the magnetic field, perpendicular to that of the field magnets, and the resultant field is as shown in Fig. 932.

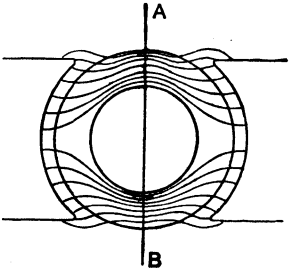


FIG. 931.—Magnetic field with armature at rest.

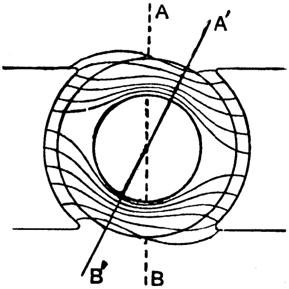


FIG. 932.—Magnetic field when current flows in armature.

The brushes must now be placed at the ends of the diameter A′B′. The coil short circuited by the brush will then be moving parallel to the magnetic field, and the e.m.f. in it is zero, and excessive sparking will be avoided. The angle by which the line joining the brushes must be advanced from AB to A′B′ to stop sparking is called the **lead of the brushes.**

To facilitate this adjustment the brushes are carried in brush holders (Fig. 933) attached to a framework which can be rotated through any desired angle, the position for sparkless running being found by trial.

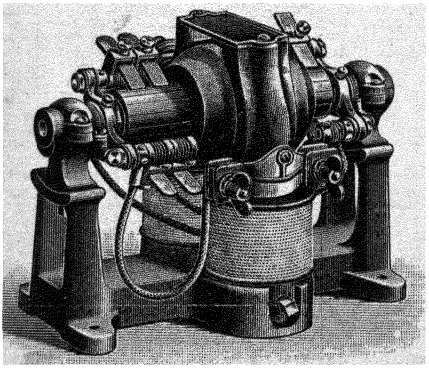


FIG. 933.—Direct current dynamo.

Drum armature.—In the ring armature, the conductors lying outside the iron core cut the magnetic field as the armature rotates, but those lying on the inside of the ring do not cut any appreciable magnetic field, and therefore they do not give rise to any e.m.f. They are merely idle conductors, and for this reason, the ring form of armature is only used in certain small machines. The simple rotating coil (Fig. 926) does not have any idle conductors except at the ends, but the electromotive force in it is not sufficiently steady for industrial purposes. If, however, a number of such coils in series arranged at various angles to each other can be employed, their resultant e.m.f. will be much more nearly constant than when one is used alone. The connecting up of such coils presents difficulties, but the arrangement is practically realised in the **drum armature** (Fig. 934).

A number of straight copper conductors, properly insulated, lie in slots cut in the cylindrical surface of the armature, and are properly insulated from it. The simple case of eight such conductors will now be considered, although of course the number would be much greater in practice. There is also great variety in the methods of connecting the conductors to each other and to the commutator bars. When there are eight conductors *a*, *b*, *c*, *d*, *e*, *f*, *g* and *h* (Fig. 934) there will be four commutator bars A, B, C and D. With the direction of rotation shown, the e.m.f. in *b*, *c* and *d* is from back to front of the armature, while in *f*, *g* and *h* it is from front to back. The conductors are joined to the commutator bars as indicated by the thick lines, while the dotted lines show how the conductors are joined together at the back of the armature.

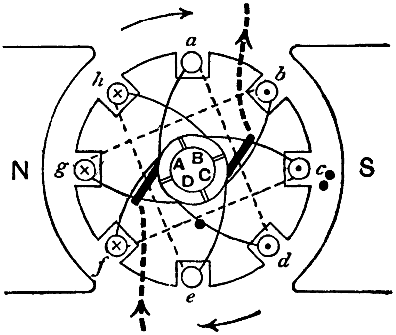


FIG. 934.—Drum armature.

Starting with the bar A, the current enters the armature from the external circuit and divides into two circuits in parallel. Thus one part passes to *a*, and flowing down this comes at the back of the armature to *d*, thence it passes across the front, down *g* and across the back to *b*, and then to the brush at C. It then flows out to the external circuit. The other part of the current flows from A to *f*, thence by way of *c*, *h* and *e* to the brush at C.

We see, then, that there are two circuits in parallel. Also, as the brushes pass from one commutator bar to the next, one coil is short circuited and the brushes must have such an angle of lead that the e.m.f. in this coil is zero at this particular moment (p. 992).

Electromotive force in armature.—Since there are two sets of conductors in parallel, the e.m.f. of the dynamo is equal to the e.m.f. in either one of these sets. In the simple case of eight conductors, there will be four in each set, but in practice the number will, of course, be very much greater.

Let F be the total magnetic flux, or number of lines of force that each conductor cuts across in passing from top to bottom of the field. Again, let Z be the total number of conductors round the armature. Then $\frac{1}{2}Z$ is the number of conductors in series. Each conductor cuts F lines in a half turn, so that if the armature makes N revolutions per second, it makes $2N$ half revolutions per second, and $2NF$ is the number of lines cut per second by each conductor. It is therefore the average e.m.f. in each conductor. As the number of conductors in series is $\frac{1}{2}Z$, $2NF \times \frac{1}{2}Z = FNZ$ is the e.m.f. between brush and brush. This is, of course, in absolute c.g.s. units of e.m.f.

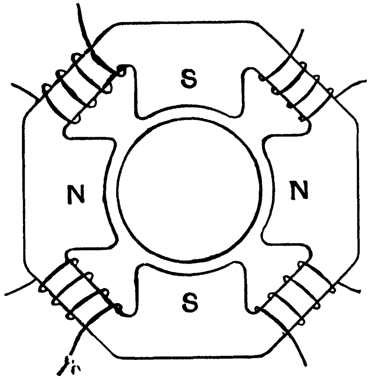


Fig. 935.—Four-pole field magnet.

$$\therefore \text{ e.m.f. of machine} = FNZ \times 10^{-8} \text{ volts.}$$

In some machines a number of pairs of poles are arranged round the circumference of the armature, and the above result gives the e.m.f. for **each pair of poles.** For the resultant e.m.f. we must multiply by the number of pairs of poles.

Field magnets.—For the production of the magnetic field of the dynamo, powerful field magnets are necessary. These have various forms, one form being that given in Fig. 936. Here the current flowing in the field coils, which causes the magnetisation, flows also in the external circuit. The field coils are thus in series with the armature and the external circuit. When this arrangement is adopted the dynamo is said to be '**series wound.**' Series winding has this advantage, that as the current in the external circuit increases,

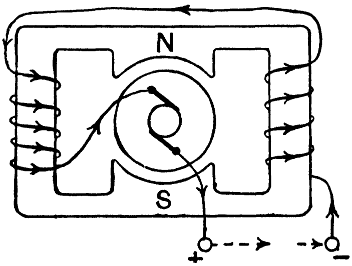


Fig. 936.—Series wound field magnet.

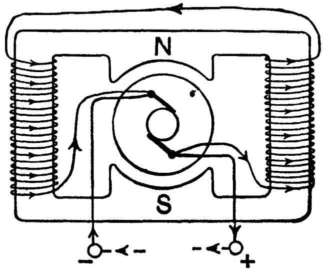


Fig. 937.—Shunt wound field magnet.

the magnetisation of the field magnets is thereby increased. There is usually enough residual magnetisation in the iron cores of the field magnets to produce a feeble current when the machine starts, provided that the external circuit is closed. The voltage and current then mount up rapidly as the magnetisation increases. The series winding consists of few turns of low resistance, since the resistance of the coils is included in the internal resistance of the machine.

In Fig. 937 the field coils consist of a great many turns of fine wire and are placed in parallel with the armature and external circuit. For this reason this is called a '**shunt wound machine.**'

If the resistance of the external circuit decreases, so that the external circuit takes more current, the current in the field coils drops, and hence the e.m.f. of the machine falls. The shunt wound machine is, however, useful for charging accumulators; since on connecting the machine to the accumulators, current flows in the field coils and produces excitation of the field magnet in the proper direction. Also, as the cells become more fully charged, their e.m.f. rises, and hence more current flows in the shunt coils, thereby increasing the magnetisation and the e.m.f. of the dynamo. Series winding must not be exclusively employed in a machine for cell charging, as on connecting the machine to the cells, they cause a current which produces the magnetisation in the wrong direction in the field magnets, and the e.m.f. of the dynamo becomes reversed, with disaster to both dynamo and battery.

Characteristic curve of a dynamo.—Considerable information can be deduced from a curve of current and voltage for a dynamo running at constant speed. Such a curve is called a **characteristic curve** for the machine. On running the machine at constant speed with an ammeter in series and a voltmeter across the machine terminals, current and voltage can be observed. On varying the external load the current can be changed. OAB (Fig. 938) is the type of curve obtained for a series wound machine. The curve rises at first owing to the rapid increase in magnetisation of the field coils with increasing current. At high currents it drops slightly, as the external resistance becomes less in proportion to the internal resistance of the machine.

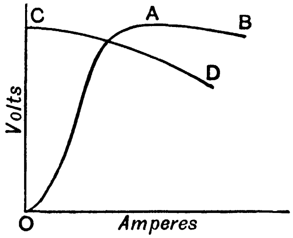


FIG. 938.—Characteristic curve of a dynamo.

For a shunt wound machine the characteristic is of the form CD (Fig. 938). The e.m.f. is greatest when there is no external current, the whole current then going through the field coils. As the external current becomes greater, that flowing in the shunt coils gets less, and the e.m.f. therefore drops.

By suitably combining these types of winding, a machine having a nearly flat characteristic may be produced, which therefore has nearly constant voltage at all loads. The machine has an ordinary shunt winding consisting of many turns of fine wire, but upon the outside of this a few series turns are wound. Such an arrangement is called **compound winding.**

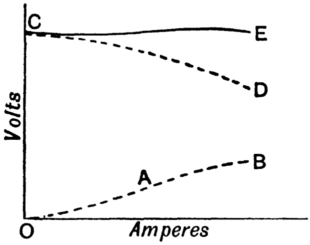


Fig. 939.—Characteristic curve of a compound wound dynamo.

If the curve OAB (Fig. 939) gives the e.m.f. due to the series turns, and CD that due to the shunt turns, the curve CE, which is the sum of the other two, will be the characteristic when both windings are employed.

Such compound wound dynamos are largely used for purposes of public supply, and in all cases where it is desired to maintain automatically a constant voltage of supply at all loads.

Efficiency of dynamos.—There are several points of view from which the efficiency of a dynamo may be regarded. The fraction of the electric power generated in the machine which is available in the external circuit is called the **electrical efficiency.** Thus,

$$\text{Electrical efficiency} = \frac{\text{watts in external circuit}}{\text{total watts generated}}. \quad \ldots\ldots\ldots\ldots(1)$$

This quantity will, of course, vary with the current produced, and will be determined by the p.d. between terminals E, the current I, and the resistances of the armature R_a, and of the field coils, R_m for a series machine, or R_s for a shunt machine.

Thus, for a series machine,

$$\text{Electrical efficiency} = \frac{EI}{EI + I^2(R_a + R_m)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and for a shunt machine,

$$\text{Electrical efficiency} = \frac{EI}{EI + \frac{E^2}{R_s} + \left(I + \frac{E}{R_s}\right)^2 R_a}. \quad \ldots\ldots\ldots\ldots(3)$$

The user of the dynamo is usually chiefly concerned with its **commercial efficiency**, which may be expressed as follows:

$$\text{Commercial efficiency} = \frac{\text{watts in external circuit}}{\text{mechanical power supplied to dynamo}}. \quad \ldots(4)$$

The mechanical power supplied to the dynamo is used in supplying the energy in the form of current, and also in providing for certain unavoidable losses, such as friction at the bearings, magnetic hysteresis losses in the armature core (p. 823), etc. If these losses constitute a total of W watts, the equation for a series machine becomes,

$$\text{Commercial efficiency} = \frac{\mathsf{EI}}{\mathsf{EI} + \mathrm{I}^2(\mathsf{R}_a + \mathsf{R}_m) + \mathsf{W}}, \quad \text{............(5)}$$

with a corresponding equation for the shunt machine.

To the mechanical engineer the chief question of interest in the dynamo is the ratio of the total watts generated to the mechanical power supplied, which is called the **mechanical efficiency.** Thus,

$$\text{Mechanical efficiency} = \frac{\text{total watts generated}}{\text{mechanical power supplied}}. \quad \text{........(6)}$$

It will further be seen that

$$\text{Mechanical efficiency} = \frac{\text{commercial efficiency}}{\text{electrical efficiency}}. \quad \text{............(7)}$$

Electromotors.—Although it is a general rule that any dynamo can be run as a motor, still the design of the motor is usually different from that of the dynamo, to suit the purposes to which the motor is to be put. That a dynamo will run as a motor if supplied with current may be seen from Fig. 940, which is similar to Fig. 934. In this case, however, current produced by some external source enters at the brush A and leaves at C. Hence the current flows in the conductors *a*, *b*, *c*, etc., exactly as in the previous case. That is, in *b*, *c* and *d* the current flows from back to front and in *f*, *g* and *h* from front to back. Now, taking into account the direction of the magnetic field and applying the **left-hand** rule (p. 968), it will be seen that the forces upon the armature conductors are in such a direction that the armature is urged to rotate in an anticlockwise direction. Hence the brushes must be set for this direction of rotation.

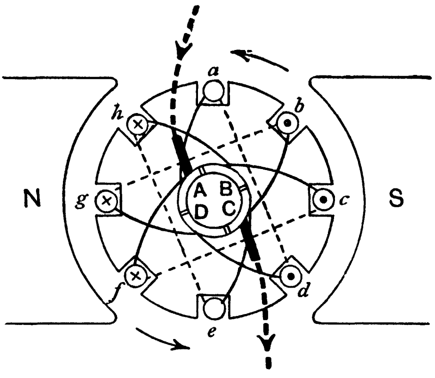


FIG. 940.—Drum wound motor armature.

It should further be noted, that if the directions of the current in the armature and in the field magnets be both reversed, the direction of rotation will remain the same. In order to reverse the direction of rotation of the motor **either** the direction of the field, **or** that of the current in the armature must be reversed, **but not both.**

For the direction of rotation of the motor to be the same as that for the dynamo, the field having the same direction, the current in the armature is in the **opposite** direction in the motor to that in the dynamo, and the brushes will require to have a **lag**, if sparkless running is to be attained, instead of the **lead** required in the case of the dynamo (p. 992).

Back e.m.f. in motor.—In the conductors of the armature of a motor, an e.m.f. is produced, because these conductors cut across the magnetic field. Its direction is such that it opposes the current driving the motor. It therefore tends to reduce the current in the armature and is known as a **back e.m.f.** Hence the current in the armature may be calculated from the expression $(E - E')/R$, where E is the e.m.f. applied to the armature terminals, E' the back e.m.f. in the armature, and R its resistance. The power IE watts is thus applied to the armature, and of this $I(E - E') = I^2R$ is wasted as heat in the armature, while IE' is the power in watts converted into work in turning the armature.

It is clear that E' depends upon the speed of rotation of the motor. When the motor is at rest $E' = 0$, and if the e.m.f. E were then applied to it, a very great current would flow, sufficiently great to injure the insulation of the conductors and perhaps melt the solder used in making the joints. For this reason it is necessary to employ a **starting resistance** in series with the motor. By means of a switch, the coils of the starting resistance can be cut out one at a time, so that the current never rises to an excessive amount. As the speed rises, the back e.m.f. increases, and at full running speed the starting resistance is entirely cut out.

Efficiency of motors.—Just as in the case of the dynamo (p. 996) the efficiency of a motor may be considered from several points of view. Three of the most important are as follows :

Electrical efficiency

$$= \frac{\text{Electrical power spent in producing rotation}}{\text{Total electrical power supplied}}. \quad \ldots\ldots(1)$$

Commercial efficiency

$$= \frac{\text{Brake output}}{\text{Total electrical power supplied}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Mechanical efficiency

$$=\frac{\text{Brake output}}{\text{Electrical power spent in producing rotation}}. \quad \ldots\ldots(3)$$

$$=\frac{\text{commercial efficiency}}{\text{electrical efficiency}}. \quad \ldots\ldots(4)$$

The brake output is the power which the motor is capable of giving out for driving purposes, and may be measured by means of a brake (p. 513).

If the back e.m.f. E' be known (p. 998), these efficiencies may be simply expressed; for $E'I$ is the rate at which electrical power is converted into mechanical power, while EI is the total electrical power supplied to the motor, E being the p.d. between terminals;

$$\text{Electrical efficiency}=\frac{E'I}{EI}=\frac{E'}{E} \text{ (for a series motor)} \quad \ldots\ldots(5)$$

$$=\frac{I_aE'}{(I_s+I_a)E} \text{ (for a shunt motor),} \quad \ldots\ldots(6)$$

where I_a and I_s are the currents in the armature and the shunt coils respectively.

Again, if W is the waste of power in friction, hysteresis loss, etc.,

$$\text{Commercial efficiency}=\frac{I_aE'-W}{I_aE} \text{ (for a series motor)} \quad \ldots\ldots(7)$$

$$=\frac{I_aE'-W}{(I_s+I_a)E} \text{ (for a shunt motor).} \quad \ldots\ldots(8)$$

The most profitable speed at which to run a motor is not necessarily the speed at which its efficiency is greatest. For, take a series motor in which the back e.m.f. is E' at any given speed. This, of course, depends upon the speed, increasing with it, and R is the resistance of the machine.

Now, electrical efficiency $=E'/E$, and the greatest possible value of this is unity, when, of course, $E'=E$, and since $I=(E-E')/R$ (p. 998), $I=0$. This means that the efficiency increases as the speed increases; but as the speed for which $E'=E$ is approached the current becomes so small that the output of the motor is negligible.

To find the speed for greatest output, note that,

$$\begin{aligned}\text{Electrical power spent in producing rotation} &= E'I\\ &= I(E-RI)\\ &= IE-I^2R.\end{aligned}$$

Draw a line OB (Fig. 941) such that $CB/OC = R$, then the greatest possible current that the motor will take is $E/R = OC$, if $CB = E$.

Now complete the rectangle OABC. For any other current OH, the electrical power spent in producing rotation is represented by the area of the rectangle AF, for

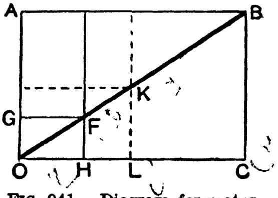


FIG. 941.—Diagram for motor efficiency.

$$\begin{aligned} IE - I^2R &= (OH \times OA) - \left(OH^2 \times \frac{FH}{OH}\right) \\ &= (OH \times OA) - (OH \times FH) \\ &= \text{area of rectangle } AF. \end{aligned}$$

This area is very small when the current I represented by OH is small, and vanishes when $I = 0$. It vanishes again when $I = OC$, since in this case the motor is at rest, and it has its maximum value equal to the area AK when the current has the value OL, which is half its maximum value. Also, the speed for greatest output is half the speed for which the back e.m.f. would equal the applied e.m.f. if the field be constant, for

$$\frac{E - E'}{R} = \frac{1}{2}\frac{E}{R}.$$

or,

$$E' = \tfrac{1}{2}E.$$

Alternators.—It was seen (p. 988) that a single coil rotating in a magnetic field gives rise to an alternating electromotive force. To produce a sufficiently high e.m.f. for any useful purpose, the coil would require to have an immense number of turns, or else its rate of rotation would have to be very high. Also, in using a single turn, a large amount of space would be wasted. In order to get over these difficulties, many coils are arranged in series round a cylindrical core, and pole pieces lie outside the coils. The **alternator** shown in Fig. 942 has four poles and four rings. The field magnets in the case of an alternator must be separately excited by means of a continuous current. In small machines this current is supplied by a battery, but in large machines a separate direct-

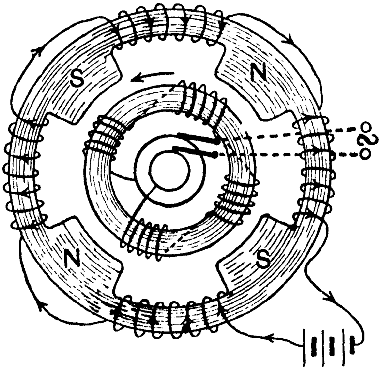


FIG. 942.—Four-pole alternator.

current dynamo is used for producing the field current. The regulation of the alternating voltage to secure constancy is performed by altering the resistance in the field circuit by means of a rheostat.

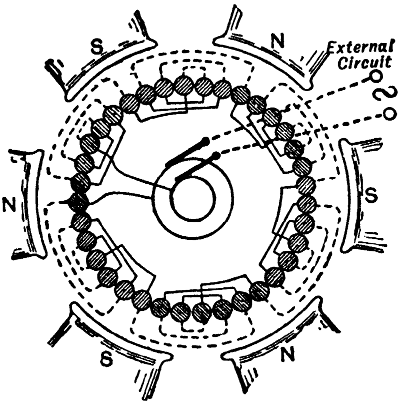


Fig. 943.—Rotor connections for a six-pole alternator.

The e.m.f. in the coils depends only upon the relative motion of the coils and the field magnet. In the case given (Fig. 942) the field magnets are fixed, and the coils rotate in the direction of the arrow. Some machines, however, are designed so that the coils are fixed and the field magnets rotate. For the e.m.f. in the above case to remain unchanged, the magnets would have to rotate in a direction opposite to that indicated by the arrow. Whichever part is fixed is called the **stator.** The rotating part is called the **rotor.**

As was seen on p. 992, the ring form of armature is wasteful in copper, hence it is desirable to arrange the conductors upon the outside only of the iron core. In Fig. 943 an arrangement of thirty-six conductors round the surface of the rotor is shown. These are supposed to be connected at the front as shown by the thick lines and at the back by the dotted lines. There are three pairs of poles in the stator. It will be seen that all the conductors of the rotor are in series and that they are grouped into sets in such a way that the e.m.f. in the sets at any one instant is the same, but in passing from one

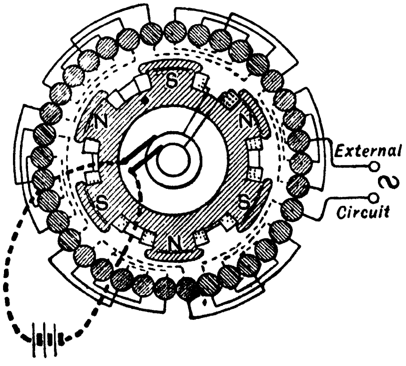


Fig. 944.—Stator connections for a six-pole alternator.

pole to the next of the same kind a complete alternation in e.m.f. is made.

In Fig. 944 is shown a similar arrangement, but with the field magnets as rotor, and the conductors, in which the alternating e.m.f. is being produced, as stator. The field is shown as though it were excited by a battery, but a separate direct-current dynamo is usually employed for this purpose.

EXERCISES ON CHAPTER LXXVII.

1. Explain the principle of the dynamo, and give sketches of (*a*) a series-wound, (*b*) a shunt-wound dynamo.

2. Give the laws of production of an electromotive force in a circuit when this is cutting across a magnetic field.

A closed coil of wire rotates slowly about a vertical axis, and a magnetic needle is suspended at its centre. As the coil rotates it cuts the earth's magnetic field. Represent by a curve, or other diagram, the deflecting couple acting on the needle during one revolution of the coil. L.U.

3. Describe some form of commutator by which the alternating e.m.f. in a coil rotating in a magnetic field may give rise to a current always in the same direction in an external circuit.

4. Describe the Gramme ring armature, and account for the fact that the brushes must be given a 'lead,' in order to obtain sparkless running.

5. Give a sketch of the connections of a drum armature, and trace the current through *one* set of conductors from one brush to the other.

6. What is 'compound winding'? Account for the advantages obtained by means of it.

7. Describe some form of alternator, giving the connections of the conductors of the armature.

8. An electromotor is supplied with current from the mains at a fixed difference of potential. When heavily loaded and running slowly the current passing is about 5 amperes. As the load is progressively diminished and the speed increased the current diminishes, attaining its minimum value when the speed of the motor is greatest. Explain how this occurs.
Madras University.

9. A continuous current shunt motor on being installed is found to rotate in the wrong direction. Describe what you would do in order to reverse the direction of rotation, and give reasons. C.G.

10. Describe some form of armature for a continuous current dynamo, stating the material of which each part is made and the reason why that material is used. Why is the armature core made up of a number of thin plates instead of a solid cylinder? C.G.

11. What is the "counter" or "back" electromotive force of a motor and on what does it depend? What change takes place in the speed of a series motor if the load is reduced? C.G.

12. In what forms may the efficiency of an electro-motor be expressed? How will the efficiency depend upon the speed in the case of a series motor?

13. Describe some form of electro-motor, and state how you would calculate the horse-power, neglecting friction and other losses, given that there are n straight conductors in the armature, each of which cuts across a magnetic field consisting altogether of 20,000 lines, N times per second, if 1 ampere flows in each conductor. L.U.

14. Find the electrical and commercial efficiencies of a series dynamo, if p.d. at terminals is 125 volts, current = 100 amperes, resistance of dynamo is 0 04 ohms and power supplied is 15·5 kilowatts.

15. Explain the terms electrical, commercial, and mechanical efficiencies of a dynamo.

Calculate the electrical and mechanical efficiency of a shunt dynamo when the p.d. between terminals is 150 volts, the external current 70 amperes, if the resistance of the armature is 0·06 and of the shunt coils 30 ohms, and the power wasted in friction etc. is 500 watts.

CHAPTER LXXVIII

THE TELEGRAPH, TELEPHONE, AND ELECTRIC LAMPS

The electric telegraph.—One of the most important applications of the electric current is its use for conveying messages. When the message is conveyed by means of a number of intermittent currents passing along a wire or cable from the transmitting to the receiving station, the arrangement is generally called a **telegraph.** A simple form of telegraph is shown in Fig. 945. A magnetic needle NS

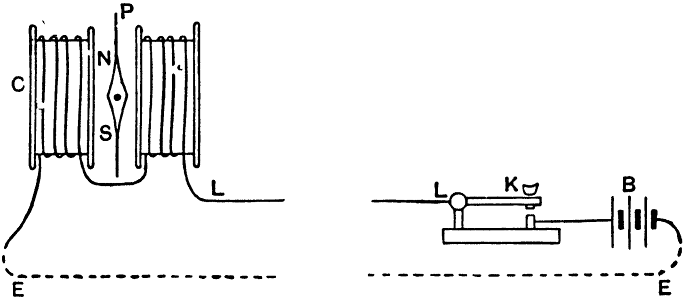


Fig. 945.—Simple telegraph.

is pivoted on the axis of a coil C, the arrangement being a simple form of galvanometer. Attached to the axis of rotation of the needle is a pointer P, so that the movement of the needle may be seen from outside the coil. This simple galvanometer, or indicator, is situated at the receiving station. At the transmitting station is a battery in circuit with a key K. The circuit between the two stations is completed by the line or cable LL, the return part of the circuit being either another line EE, or in most cases the earth itself. For this purpose a plate is buried in the earth at each station and the end of the circuit E connected to the plate.

It will thus be seen that on closing the key K, the needle and pointer are deflected, and by a code of signals any desired message may be transmitted. The code most frequently used is the **Morse code.** A long depression of the key corresponds to a dash and a short one to a dot. The Morse code is as follows :

		Short numerals preceded by FI.	Long numerals.
E .	T —		
I . .	M — —		
S . . .	O — — —	1 . —	1 . — — — —
H		2 . . —	2 . . — — —
A . —	N — .	3 . . . —	3 . . . — —
U . . —	D — . .	4 . . . . —	4 . . . . —
V . . . —	B — . . .	5 .	5 . . . . .
W . — —	G — — .	6 —	6 — . . . .
C — . — .		7 — — . . .	7 — — . . .
R . — .	K — . —	8 — . .	8 — — — . .
L . — . .	Y — . — —	9 — .	9 — — — — .
F . . — .	Q — — . —	0 —	0 — — — — —
P . — — .	X — . . —		
J . — — —		Followed by FF.	
Z — — . .			

With the arrangement shown in Fig. 945 it is only possible to send messages in one direction. It must therefore be understood that the apparatus is duplicated, so that at each end there is a transmitting battery and key, and also a receiving indicator. The line can only be used for sending messages in one direction at a time.

Sounders and ink-writers.—It is generally preferable in receiving messages to take them by ear rather than by sight. Thus the pointer P (Fig. 945) strikes a bell when the needle is deflected. Also, it is usually arranged that the current in the line can be sent in one direction or the reverse so that the deflections are to right or left, right corresponding to a dash and left to a dot in the code. If there are two bells of different tones, one struck by P when the deflection is to the right and the other when it is to the left, this does away with the necessity of watching the instrument. The observer recognises the dots and dashes by the tones of the two bells.

There are also special **sounders** that can be used in conjunction with the galvanometer at the receiving station. The Morse sounder is used in this way. A two-pole electromagnet A, only one pole of which can be seen in the diagram (Fig. 946), attracts a soft iron armature D when the current from the line flows in its coils. This pulls down the arm to which D is attached, the sound being produced by the screw C coming into contact with a stop. The screw B enables the distance of the soft iron D above the pole face to be adjusted to the proper amount. When the current ceases the arm is pulled up again by a spring which is not shown in the diagram.

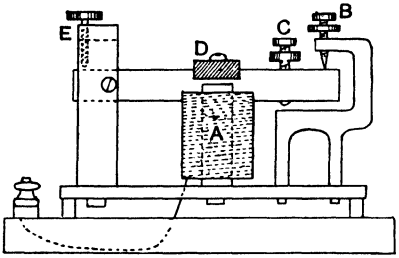


FIG. 946.—Morse sounder.

In the Morse ink-writer a wheel which is kept inked is carried by the pivoted arm. When the current from the line flows through the coils of the electromagnet the arm is pulled down and the inky wheel is brought into contact with a strip of paper which is driven past it by clockwork. A dash then corresponds to a line inked by the wheel upon this paper, a dot being a very short line.

For the purpose of signalling, a small **buzzer** is often employed. This is a small electro-magnet with a soft iron armature arranged as a make and break in the same manner as the interrupter of the induction coil (Fig. 918). A long buzz is used for a dash and a short one for a dot.

Duplex telegraphy.—In the simple system described above, the line can only be used for transmission of messages in one direction at a time. This means very low efficiency for the line, so that it is desirable to increase the efficiency by arranging that messages can be sent in both directions at the same time. This is called a **duplex** arrangement. There are also **multiplex** arrangements, but only the duplex telegraph will be described here. There are two widely used duplex systems, namely, the **differential** system and the **bridge** system.

The **differential** method is illustrated in Fig. 947. G_1 and G_2 are the galvanometers at the two stations. These galvanometers are wound in two similar sections so that the current from the battery B_1 will flow through these two sections in opposite directions when the key K_1 is depressed. If these two currents are equal, then the effect upon the galvanometer G_1 is zero. In order to make the currents equal, a resistance R_1 is placed in series with one of the coils, and the line in series with the other, the resistance R_1 being so adjusted that it is equal to the joint resistance of the line and the instruments at the far end. It follows that, on depressing K_1, current goes to the line, but the instrument G_1 is not affected. However, the instrument G_2 at the other end of the line is affected by this current, since it flows through the first coil and the key K_2 to earth, and also partly through the other coil and the resistance R_2. But in this case it flows through the two coils in series, so that the effect is the sum of the two, and the instrument G_2 therefore indicates the signals from the first station.

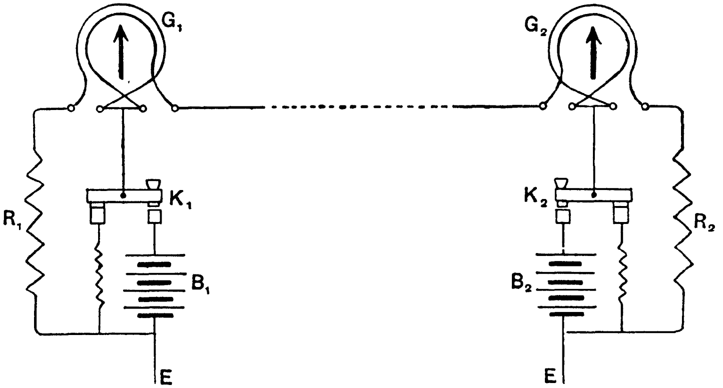


FIG. 947.—Duplex telegraph (differential).

With this arrangement the line is available for sending messages in both directions simultaneously, since at either station a depression of the key does not affect the galvanometer at that station, but does affect the galvanometer at the other end of the line. This differential system is chiefly used for land telegraphs.

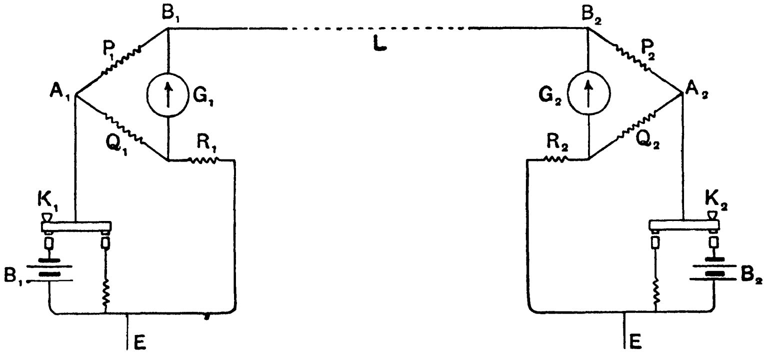


FIG. 948.—Duplex telegraph (Wheatstone's bridge).

The **bridge method** is employed for submarine cables. At one station the battery B_1 (Fig. 948) is connected by means of the key to one corner, A_1, of a Wheatstone's bridge. The four arms of the bridge are P_1, Q_1, the line L, and the resistance R_1.

Then, if R_1 is so adjusted that

$$\frac{P_1}{Q_1}=\frac{L}{R_1},$$

no current flows through the galvanometer G_1 on depressing K_1. The current from the line on reaching B_2 will pass to earth, partly through P_2 and partly through G_2, so that the signal is recorded at this end. A similar bridge arrangement is at each end of the line, so that messages can be sent simultaneously in both directions.

The galvanometer must be of a sensitive type, either a reflecting galvanometer, or the siphon recorder described below being used.

Relays.—For working an ink-writing machine, or a sounder, the currents from the line are, in many cases, too feeble. A **relay** is then employed. This consists of an electromagnet whose coils are in the circuit of the current at the receiver. These coils consist of a great many turns of wire so that a feeble current is able to magnetise the core. The arrangement of the Post Office standard relay is shown in Fig. 949.

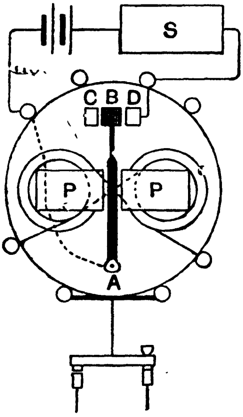


FIG. 949.—Post Office relay.

PP are the soft iron pole pieces of the electromagnet, the coils of which are wound differentially and connected up in the same way as the galvanometer coils in Fig. 947. At A is pivoted a soft iron arm which lies between P and P. This soft iron arm is maintained permanently magnetised by a steel permanent magnet, so that it is deflected to left or right according to the direction of the current from the line. Mounted upon the same axle as the soft iron arm is a light lever ending in the contact piece B, which makes contact with C or D. This closes the local circuit, consisting of a battery and the sounder or ink-writer S. Thus the current in the line, which is not itself strong enough to work the sounder, can close the local circuit by means of the much more sensitive relay, and thus enable the strong local current to work the sounder.

Siphon recorder.—In long-distance submarine telegraphy the currents are so feeble that a very sensitive recorder is necessary. The **siphon recorder** due to Lord Kelvin is commonly employed. It is really a suspended coil galvanometer. The coil A (Fig. 950) is suspended between the poles of a strong permanent magnet. *ab* and *cd* are two fibres connecting the suspended coil to a small block which can turn about the wire *ef* as axis. *s* is a fine tube fixed to the block *bd*, having one end dipping into an ink-pot and the other resting upon the paper at *p*, which is caused by rollers to travel under it. It will then be seen that a current in A will cause the point *p* to travel laterally upon the paper and so record the signal. *g* is a fibre attached to *s* at one end and to a vibrator at the other. This causes the point *p* to be continually lifted off and allowed to touch the paper, producing a fine dotted line as the paper travels forward. This reduces to a great extent the friction between *p* and the paper.

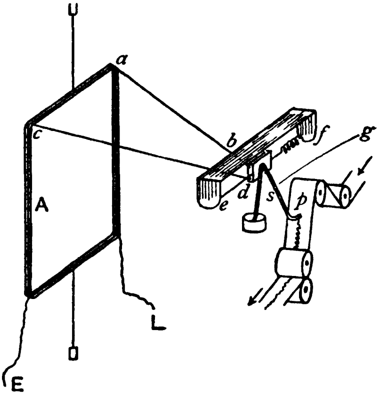


FIG. 950.—Siphon recorder.

The telephone.—Sounds arising at one place may be reproduced at another place by the arrangement due to A. Graham Bell known as the **telephone.** The sound waves arriving at the thin sheet-iron diaphragm A (Fig. 951) causing it to vibrate, a compression of the air (p. 682) driving it inwards and a rarefaction causing it to bulge outwards. Behind the diaphragm is a permanent horseshoe magnet NS, provided with a pair of soft iron pole pieces which nearly but do not quite touch the diaphragm. Coils having a great number of turns are situated upon these pole pieces, and are in series with the line LL, and a similar instrument acting as receiver at the other end. Magnetic flux, therefore, passes from the N pole of the permanent magnet through one pole piece, and by way of the soft iron diaphragm to the other pole piece, and the S pole of the magnet. The less the air space between the pole pieces and the diaphragm the greater will be this magnetic flux, and since this magnetic flux is threaded through the coils upon the pole pieces, it follows that any variation in position of the diaphragm causes an alteration in the magnetic flux, and hence produces an electromotive force in the coils. As the diaphragm follows the compressions and rarefactions of the air, a corresponding variation of the e.m.f. and therefore of current in the coils and in the line will take place.

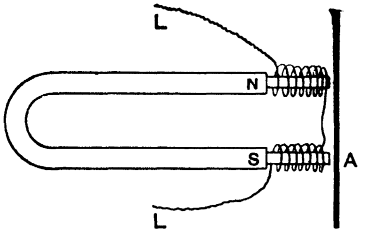


FIG. 951.—The Bell telephone.

On arrival at the receiving end, the current passes through a similar pair of coils upon the pole pieces, and the variation in magnetic flux causes a varying attraction between the pole pieces and the soft iron diaphragm. Hence the diaphragm of the receiver copies the motion of the transmitter and in doing so sets the air in motion, and reproduces the sounds which caused the motion of the diaphragm of the transmitter.

It is clear that the permanent magnet of the transmitter is necessary, otherwise there would be no magnetic flux at all, and consequently no current produced by the movement of the diaphragm. That the permanent magnet of the receiver is necessary is not at first sight obvious, since the varying current would produce a varying magnetic flux even without the presence of the permanent magnet. But it must be remembered that the movement of the diaphragm of the receiver depends upon the **variation** in the pull of the magnets upon it. Now, the pull per square centimetre is $B^2/8\pi$, where B is the magnetic induction (p. 815). If, now, B changes by the small amount b, it becomes $B+b$, and the pull is $(B+b)^2/8\pi$, or $(B^2+2Bb+b^2)/8\pi$. The increase of the pull is therefore $(2Bb+b^2)/8\pi$, or $2Bb/8\pi$ approx. Thus it is proportional to the increase in induction b, **and also to the value of the induction B.** Hence the permanent magnet maintains a fairly high value of B. This must not, however, be too high, for if the iron approaches saturation, the current will produce very little alteration in B, as will be seen from the curve, Fig. 763, p. 823.

The carbon microphone.—The Bell receiver can be used as a transmitter, as described above. But, owing to the feebleness of the currents produced, the intensity of the sound produced by the receiver is small. For this reason the **microphone** is nearly always used for the transmitter. This employs the fact that the resistance of a carbon contact is considerable, but is highly sensitive to mechanical disturbance. Thus, if a pointed carbon rod C (Fig. 952) is supported by two carbon blocks A and B, and the circuit is completed through a few cells and a Bell receiver, any disturbance of the carbon rod causes a loud noise to be heard in the receiver. The carbon contacts at A and B vary so much in resistance with any slight mechanical disturbance, that the current in the circuit varies sufficiently to produce a loud sound in the receiver.

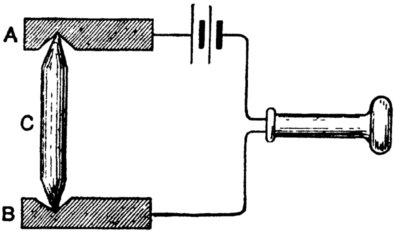


FIG. 952.—Carbon microphone.

In telephoning, single carbon contacts are not used, but granulated carbon is placed between a carbon diaphragm and a carbon plate. The arrangement of the transmitter is given in Fig. 953. D is a thin carbon diaphragm which vibrates in accordance with the sound waves arriving by the mouthpiece M. A is a carbon block having corrugations or protuberances. Between D and A granules of carbon, G, are situated. The movement of the diaphragm thus causes corresponding changes in the resistance of the carbon contacts between D and A, and therefore corresponding variations of the current going to the receiver by way of the line L.

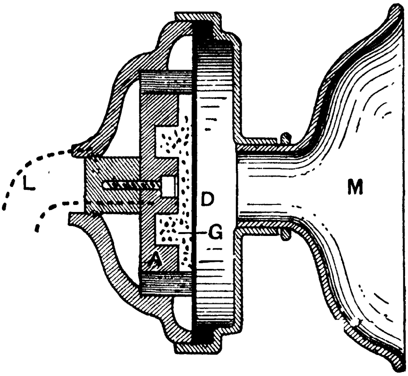


FIG. 953.—Carbon microphone transmitter.

Use of a transformer, or induction coil.—In order to keep the resistance of the microphone circuit low, so that any variation in resistance of the carbon contacts shall be as large a proportion of the total resistance as possible, and so produce large variations in current, it is usual to use a transformer or induction coil, as shown in Fig. 954. C is the induction coil, the low resistance primary of which is in series with the microphone M and a battery. The secondary, consisting of many turns, is in series with the line and the receiver at the other end of the line. In the position shown, the receiver R is supposed to be in use. When not in use it is hung up on the key K, which then throws the induction coil out of circuit and joins the line to the key K_2, and the distant station can then call up, since the line is in series with the electric bell B. To call up the distant station, the button of the key K_2 must be pressed. This puts the battery in series with the line and the electric bell at the distant station.

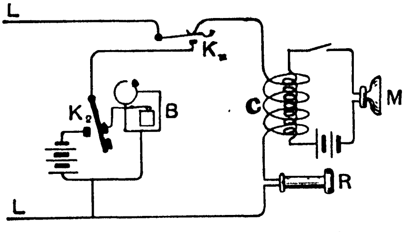


FIG. 954.—Transformer in telephone circuit.

Arc lamps.—Whenever an electric circuit is suddenly broken a spark occurs, owing to the inductance in the circuit (p. 986). As a rule this spark is very soon quenched; but if the electromotive force in the circuit is sufficiently high, and the materials where the break occurs are suitable, the current persists, the spark becoming transformed into an arc. The most suitable material for forming the arc is carbon, since this does not melt, even at the very high temperature of the arc.

The arc between carbon rods cannot exist permanently unless the e.m.f. in the circuit exceeds a certain value, about 40 volts. When the carbons are allowed to touch, and are then drawn apart, the positive carbon soon burns out into the form of a crater C (Fig. 955). Both positive and negative carbons burn away, but the positive more rapidly than the negative. There is a small area within the crater, at which the temperature is the highest, and reaches 3500° C. to 4000° C. It is from this point that most of the light of the arc is emitted. The crater forms most readily if the positive carbon has a soft core, and for this reason the positive carbons are usually 'cored,' the negative carbons being solid.

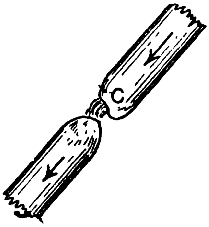


FIG. 955.—Electric arc.

It is usual to run arc lamps with a resistance in series. Thus, to run a 10 ampere arc lamp on a 100 volt circuit, a resistance must be placed in series with it. To calculate the resistance required, remember that 40 volts is required to maintain the arc, apart from its resistance. Then, $(100-40)/10=6$ ohms resistance must be placed in series.

Owing to the very high temperature of the arc, it has now many industrial applications. Silica may be fused in it; welding of iron and steel is performed by it; and the calcium in lime can be caused to unite with carbon forming the calcium carbide used for the production of acetylene gas.

Automatic arc lamp.—There are many arrangements for the maintenance of the arc, but that illustrated in Fig. 956 is a good example. Two soft iron plungers, A and B, are attached to a rocking arm pivoted at P. A lies partly within a solenoid through which the main current in the arc passes, while B has a high resistance and is in parallel with the arc. Before switching on the current, the carbons are in contact, but on completing the circuit the current flows in the series coil through the arc. This pulls down the plunger

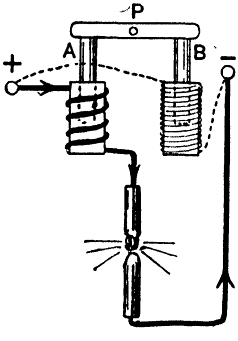


FIG. 956.—Principle of the automatic air lamp.

A, which by a suitable mechanism draws the carbons apart, and the arc is 'struck.' As the carbons burn away, the resistance of the arc increases so that more current will flow in the shunt coil and the plunger B is pulled down. This releases a brake and allows the mechanism to bring the arcs closer together. If the mechanism, which is not shown in the diagram, be properly adjusted, the differential effect of these two processes keeps the carbon the proper distance apart.

Incandescent lamps.—For the purpose of ordinary lighting, the **incandescent or glow lamp** is the most convenient. It consists of a filament of fairly high resistance which is rendered incandescent by the heat produced by the current in it. The filament is attached at the ends to pieces of platinum wire sealed into the walls of a glass bulb. The air is exhausted from the bulb containing the filament, for two reasons ; the air would cause oxidisation of the filament when hot, and would also remove heat so rapidly by conduction and convection that the lamp would have very low efficiency.

The older type of incandescent lamp has a **carbon filament.** The filament is constructed by dissolving cotton wool in zinc chloride solution, to form a paste, which is squirted through small holes into alcohol. On remaining in the alcohol for several hours the filaments harden. They are then removed, cleaned, and bent into the shape required for the lamp. This is done upon a carbon "former," and they are then packed round with carbon powder and heated to a temperature of about 550° C. in a carbon crucible, which converts them into the well known hard carbon filaments. The final treatment is the **flashing**, which consists in heating in benzine vapour by means of a current. Where the filament is thin, the temperature rises most and the benzine vapour is decomposed, and carbon is deposited upon the filament, thus rendering the thickness uniform. The filament is then fixed in its bulb which is exhausted and sealed off.

Carbon filament lamps have now been largely superseded by **metallic filament lamps,** the chief advantage of which is their high efficiency. The efficiency of the carbon lamp varies from about 4 watts per candle power for small lamps to 2·5 watts per candle power for large lamps, while the metallic filament lamp can be run at a higher temperature, and has an efficiency of about 1·0 to 1·5 watts per candle power. Recently, lamps taking only 0·5 watt per candle power have been produced, but these are only made in candle powers of 300 and upwards.

The metallic filaments have been constructed of various metals. Platinum is unsuitable as its temperature when running is not much below the melting point. Filaments constructed of the metal tantalum have proved of considerable efficiency. The later lamps, of which the Osram lamp is a type, have filaments of tungsten, annealed and drawn into wire.

Resistance and efficiency of incandescent lamps.—From Expt. 190, p. 878, it will be seen that the resistance of a carbon filament lamp falls as the temperature rises. Carbon is, in this respect, unlike the metallic conductors, whose resistance increases with rise in temperature. The resistance of carbon when white hot is about half that when cold. Thus a carbon filament lamp whose resistance when incandescent is 150 ohms, will have a resistance of about 300 ohms when cold. It follows that the luminosity of carbon filament lamps varies considerably for small fluctuations in the voltage of the supply. A slight drop in voltage reduces the current in the lamp and the consequent cooling causes an increase in resistance with further drop in current. The reverse effect is observed with a slight rise in voltage. On the other hand metallic filaments are much more steady in running, as a slight increase in voltage produces a rise in temperature and therefore an increase in resistance in the filament. This prevents any considerable rise in the current.

The term **efficiency of a lamp**, in its common use, is not logically applied. It is usually given in terms of **watts per candle power.** Thus the higher the number of watts per candle power, the less will be the real efficiency of the lamp. The measurement of candle power has already been described (p. 547), and to determine the efficiency, the watts must be measured (p. 878 or 973) at the same time.

The **life of the lamp** is also of great importance ; it should be at least 1000 hours for an efficient lamp. The watts per candle power in the case of a carbon filament lamp rises throughout the life of the lamp, owing to deposition of carbon upon the glass bulb with consequent absorption of light and lowering of the candle power. In the metallic filament lamp, there is a slight drop in watts in the first 100 hours with gradual increase afterwards, but the change is not nearly so great as in the carbon lamp.

EXERCISES ON CHAPTER LXXVIII.

1. Describe some simple form of telegraph for the transmission of messages electrically.

2. Explain clearly the use of ' Relays ' in the Morse telegraph, giving a diagram showing the connections. Allahabad University.

3. In what ways does the presence of a permanent magnet in a telephone receiver enhance its working efficiency and how is the result brought about ? C.G.

4. Give an account of some system in which messages may be sent in both directions simultaneously through a telegraphic cable.

5. Give a skeleton diagram and explain the working of
(*a*) A single current differential duplex circuit;
(*b*) A single current bridge duplex circuit. C.G.

6. Describe some arrangement for the automatic feeding of an arc lamp.

7. Sketch and describe the construction of any type of galvanometer used on a telegraph circuit, and state its simple function. C.G.

8. Describe briefly the process of making the carbon filament for an incandescent lamp. What is the reason for the process of "flashing"?

9. Describe some form of incandescent electric lamp, giving figures for volts, amperes, and candle power, and state how the resistance varies with change of temperature. C.G.

10. Why are metal filament lamps more efficient than carbon filament lamps? A metal filament lamp costing 2s. 3d. gives 25 c.p. with 30 watts, and is discarded after 500 hours; a carbon filament lamp costing 8d. gives 25 c.p. with 100 watts and is discarded after 800 hours. Find the cost per 1,000 candle hours, inclusive of lamp renewals, in each case, the price of current being 1½d. per B.O.T. unit. C.G.

CHAPTER LXXIX

THERMO-ELECTRICITY

Thermo-electric couples.—When all parts of a circuit, composed entirely of metals, are at the same temperature, there is no resultant electromotive force in the circuit and therefore no current. If, however, any point at which two different metals are in contact be at a different temperature to the rest of the circuit, there will be an electromotive force, and if the circuit be complete, a current will flow in it.

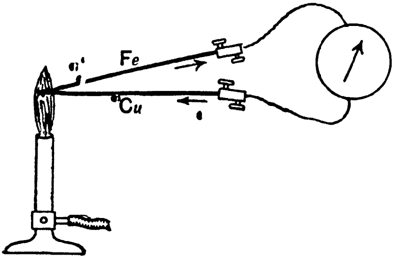


FIG. 957.—Copper-iron thermal couple.

This may easily be shown by joining a piece of copper and a piece of iron wire (Fig. 957), either by twisting them together, or, better still, by soldering them, the free ends being connected to a sensitive galvanometer. Such a pair of metals is called a **thermal couple.** On heating the copper-iron junction a deflection will be observed, showing that a current is flowing, and therefore that an electromotive force exists in the circuit. This **thermo-electric effect** was discovered by Seebeck in 1826, and bears his name. He arranged a number of metals in a list so that the current flows across the hot junction from the earlier to the later metal in the list. Amongst these metals were :

> Bismuth (Bi), Platinum (Pt), Copper (Cu), Lead (Pb), Tin (Sn), Silver (Ag), Zinc (Zn), Iron (Fe), Antimony (Sb), Tellurium (Te).

That the electromotive force, for a copper-iron couple, is not proportional to the excess of temperature of the hot junction over the rest of the circuit may easily be seen. For, as the temperature rises the current gets greater for a time and then ceases to grow. After this it decreases to zero and eventually becomes reversed, after which it gets greater and greater in this reverse direction.

Measurement of thermo-electric e.m.f.—If the deflection produced by a known e.m.f. in the circuit of Fig. 957 be determined, the galvanometer deflections can then be converted into volts or microvolts. A more satisfactory way of determining the relation between e.m.f. and temperature consists in employing a potentiometer method. The two similar resistance boxes A and B are placed in series with a cell C (Fig. 958) of known e.m.f. A Daniell's cell will do, if great accuracy is not required, the e.m.f. being taken as 1·1 volt. In making the measurements, the circuit ABC must have constant resistance, so that the current may remain constant. This is attained by starting with all the plugs out of B and only transferring plugs from A to their similar positions in B for producing a balance.

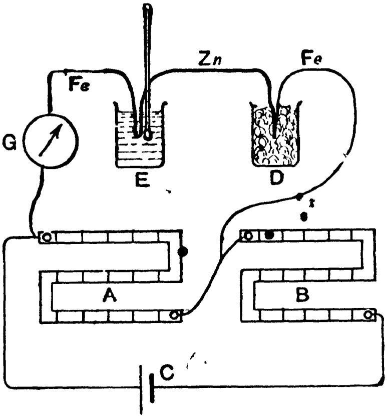


FIG. 958.—Potentiometer method of measuring thermo-electric e.m.f.

The thermo-electric couple is connected across the box A, the galvanometer G being in its circuit. One junction of the couple is situated in ice at D to keep its temperature constant, the other junction being raised to various temperatures in E. This is the potentiometer arrangement (p. 895) in which the pair of boxes A and B replace the stretched wire of Fig. 827. The e.m.f. in the thermo-electric couple is proportional to the resistance in A when the galvanometer deflection is zero, or

$$\frac{\text{e.m.f. of couple}}{\text{e.m.f. of cell C}} = \frac{\text{resistance in A}}{\text{total resistance in A and B}}.$$

The resistance of the cell may be neglected in comparison with that in A and B, which is usually of the order of 10,000 ohms. It is desirable, if possible, to attain a fairly high temperature for the hot junction. For the zinc-iron couple, the vessel E may be a crucible containing solder with the couple in an oil capsule. The e.m.f. is usually small, and is therefore conveniently measured in micro-volts.

The curve in Fig. 959 is that for a zinc-iron couple, one junction being at 0° C.

Thermo-electric diagram.—It will be seen that to include the e.m.f.'s for all possible pairs of metals upon one diagram, such as

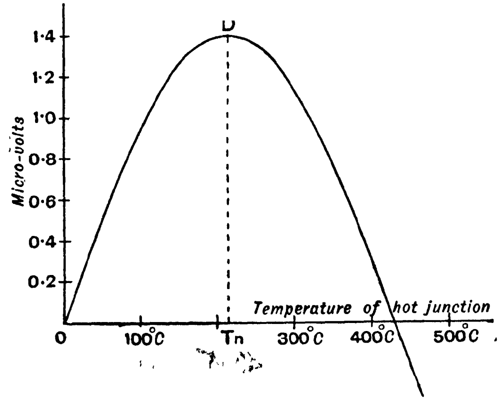


FIG. 959.—e.m.f.-temperature curve for zinc-iron couple.

Fig. 959, would lead to great complexity. Another method of representation due to Prof. Tait is therefore usually adopted. Instead of plotting e.m.f. against temperature, **the e.m.f. with unit difference of temperature between the junctions** is plotted. Thus, in Fig. 960, the ordinate tC represents the e.m.f. in the circuit of a couple when one junction is half a degree below t° and the other junction is half a degree above t°, so that the average temperature is t°. For nearly all metals in pairs, such a curve for any pair is a straight line AB.

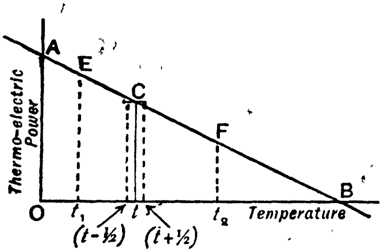


FIG. 960.—Principle of the thermo-electric diagram.

On such a diagram, the e.m.f. with one junction at t_1°, and the other t_2°, the area Et_1t_2F represents the corresponding e.m.f. in the circuit. For, the area of a strip such as tC is equal to the e.m.f., since the width of the strip is unity. In adding up such strips between t_1E and t_2F, the whole area is the sum of the areas of the corresponding strips, and thus represents the total e.m.f.

For a reason to be given later, curves, such as AB, are drawn for all the metals, using lead as one element of the couple. When this is done, a diagram such as Fig. 961 is obtained. This is known as the **thermo-electric diagram**, and the ordinate of each point on a line is known as the **thermo-electric power** of the metal at each temperature. The advantage of such a diagram as this, lies in the fact that the e.m.f. for any pair of metals for any difference of temperature between the junctions can be found.

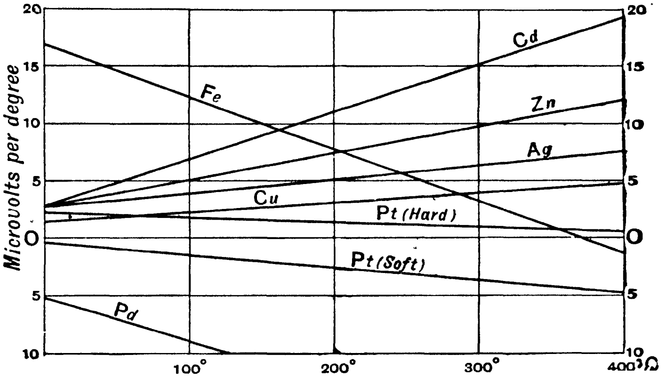


FIG. 961.—Thermo-electric diagram.

Let us suppose that $A_1B_1C_1DE_1$ (Fig. 962) is the thermo-electric line for one metal and $A_2B_2C_2DE_2$ that for another metal, plotted with respect to lead. Then, for temperatures represented by B and C, the e.m.f. for a couple consisting of the first metal and lead is represented by the area BB_1C_1C. Again, the e.m.f. for the second metal and lead is represented by the area BB_2C_2C. Hence the e.m.f. for a couple consisting of the first metal and the second is represented by the difference of these two areas, that is, by area $B_1B_2C_2C_1$.

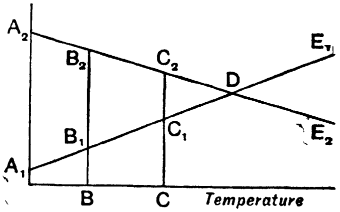


FIG. 962.—Explanation of the thermo-electric diagram.

Neutral point.—We can see from the thermo-electric diagram why there should be a diminution, followed by a reversal, of the e.m.f. in the case of a thermal couple (p. 1017). The thermo-electric diagram is so drawn that for any pair of thermo-electric lines such as $A_1B_1C_1$ and $A_2B_2C_2$ (Fig. 963) the e.m.f. acts one particular way round the circuit. This direction is anticlockwise. Thus for the two given metals, if the junctions are at temperatures A and B the e.m.f. is represented by the area $A_1B_1B_2A_2$ and acts across the hot junction from the metal 1 to the metal 2.

Since the thermo-electric lines are in general inclined to each other, they intersect at some point such as D. Hence, if the temperature of the hot junction be gradually raised, the lower junction being kept at fixed temperature, the area $A_1B_1B_2A_2$ gradually increases, which means that the e.m.f. in the couple increases. But the rate of increase gets less as the point D is approached. When the hot junction is at a temperature above that represented by the point D, as at C, the area DC_2C_1 must be deducted from A_1DA_2 in order to obtain the e.m.f. in the circuit. It will be seen from the arrows that the e.m.f. corresponding to the part DC_2C_1 must be deducted from A_1DA_2 in order to obtain the e.m.f. in the circuit. The arrows show that the e.m.f. corresponding to the part DC_2C_1 is acting the opposite way round the couple to that corresponding to A_1DA_2. D therefore corresponds to the turning point D of the e.m.f. curve in Fig. 959. The temperature corresponding to the point D is called the **neutral temperature**, T_n, for the two given metals.

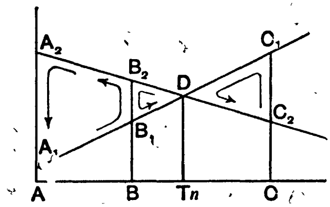


FIG. 963.—Neutral point.

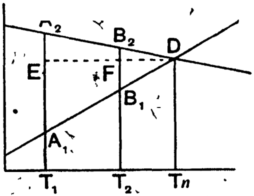


FIG. 964.—Calculation of e.m.f. from the thermo-electric diagram.

Thus, **when the temperature of one junction passes the neutral temperature, the e.m.f. begins to decrease.** Also, if one junction is as much above the neutral temperature as the other is below it, the e.m.f. in the circuit is zero. Again, when the difference of temperature between T_n and C (Fig. 963) becomes greater than the difference between A and T_n, the e.m.f. becomes reversed.

It is possible to find from the thermo-electric diagram, an equation for the e.m.f. in any couple, in terms of the neutral temperature and one other constant, depending upon the metals forming the couple. All that is necessary is to find the area of $A_1B_1B_2A_2$ (Fig. 964). Thus,

$$\text{Area of triangle } A_1A_2D = \tfrac{1}{2}A_1A_2 \times ED.$$

$$\text{Area of } B_1B_2D = \tfrac{1}{2}B_1B_2 \times FD;$$

$$\therefore \text{ e.m.f.} = \tfrac{1}{2}(A_1A_2 \times ED) - \tfrac{1}{2}(B_1B_2 \times FD).$$

Again, from similar triangles, the sides A_1A_2 and B_1B_2 are proportional to ED and FD respectively.

$$\therefore \text{ e.m.f.} \propto (ED^2 - FD^2).$$

But
$$ED = T_n - T_1 \quad \text{and} \quad FD = T_n - T_2;$$

$$\begin{aligned}\therefore \text{ e.m.f.} &\propto (T_n - T_1)^2 - (T_n - T_2)^2\\ &\propto T_1^2 - 2T_1T_n - T_2^2 + 2T_2T_n\\ &\propto (T_1^2 - T_2^2) - 2T_n(T_1 - T_2)\\ &\propto (T_1 - T_2)(T_1 + T_2 - 2T_n),\end{aligned}$$

or
$$\text{e.m.f.} = K(T_1 - T_2)\left(\frac{T_1 + T_2}{2} - T_n\right),$$

where K is some constant, which together with T_n must be found by experiment for each pair of metals.

The above equation shows that **the e.m.f. in any couple is proportional to the difference of temperature of the junctions and also to the difference between the neutral temperature and the average temperature of the junctions.** It thus appears that **the e.m.f. vanishes either when the junctions are at the same temperature,** or when **their average is the neutral temperature.**

Peltier effect.—When we seek for the origin of the energy required to maintain the current in a thermo-electric couple, we see that since the metals are unchanged in any way, the heat supplied by external agencies, such as the bunsen burner (p. 1016), is the only source of energy available. It is natural then to turn to the junction of the two metals as the place where heat energy is converted into current energy. This implies that there is an electromotive force acting across the junction from one metal to the other. It is clear that if both junctions are at the same temperature the e.m.f.'s at the two junctions are directed in opposite ways round the circuit and the resultant e.m.f. is zero. But if the e.m.f. at the junction changes with temperature, it follows that when there is a difference of temperature between the junctions, these e.m.f.'s do not balance, and there will be some resultant e.m.f. available for driving a current.

This e.m.f. at the junction of two metals was discovered by Peltier and is known as the **Peltier coefficient.** Like all other electromotive forces (p. 853), it implies a reversible condition. If the current flows in the direction in which the e.m.f. tends to drive it, heat energy is converted into electrical energy, but if it flows in the reverse direction, electrical energy is converted into heat. Thus, at a copper-iron junction the e.m.f. causes a current to flow from copper to iron at the hot junction, that is, where heat is being absorbed. This is the state of affairs in the upper diagram (Fig. 965). Indeed, if the current be caused to flow by applying some external source of e.m.f., such as a cell, heat will still be absorbed at this junction, and if this heat is not supplied by some agent, such as a burner, the heat is absorbed from the metal itself, which is thereby cooled. In the lower diagram the current is reversed, and in this case the junction becomes warmed.

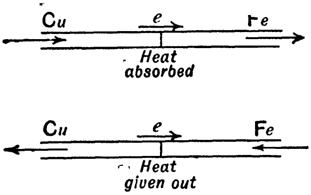


FIG. 965.—Peltier effect.

If the copper be replaced by bismuth and the iron by antimony, the same effect will follow, but it is much greater in this case, since a bismuth-antimony junction has a greater Peltier coefficient than one of any other pair of the common metals.

Even in the case of bismuth and antimony, the Peltier coefficient is only of the order of 0·03 volt, so that for a current of 1 ampere the work done at the junction per sec. is only $0{\cdot}03 \times 1$ joule or $0{\cdot}03 \times 0{\cdot}24 = 0{\cdot}007$ calorie. The amount warming or cooling is therefore not easy to observe. It may, however, be detected by soldering a bismuth bar between two antimony bars, as in Fig. 966, and passing a few amperes through. Then one junction is cooled and the other warmed, as may be shown by means of two similar pieces of fine platinum wire, A and B, wrapped round near the junctions. The difference in temperature produced by the Peltier effect causes one platinum wire to have a different resistance from the other (p. 851). Hence, if the two resistances are balanced upon a metre bridge before the main current flows, the heating and cooling will disturb the bridge balance, and we can see which junction is warmed and which cooled. With the current as shown in Fig. 966, A is warmed and B cooled.

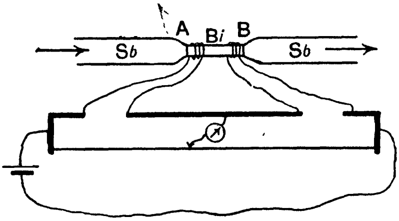


FIG. 966.—Demonstration of the Peltier effect.

This heating and cooling, known as the **Peltier effect,** must not be confused with the heating produced by all currents due to the resistance of the conductor. The latter is proportional to the square of the current, and is not reversible. If r be the resistance of the junction, i^2r is the energy in ergs converted into heat per second. Then, if π be the Peltier coefficient, πi is the energy in ergs converted from electrical energy to heat or *vice versa* in one second. Thus the total heating of any junction per sec. is $i^2r \pm \pi i$ ergs. The positive or negative sign must be taken, according to the direction of the current. In observing the Peltier effect, i^2r is made small by making r as small as possible, although in the above experiment i^2r will be approximately the same for both junctions, so that the difference of temperature is due to the Peltier effect only.

Thomson effect.—It was pointed out by Lord Kelvin (then Sir Wm. Thomson), on theoretical grounds, that there must be thermo-electric effects in a couple other than those occurring at the junctions. He found that **there is an electromotive force between different parts of the same metal when at different temperatures.** Thus, in copper there is an e.m.f. acting from the parts of lower to those of higher temperature, and in iron from parts of higher to those of lower temperature. Hence, if a copper bar be heated in the middle and a current passed through it, heat is absorbed as the current flows from colder to hotter parts, and given out as the current flows from hotter to colder parts. In fact, the current behaves exactly as a stream of liquid would do on account of its specific heat. On the other hand, in the case of iron the current gives out heat in passing from colder to hotter parts and absorbs heat in passing from hotter to colder parts. It is sometimes said that in iron the current behaves like a liquid of negative specific heat. The metals, cadmium, zinc, and silver behave like copper, while palladium behaves like iron. In fact, the direction of the slope of the thermo-electric line (Fig. 961) gives the sign of the Thomson effect.

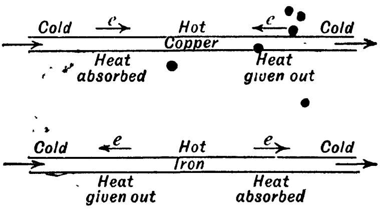


FIG. 967.—Thomson effect.

In lead the Thomson effect is zero, and it is for this reason that thermo-electric powers are always plotted with respect to lead (p. 1019).

Both the Peltier effects and the Thomson effects may be identified upon the thermo-electric diagram. For, if the temperature be reckoned from the absolute zero (p. 402) and A_1B_1, A_2B_2 (Fig. 968) are the thermo-electric lines for two metals, we have already seen that the e.m.f. of the couple is represented by the area $A_1B_1B_2A_2$. Further, it may be shown from thermo dynamic reasoning that the Peltier coefficients at the two junctions are represented by the areas HA_1A_2E and GB_1B_2F respectively. Also the e.m.f. due to the Thomson effect in the first metal is represented by HA_1B_1G and in the second by FB_2A_2E.

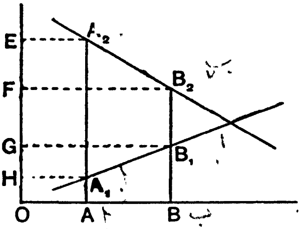


FIG. 968.—Peltier and Thomson effects upon the thermo-electric diagram.

Also,

$$FB_2A_2E + GB_1B_2F + HA_1B_1G - HA_1A_2E = A_1B_1B_2A_2.$$

That is, **the resultant e.m.f. in the couple is the algebraic sum of all the Peltier and Thomson e.m.f.'s.**

The thermopile.—These electromotive forces due to thermal effects are used for the measurement of radiant heat. In the **thermopile** a number of rods of antimony and bismuth are connected in series, the two metals alternating (Fig. 969). One set of junctions, B, is protected from external variations in temperature by means of a brass cap, while the radiant energy to be measured is allowed to fall upon the other set of junctions A. The junctions A, therefore, rise slightly in temperature, and the resulting electromotive force is equal to that for one bismuth-antimony couple multiplied by the number of couples. The resistance of the thermopile being small, it is desirable to use a low-resistance galvanometer to obtain as large a current as possible. The galvanometer in series with the thermopile then gives a deflection proportional to the difference of temperature of the junctions.

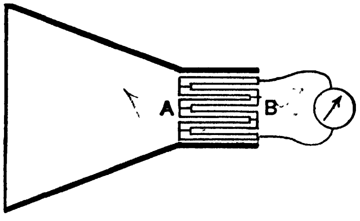


FIG. 969.—The thermopile.

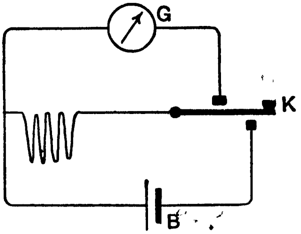


FIG. 970.—Demonstration of thermal e.m.f.'s by the thermopile.

By means of the thermopile, the existence of the thermal e.m.f.'s may be shown indirectly. Connecting it up to a cell B, galvanometer G, and a key K, as shown in Fig. 970, then on depressing the key a current flows through the thermopile. Owing to the Peltier effect, one set of junctions becomes warmed and the other set cooled. On raising the key the battery is cut out, and the galvanometer placed in series with the thermopile. The difference of temperature between the junctions produced by the current in the first part of the experiment now causes the thermo-electromotive force, and a current will flow in the galvanometer until the junctions are all brought again to uniform temperature.

Radio-micrometer.—In the radio-micrometer, due to Prof. C. V. Boys, the thermo-electric couple and the galvanometer are combined in one instrument. Two small bars, one of antimony and the other of bismuth, touch at their lower ends, the upper being connected through a loop of wire which hangs in the field of a powerful magnet NS (Fig. 971). The radiation to be detected falls upon the bismuth-antimony junction, and the resulting current in the coil, which is suspended by a quartz fibre and is provided with a mirror M, produces a large deflection for a very small amount of radiant energy.

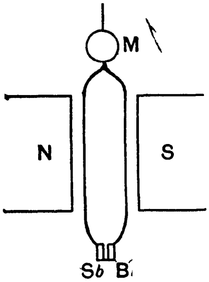


FIG. 971.—Boys's radio-micrometer.

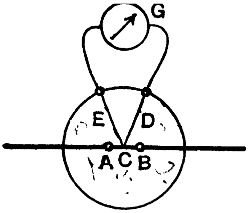


FIG. 972.—Thermo-milliammeter.

Thermo-milliammeter.—Several forms of **thermal galvanometer**, or **thermo-milliammeter**, have come into use. The current to be measured passes through a fixed wire of constantan AB (Fig. 972), which therefore becomes heated. A couple consisting of bismuth and tellurium has one junction soldered to the constantan wire at C, the other ends being connected to a galvanometer G by means of the leads E and D. The galvanometer scale can be calibrated by sending known currents through AB, and observing the deflection. The instrument is very sensitive and can be used equally well for direct or alternating currents. The sensitiveness is increased by enclosing the thermo-electric part of the apparatus in an exhausted glass bulb.

Pyrometers.—It will be seen, on referring to Fig. 959, that on attempting to use the thermal electromotive force for the measurement of temperature, the result is ambiguous, for each e.m.f. corresponds to two different temperatures. If, however, we could find two metals for which the thermo-electric lines are parallel, or very nearly parallel, it would follow from the reasoning on p. 1020 that the e.m.f.-temperature curve would be a straight line, and each e.m.f. would correspond to one particular temperature. The couple can then be placed in series with a galvanometer or milliammeter, and if one junction be kept at constant temperature the scale of the galvanometer can be so calibrated that it indicates the temperature of the other junction. Such an arrangement, when used for measuring high temperatures, such as furnace temperatures, is called a **thermo-electric pyrometer.** The couple for this purpose usually consists of platinum and rhodium, or of platinum and an alloy of platinum and rhodium. In Fig. 973 such a pyrometer is shown. The couple is placed in an iron or porcelain tube, and the scale of the reading instrument is graduated in degrees, up to 1000° or 2000° C.

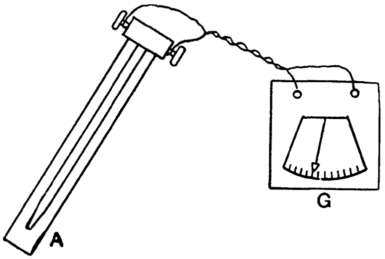


FIG. 973.—Thermo-electric pyrometer.

EXERCISES ON CHAPTER LXXIX.

1. What is meant by a thermo-electric couple? Describe the construction of a thermopile. State whether you would select a galvanometer of high resistance or one of low resistance for use with the thermopile, giving reasons for your answer. L.U.

2. Describe some method of measuring the e.m.f. of a given thermo-electric couple, and its variation with temperature.

3. How does the thermo-electric electromotive force for a given pair of metals vary as the temperature of one junction is raised while the other is kept at constant temperature?

4. Describe the Seebeck, the Peltier and the Thomson effects.

5. Show how thermo-electric powers may be represented on a diagram, and describe how the various thermal e.m.f.'s in a circuit may be represented by areas upon this diagram.

6. Give an account of thermo-electric inversion and neutral temperature.

7. Describe some form of thermo-electric pyrometer for measuring high temperatures.

8. Describe the Peltier effect, and give an account of some method by which its existence and sign may be determined.

9. What is the meaning of the term thermo-electric power? In what units is it measured?

CHAPTER LXXX

CURRENT IN GASES: X-RAYS: RADIOACTIVITY

Electric spark.—The fact that the air space between two conductors may become conducting has been mentioned several times. With air at atmospheric pressure, the difference of potential that must exist between the conductors before any current passes is considerable. It depends, moreover, upon the distance apart of the conductors, and also upon their shape. It was seen on p. 937 that the discharge takes place most readily from points. Since the shape of the conductor is of such importance in determining whether the discharge will take place, it is usual in making measurements upon the sparking potential to employ spheres as terminals.

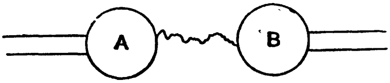


FIG. 974.—Electric sparks.

As the difference of potential between two knobs A and B (Fig. 974) is caused to rise by connecting them to an electrical machine or induction coil, the first form of the discharge to be observed is nearly silent. A faint hissing noise is heard, and in the dark a faint violet-coloured glow can be seen. This is called the **brush discharge.** On further raising the potential, a point is eventually reached at which a sudden crackling discharge takes place. This consists of a succession of discharges, each one being accompanied by a luminous streak of light between the knobs, of a form almost exactly like a flash of lightning. This is called the **spark discharge.**

The presence of points or roughness upon the conductors facilitates the brush discharge. Hence the use of **lightning conductors** to protect buildings. For, the earth being at a considerably different potential from the thunder cloud above, the lightning conductor, which consists of a strip of copper, earthed at its lower extremity and ending in a number of sharp points above, facilitates a quiet or brush discharge between earth and the cloud. This reduces the difference of potential and lessens the probability of a disruptive discharge taking place.

Current in gas at low pressure.—If the pressure of the air is continually reduced, the character of the discharge undergoes a number of changes. At about one-half to one-third of the atmospheric pressure the crackling nature of the discharge ceases, and it assumes a silent streamer-like form, and becomes coloured. The colour depends upon the nature of the gas in the tube, being a pinkish colour in the case of air. With further reduction in pressure, the discharge spreads out until it fills the tube, and at this stage it is highly luminous. This is the form known as the Geisler tube, and the difference of potential between the ends of the tube is very much less than that for the discharge at atmospheric pressure.

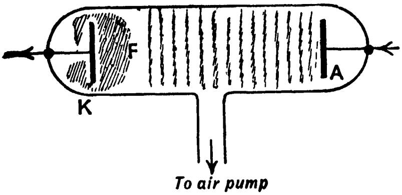


FIG. 975.—Discharge at moderately low pressure.

At this stage, a want of symmetry in the discharge becomes apparent. Near the electrode at which the current leaves, that is, at the kathode K (Fig. 975), a dark space appears and this becomes more pronounced as the pressure is reduced. It is called the **Faraday dark space**, F, and is separated from the kathode by a bluish glow. From the Faraday dark space to the anode A and extending quite up to it is the **positive column.**

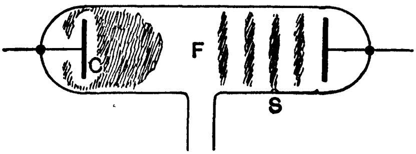


FIG. 976.—Discharge at low pressure.

When the pressure falls to about 0·1 mm. of mercury the appearance is somewhat as shown in Fig. 976. The kathode glow has increased in size and is seen to be separated from the kathode by a dark space C of constant thickness. This is called the **Crookes dark space.** The positive column is seen to consist of disc-shaped **striations** S. On reduction of pressure, the scale of the whole phenomenon grows, the place from which it grows being the kathode. Hence the Crookes dark space, the kathode glow, and the Faraday dark space all increase in size. The striations also become larger and more distinct, but, of course, fewer in number, as they always extend as far as the anode however near or distant that may be.

It will easily be seen that on further reduction of pressure, say to 0·01 mm., there will not be room in an ordinary vacuum tube for any positive column at all. The Crookes dark space will then occupy the greater part of the tube and only part of the kathode glow will be present.

Kathode rays.—In the Crookes dark space several phenomena may be observed. The boundary of this dark space is always luminous. When the boundary is the gas in the tube, the luminescent gas constitutes the kathode glow, but when the dark space extends to the walls of the tube, a bright fluorescence is produced. This is usually coloured, the colour depending upon the nature of the material. With soda glass the wall of the tube fluoresces a bright green, as may be seen in the case of the X-ray tube (p. 1033). With lead glass the fluorescense is blue. Many minerals give characteristic colours ; corundum is a bright crimson and zinc sulphide a bright blue.

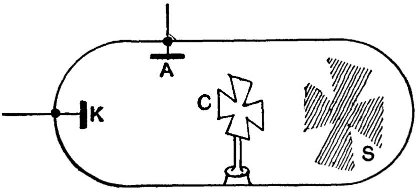


FIG. 977.—Kathode rays stopped by metallic screen.

Whatever it is that produces this effect is travelling in straight lines from the kathode and can be stopped by dense bodies; for a metallic screen C (Fig. 977) placed in front of the kathode casts a shadow upon the walls of the tube. The name **kathode rays** is given to the emission from the kathode which produces these effects.

Mechanical effect of kathode rays. If the kathode rays fall upon a little mill wheel W having light mica vanes, the wheel is caused to rotate when one half of it is shielded from the rays. Thus in Fig. 978 if A is kathode the rotation is as shown, but if B is kathode the rotation is in the opposite direction. The explanation of this is not quite so simple as might at first appear, but it is undoubtedly due to the rays from the kathode falling upon the vanes.

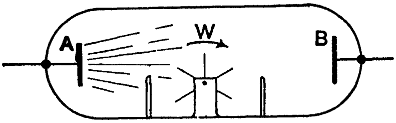


FIG. 978.—Mechanical effect of kathode rays.

Heating effect of kathode rays.—When falling upon a surface, the kathode rays always heat the material upon which they fall. If the rays, which are emitted normally from the kathode, are concentrated upon a small area of a platinum surface (Fig. 982), the platinum may be raised to red heat.

Electrical charge carried by kathode rays.—On catching the kathode rays in a hollow metallic vessel, placed within the vacuum tube, the vessel acquires a negative charge. Thus, if the vessel V (Fig. 979) be connected to an electrometer or electroscope, it will be found to acquire a negative charge when the kathode rays enter it. Thus, whatever their nature may be, the kathode rays are certainly accompanied by a stream of negative electricity.

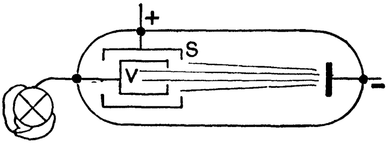


FIG. 979.—Electric charge carried by kathode rays.

Effect of magnetic field upon kathode rays.—Kathode rays are easily deflected by a magnetic field. They are driven in a direction at right angles to both their own path and the magnetic field. This may easily be shown by allowing the beam of kathode rays passing through a narrow slot in the screen S (Fig. 980) to pass along a screen AB covered with a layer of zinc sulphide. The zinc sulphide fluoresces a bright blue in the kathode rays. Their path is therefore marked by a straight streak upon the screen. On advancing the N pole of a magnet towards the vacuum tube the beam becomes curved downwards. Considering the kathode rays to be a stream of charged particles emitted by the kathode, the direction of their deflection, as indicated by the **left-hand rule** given on p. 968, shows that the charges must be **negative** electricity. This confirms the conclusion from the last experiment (Fig. 979).

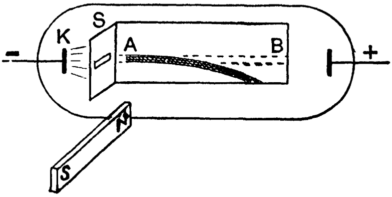


FIG. 980.—Deflection of kathode rays in a magnetic field.

Nature of the kathode rays,—electrons.—It is beyond the scope of this work to consider the methods by which the nature of the kathode rays was established. It is, however, of interest to note that they are now considered to consist of particles whose mass is $\frac{1}{1860}$ of that of an atom of hydrogen (*i.e.* $8{\cdot}8 \times 10^{-28}$ gr.), and with each particle is associated a negative charge of $1{\cdot}57 \times 10^{-20}$ electromagnetic units. They are now universally called **electrons.**

Owing to the negative charge of the electron it has an acceleration in the electric field applied to the tube, which acceleration is very great owing to the small mass of the electron. Since their discovery it has been found that they are fundamental constituents of all atoms and play a part in phenomena as widely separated as the conduction of electricity and the emission of light.

The mass and charge of the electron are constant, whatever may be the material of the electrode or gas in the tube, in fact, the charge of an electron seems to be the ultimate and indivisible unit of electricity.

Canal rays—Positive particles.—Electrons, owing to their negative charges, experience forces driving them away from the kathode. There are, however, **positively charged particles which travel towards the kathode.** Since these positive particles travel towards the kathode and eventually strike it, they were not discovered until a perforated kathode was used. This is a kathode having a number of holes or canals bored through it (Fig. 981). Many of the positive particles, travelling towards the kathode pass through these canals, and form faintly luminous streamers called **positive rays** or **canal rays** upon the side remote from the anode.

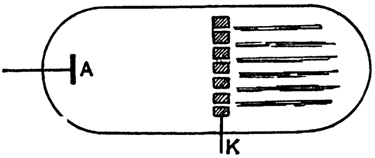


FIG. 981.—Canal rays.

These positive rays can be deflected only by very powerful magnetic fields. The direction of deflection is then found to be **opposite to that of the kathode rays**, which indicates that their charges are positive. They are much more complex than electrons, and have masses depending upon the nature of the gas in the tube. The smallest mass of any particle is equal to the mass of an atom of hydrogen, and their masses are proportional to the atomic weights of the various elements. Their charges are always multiples of that of the electron, the lowest being equivalent to an electronic charge, but, of course, of opposite sign. It seems probable that the positive rays consist of ordinary atoms which have lost one or more electrons.

Röntgen rays, or X-rays.—One of the most important properties of the kathode rays is that whenever they strike any material substance a new form of radiation is emitted. This radiation has great penetrating power and produces photographic and electrical effects. It was discovered by Röntgen and was called by him **X rays.** A modern form of X-ray tube is shown in Fig. 982. It is exhausted until the Crookes dark space is larger than the tube itself, so that there is no kathode glow present.

The kathode K is of aluminium and is concave, so that the kathode rays come to a focus at a small spot upon a sheet of platinum B placed at an angle of 45° to the axis of the kathode. B is sometimes called the antikathode and is connected to an aluminium anode A. The presence of A is not essential.

The X-rays arise from the small spot upon B, upon which the kathode rays impinge. They have so great a penetrative power that they pass through the walls of the tube. Their presence may be detected by the fluorescence they produce in certain substances. Barium platino-cyanide is one of the best substances for this purpose, and it is usually spread upon a cardboard screen C. On looking at the side D upon which the fluorescent material is spread, a pale blue fluorescence due to the X-rays may be seen.

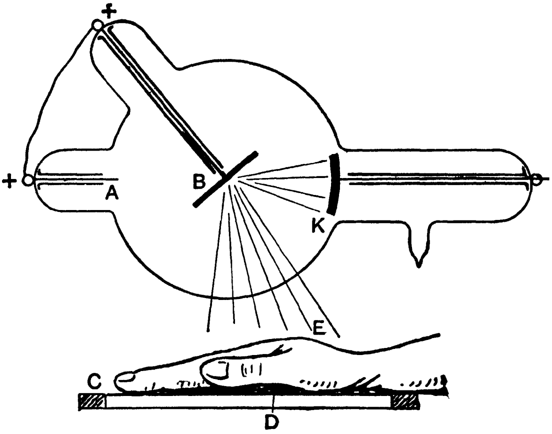


FIG. 982.—Production of X-rays.

The penetrative power of the X-rays depends upon the vacuum in the tube and also upon the density of the material through which they are passing. Thus the higher the vacuum, the greater the penetrative power of the X-rays. With very high vacuum and great penetrative power of the X-rays, these are said to be **hard** X-rays. The reverse are said to be **soft** X-rays. Since the penetrative power depends upon the density of the material, a body such as E (Fig. 982), placed in the path of the X-rays will cast a shadow upon the screen, indicating the variation in density of structure of the body. If the body E be the human hand, the bones being dense, obstruct the rays and cast a dense shadow. The fleshy parts are not so dense, and the parts of the screen illuminated by the rays passing through these parts are brighter.

If the screen C be replaced by a photographic plate, the plate is acted upon by the rays to an extent depending upon the intensity of the rays. On developing the plate a negative of the object is obtained and a positive may be printed from it in the ordinary way. Such a photograph is called a **radiograph**. A modern form of X-ray tube is shown in Fig. 983.

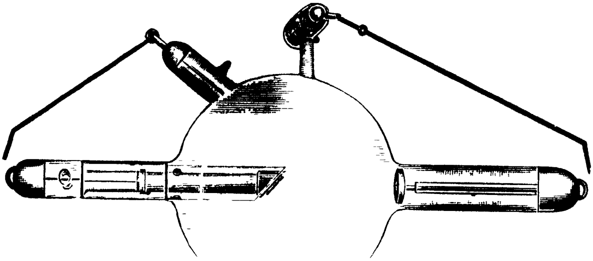


FIG. 983.—X-ray tube.

Ionisation produced by X-rays.—Even more important than their fluorescent and photographic effects is the power which X-rays possess of rendering the gas through which they pass a conductor of electricity. If a battery consisting of a number of cells be joined to two conductors between which is situated a gas, an appreciable current will flow through the gas when X-rays are passing through it. The reason is that the X-rays have the power of liberating electrons from the atoms of the gas. These electrons, having negative charges, the remaining parts of the atoms are therefore positively charged, and these **ions**, as they are called, experience forces in the electric field, causing the positive ions to move towards the kathode and the negative ions towards the anode. These drifts of charges constitute an electric current. The process is called **ionisation** and the gas is said to be **ionised**.

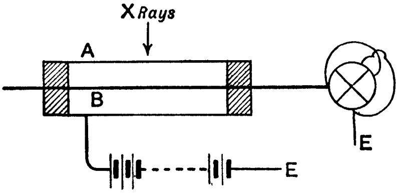


FIG. 984.—Ionisation current.

An ionisation current may be exhibited by insulating a wire B (Fig. 984) which passes axially down a thin aluminium tube A, also insulated. B is connected to one pair of quadrants of an electrometer, the other pair being earthed. A is connected to one terminal of a battery of cells of which the other terminal is earthed. Starting with B at zero potential, by momentary earthing, it will be seen that on passing a beam of X-rays through the tube A, the needle of the electrometer will have a continually increasing deflection. This shows that a current is passing through the gas and is charging the wire B and the quadrants of the electrometer.

Nature of X-rays.—It is now known that X-rays are identical in character with light, that is, they consist of a wave motion. But whereas the shortest wave-length of light waves is about $0{\cdot}4 \times 10^{-4}$ cm., that of X-rays is of the order of 10^{-8} cm. For this reason their penetrative power is much greater than that of light waves. The wave-length of X-rays has been determined by observing their mode of reflection from certain crystals, and has given rise to a more intimate knowledge of the nature of crystalline structure than had hitherto been attainable.

Secondary X-rays.—When X-rays fall upon any material, other X-rays are emitted. These are of various types. Some are merely the incident X-rays scattered by the material, but others are of an extremely homogeneous form, and have a wave-length characteristic of the material upon which the primary X-rays fall. There are two types of these homogeneous secondary X-rays, one the "**Series K**" being very '**hard**' or penetrating, and the other "**Series L**" being '**soft**' or less penetrating. The quality of secondary X-rays emitted depends only upon the material emitting them and not upon the quality of the primary X-rays which cause their emission, but the primary X-rays must always be 'harder' than the secondary rays emitted. The higher the atomic weight of the element constituting the material, the harder will be the secondary X-rays emitted.

Radioactivity. There are a few rare substances which emit rays which are in some respect similar to X-rays. That is, they can penetrate ordinary materials and can produce fluorescence, photographic effects and ionisation. Such substances are said to be **radio-active**, and are uranium, radium, thorium and actinium. Of these substances radium is by far the most active, but the property of radioactivity was discovered by observations on uranium.

Of the various effects produced by the rays emitted by radioactive substances, that of ionisation lends itself best to a study of the properties of different substances. To measure the ionisation, the gold-leaf electroscope is very convenient, and several forms of this instrument have been devised for the purpose. That described on p. 946 (Fig. 882) was devised by C. T. R. Wilson. The motion of the leaf D is observed by means of a short focus telescope with a scale in the eyepiece. The rate at which the image of the leaf passes the divisions of the scale is then a measure of the rate of leak to earth of the charge upon the leaf, and this in turn is a measure of the conductivity of the air, or the ionisation due to the rays entering the chamber.

If a minute amount of radium be situated below B, the leaf will collapse rapidly, but if some salt of uranium or thorium be spread upon a piece of paper and placed below B the leaves will collapse comparatively slowly.

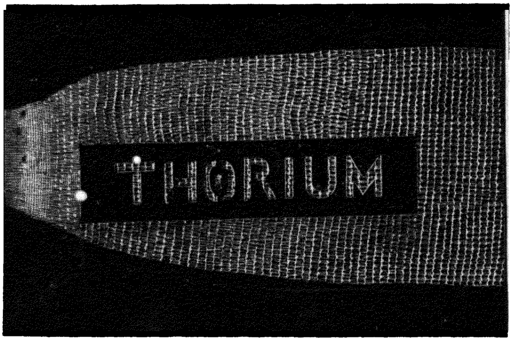


FIG. 985.—Effect produced by thorium upon a photographic plate.

The photographic effect of the rays from thorium may be shown in an interesting manner. If an ordinary incandescent gas mantle be placed in contact with a photographic plate and left for about a week, then on developing the plate the pattern of the mantle will be seen. The mantle is impregnated with thorium and the radiation from the thorium affects the plate. Fig. 985 has been obtained in this way, the letters being produced by placing a piece of tinfoil, from which the letters have been cut, between the mantle and the plate. This tinfoil shields the plate from the rays.

α, β and γ rays.—These rays, emitted by a radioactive substance, are complex. They may be distinguished, one from the other, by measuring, by means of an electroscope, their penetrating power for various layers of material. Some thorium oxide may be spread upon a sheet of glass and placed immediately under the aperture B of the electroscope (Fig. 882), the cover being removed. The time required for the leaf to pass from some particular scale division to another is then noted. A thin sheet of paper is then placed upon the thorium oxide and the observation repeated. Other layers of paper are then added, and it will be found that the ionisation is cut down to a small fraction by the first sheet, say to $\frac{1}{4}$, but the second sheet will not produce much further reduction. The reason for this is that some of the rays have very little penetrating power and are nearly all absorbed by the first layer of paper. These are called the α **rays.**

If now a sheet of paper be placed upon the thorium oxide, to cut off the α rays, and a similar experiment be performed by adding layers of thin aluminium foil, it will be found that the first layer produces a disproportionately large decrease in the ionisation. This shows as before that the remaining rays, when the α rays have been removed, are still complex. The most easily absorbable of these remaining rays are called β **rays** and the most penetrating of all are called the γ **rays.**

The following table illustrates the relative penetrating powers of the α, β and γ rays :

Rays.	Thickness of aluminium required to reduce the intensity to one-half.	Relative penetrating power.
α	0·0005 cm.	1
β	0·05 cm.	100
γ	8 cm.	10000

The methods by which the properties of these rays have been studied cannot here be entered into, but it should be noted that the α rays are of the same nature as the positive rays in the discharge tube, the β rays are like the kathode rays and the γ rays are identical in kind with the X-rays. Thus the α rays consist of positively charged particles, emitted with a velocity which sometimes reaches $2{\cdot}5 \times 10^9$ cm. per sec. The β rays consist of electrons with velocity, in some cases, as high as $2{\cdot}85 \times 10^{10}$ cm. per sec., which is very near the velocity of light.

The spinthariscope.—Many minerals fluoresce when α rays fall upon them. Thus, the diamond exhibits a blue fluorescence which enables a true stone to be distinguished from a false one. Zinc sulphide fluoresces brightly in α rays. This fact has been made use of in constructing the **spinthariscope**, which is a thin layer of zinc sulphide behind which a speck of radium bromide or some other material which emits α rays is placed. The zinc sulphide is examined by a short focus lens, and a bright and continuous shower of sparks is seen when the observation is made in the dark. Each flash is due to the rupture of a minute crystal of zinc sulphide when struck by an α ray particle.

Emission of heat by radium.—It is a well-established fact that radioactive substances emit heat. In the case of radium bromide, the temperature is always about 2° C. above that of its surroundings. The rate of emission of heat has been measured by means of the Bunsen's ice calorimeter (p. 448), and it has been found that 1 gram of radium emits 100 calories per hour, and that the rate of emission is constant whatever the temperature of the radium. That there is an immense store of energy in radioactive materials is evident from the fact that during its whole life 1 gram of radium emits about 10^{10} calories. This store of energy accounts to some extent for the energy continually emitted from the sun in the form of heat.

Radioactive changes.—An examination of the changes occurring in a radioactive material has shown that the process of radioactivity accompanies a change in nature of the substance. Thus the α ray particle has an atomic weight of about 4 and is now known to be a helium atom which has lost two electrons and is therefore positively charged This is in accordance with the fact that the radioactive elements are continually producing helium, and that helium is always found associated with minerals containing radioactive substances.

There are two series of radioactive substances, one series beginning with uranium, the heaviest known atom, and the other series with thorium. The uranium atoms each lose an α particle and become uranium X_1, but the process is so slow that in 5×10^9 years only half the atoms in any given quantity of uranium have changed in this way. Uranium X_1 loses an electron and becomes uranium X_2, and uranium X_2 loses an electron and becomes uranium IIa, both changes being comparatively rapid, half the substance having changed in 23·5 days and 1·17 minute respectively. The atoms of uranium IIa do not all change in the same manner, some of them become ionium and some uranium Y in losing an α particle; the former lose another α particle and become radium, and the latter lose an electron and become proto-actinium, which loses an α particle and becomes actinium.

The further changes that occur to radium afford a typical example of radioactive progress. A new substance can be separated from radium on heating it or dissolving it in water. This substance is a gas at ordinary temperatures, but can be condensed at the temperature of liquid air. It is called **radium emanation** or **niton.** The emanation decays to half its quantity in 3·85 days, and in doing so changes in turn to radium A, radium B, Ra C, Ra D, Ra E and Ra F. The change from radium to the emanation is accompanied by the emission of α and β rays. That from the radium emanation to Ra A by the emission of α rays; from Ra A to Ra B by α rays; Ra B to Ra C by β rays; Ra C to Ra D by α, β and γ rays. The change from Ra D to Ra E occurs without the emission of rays; Ra E to Ra F by β rays, and Ra F in changing to some unknown product by emission of α rays. The decay to half its quantity for the successive substances, radium, etc., to Ra F occur respectively in about 1730 years, 3·85 days, 3·0 minutes, 26·8 minutes, 19·5 minutes, 16·5 years, 4·8 days and 136 days.

The other radioactive materials undergo similar changes, some being more and others less complicated than those of radium. It is interesting to note that in the course of its changes, a uranium atom emits three α ray particles, and since each has an atomic weight 4, this would reduce the atomic weight of uranium from 238·5 to 226·5, and the atomic weight of radium is 225. Further, a radium atom during its changes emits five α ray particles, which would bring its atomic weight to 206·5, and that of lead is 206·9. It is a significant fact that minerals containing uranium and radium always contain a large proportion of lead.

Whether it will be found that all elements are radioactive and are undergoing changes similar to those of the radioactive materials, but vastly slower, only the future can decide.

EXERCISES ON CHAPTER LXXX.

1. Describe the changes which occur in an electric spark in air as the pressure of the air is gradually reduced to a high vacuum.

2. Give an account of kathode rays and the effects they produce. What are canal rays?

3. Describe the effect of a magnetic field upon kathode rays, and deduce some property of these rays.

4. Describe the production of X-rays and state what you know regarding their nature.

5. Give some account of radioactivity, and describe how its intensity may be measured.

6. Give a short account of the radioactive changes which occur in the case of some one substance.

7. Describe the apparatus you would use for obtaining Röntgen rays, and show how you would arrange the apparatus to obtain a radiograph of the hand. L.U.

CHAPTER LXXXI

WIRELESS TELEGRAPHY AND TELEPHONY

Radio-telegraphy.—There are many methods of signalling in which a code of dots and dashes, or short and long sounds, such as the Morse Code (p. 1005) is employed. In ordinary telegraphy a buzzer, sounder, or galvanometer at the receiving station is connected with a battery and key at the sending station, by means of a wire, or pair of wires, so that the circuit may be completed by the key at the sending station. But it is essential to this method that the stations should be connected by the wire or cable, which in many cases has a length of several thousand miles. In **radio-telegraphy**—or, as it is generally called, **wireless telegraphy**—this wire or cable is dispensed with, a series of electro-magnetic waves being given out at the sending station, which on arriving at the receiving station actuate a telephone, producing long or short sounds corresponding to dashes and dots, according to the length of each series of waves.

In the middle of the last century Lord Kelvin showed by calculation, that if a charged condenser (p. 940) be discharged by connecting the plates by a conductor of sufficiently small resistance, the charge does not merely disappear. It surges backwards and forwards between the plates, just as the water in a U tube does when disturbed, the discharge being then said to be oscillatory. It was shown afterwards by experiment that this is the case

The next step is the Electro-magnetic theory of James Clerke Maxwell, according to which any sudden alteration in the electrical field anywhere causes a disturbance to travel outwards through space with the velocity of light (3×10^{10} cm. per sec.). It follows that the oscillatory field due to the discharge of a condenser causes waves to travel outwards. The presence of these waves was at a later date demonstrated by Hertz and also by Lodge. It only remained to find a sensitive detector of these waves, and this was discovered by Branley, and took the form of a tube of loosely packed metal filings, whose conductivity was enormously increased when the electro-magnetic waves fell on them. The process of signalling here briefly sketched was adapted to practical telegraphy by Marconi and others.

Oscillatory discharge.—On p. 926 the electric lines of force are given for the case in which two spheres have opposite electric charges. Suppose that these spheres are now connected by a wire AB (Fig. 986 (*a*)). A being at a higher potential than B, a current immediately begins to flow from A to B, that is, the positive charges or positive ends of the electric lines of force travel from A to B and the negative ends from B to A This is of course accompanied by the production of magnetic lines of force (p. 830), which are circles surrounding the wire (Fig. 986 (*b*)). These grow until the current has reached its greatest value, at which instant the electric charges and electric lines of force have just disappeared. The magnetic lines of force now begin to collapse upon the wire, and in so doing produce an e.m.f. which causes the current to continue flowing from A to B until the magnetic lines of force have all collapsed and disappeared. This continued current means that B is acquiring a positive charge and A a negative charge (Fig. 986 (*c*)), and at the end of this stage the charges and electric field are exactly the reverse of those in Fig. 986 (*a*), except that there may be a diminution of charge due to energy being dissipated in the heat produced by the current in the conductor, and also by any radiation to be described below.

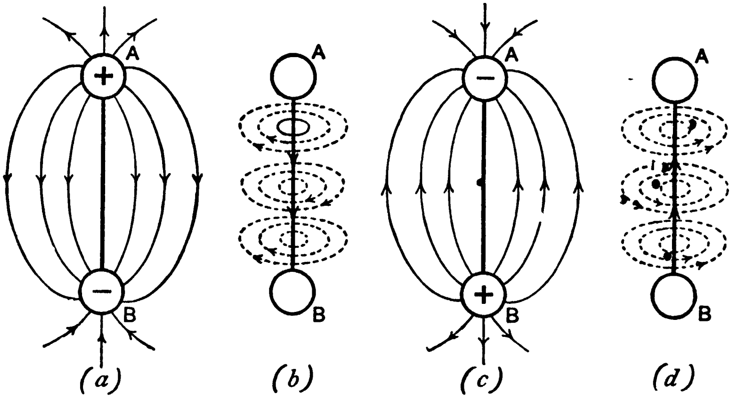


FIG. 986.—Oscillatory discharge.

The return current then immediately begins, and is shown at its greatest value in Fig. 986 (*d*).

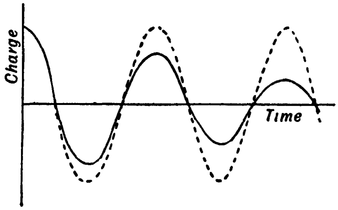


FIG. 987.—Damped and undamped oscillations.

The discharge in this case is said to be **oscillatory**, and it is easily seen that the energy exists alternately in the form of electric field and magnetic field. If there is no dissipation of energy, or if energy is supplied continuously by some source, the curve representing the amount of electric charge at succeeding intervals of time is a sine curve, shown dotted in Fig. 987, and the oscillation is said to be **undamped**. If, on the other hand, the energy dies away, the oscillation is said to be **damped**, and is shown by the continuous line curve in Fig. 987. In the case of wireless telegraphy the frequency of oscillation may vary from 2 × 10^4 to 3 × 10^6 complete oscillations per second.

Radiation.—It must be understood that the process of radiation can only be described with completeness in terms of mathematics of a higher order than is required for the rest of this book, but the following explanation may help the student to some understanding of the manner in which electric waves arise. When the difference of potential between the ends of the conductor is very great, the electric **lines of force** may be similar to ACB, (Fig. 988 (*a*)), and if the con-

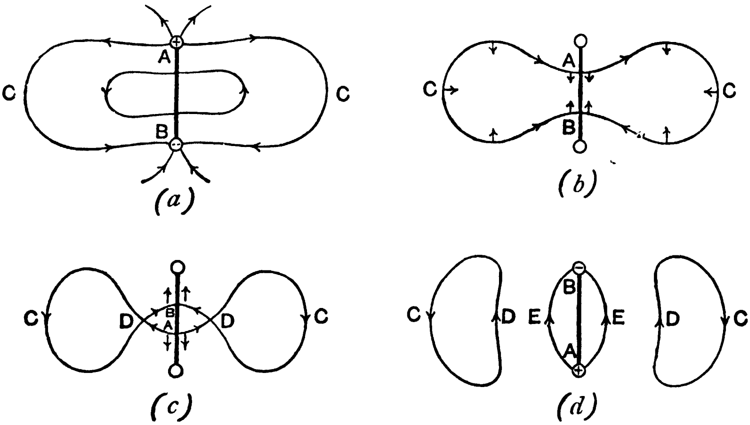


FIG. 988.—Production of electric waves.

ductivity of the conducting bridge AB, which is usually a spark (p. 1028), is great, the ends of the lines of force travel as in Fig. 988 (*b*) and may reach each other during the discharge, and even cross, before the part C reaches the conductor, thus forming a loop DC (Fig. 988 (*c*)). The intersection D is a point of instability and the lines immediately break, forming a closed loop DC and a shortened line AEB (Fig. 988 (*d*)). The loop DC travels outwards, and the part AEB continues the oscillation previously described.

In wireless telegraphy the conductor is of considerable length, in order to produce waves of sufficient size, and is then called an **aerial** or **antenna.** Aerials are of many forms, but their function is the same, that is, to increase the size of wave produced. It should be remembered that only half of each complete loop is produced by the aerial, the other part, which would be produced if the aerial were symmetrical about the spark gap S (Fig. 989), being suppressed by the earth. A few waves emitted by an aerial are illustrated in Fig 989.

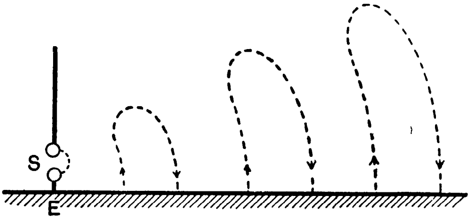


Fig. 989.—Waves from an aerial.

The waves are transmitted with the velocity of light, that is 3×10^{10} cm. per sec. as has been noted above ; in fact, they are waves of the same character as light waves, but are of vastly greater length. The average wave-length of light is about 6×10^{-5} cm., while those used in wireless telegraphy have a wave-length varying from 100 metres to 15,000 metres.

The usual relation between frequency (n), wave-length (λ), and velocity (v) holds here as in the case of all other waves (p. 680) that is,

$$\text{velocity} = \text{wave-length} \times \text{frequency}$$

or

$$v = \lambda n.$$

As an example, consider a case in which the frequency of oscillation is 120,000. Then, since the velocity is 3×10^{10} cm. per sec.

$$3 \times 10^{10} = \lambda \times 120{,}000,$$

$$\lambda = 2{\cdot}5 \times 10^{5} \text{ cm.}$$

That is, the wave-length is 2500 metres.

Production of oscillations.—There are several methods of producing electric oscillations in the aerial, but that most commonly used consists in raising the p.d. between two conductors to such a high value, by means of an induction coil (p. 983), that a spark occurs. For the transmission of powerful waves the induction coil is replaced by an alternator (p. 1000), with a step-up transformer (p. 984) to give sufficiently high p.d. for the production of the sparks. The spark gap may be in the aerial itself (Fig. 990), which shows an **open oscillatory circuit**, or the secondary coil of a transformer may be in the aerial (Fig. 991), which is an example of a **closed oscillatory circuit.** In this case the induction coil I raises the p.d. between A and B to such a magnitude that a spark occurs between A and B. This is accompanied by oscillations in the circuit consisting of the spark-gap AB, the condenser C and the primary coil L_1 of the transformer L_1L_2, sometimes called a **jiggar.** This oscillation is really an alternating current, so that an alternating e.m.f. is produced (p. 985) in the secondary coil L_2. Thus for every spark at AB a train of waves will be given out into space by the aerial. Such a train of waves is of the damped type (Fig. 987), because the energy corresponding to each discharge of the condenser C is rapidly radiated outwards in the form of waves from the aerial.

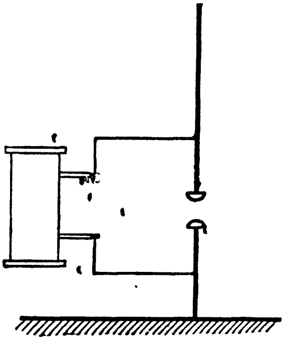


FIG. 990.—Open oscillatory circuit.

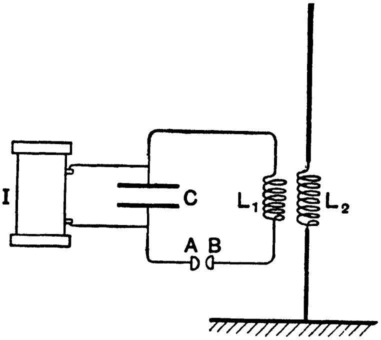


FIG. 991.—Closed oscillatory circuit.

The frequency of oscillation in any circuit in which the resistance is small is given by the equation

$$n = \frac{1}{2\pi\sqrt{LC}},$$

where L is the self-inductance (p. 986) and C the capacity (p. 943) in the circuit. As an example, let the capacity be 6×10^{-9} farads or 0·006 micro-farads, and the self-inductance 80 micro-henrys or 8×10^{-5} henrys.

Then
$$n = \frac{1}{2\pi\sqrt{6 \times 10^{-9} \times 8 \times 10^{-5}}}$$
$$= \frac{1}{2\pi\sqrt{48 \times 10^{-14}}} = 2{\cdot}3 \times 10^{5}$$
$$\therefore \text{ wave-length} = \frac{3 \times 10^{10}}{2{\cdot}3 \times 10^{5}} \text{ cm.}$$
$$= \underline{1300 \text{ metres.}}$$

Receiving.—It should be remembered that an electric line of force is really the representation of an electric field (p. 926) and that a difference of potential exists between different points on a line of force. Hence the arrival of an electric wave at a conductor causes differences of potential between the parts of a conductor and gives rise to an electric current. Thus, imagine waves travelling from left to right to arrive at the conductor AB (Fig. 992).

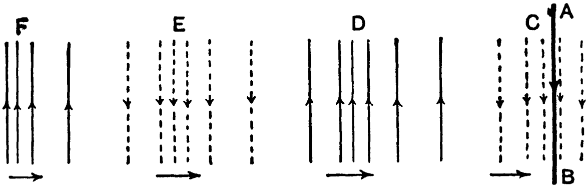


FIG. 992.—Waves arriving at a conductor.

On the arrival of the part C or E of the wave, there will be a current in the conductor from A to B, while the parts D and F will produce current from B to A. Alternating current will therefore flow in AB during the arrival of electric waves. It might perhaps be thought that by including a telephone receiver in AB a sound would be heard; but a moment's consideration will show that this is not the case. The frequency of the waves employed in wireless telegraphy is such that any note produced would be of too high a pitch to be audible (p. 672). Moreover, the oscillations are too rapid for the diaphragm of the telephone to follow to any appreciable extent. Some other means, such as the **coherer** or the **rectifier**, must be employed for their detection.

The Coherer.—The **coherer** is the earliest device used in wireless telegraphy, and consists of a tube AB (Fig. 993) into which the conductors C and D enter, the gap between which is completed by a small quantity of metal filings E, usually silver. When the feeble current produced in the aerial F by the waves received, passes through the coherer, its resistance is reduced to such an extent that the current in the local circuit due to the cell H becomes large enough to actuate the galvanometer, or sounder (p. 1006), or a relay G (p. 1008). Thus long or short trains of waves give rise to dashes or dots according to the Morse system (p. 1005).

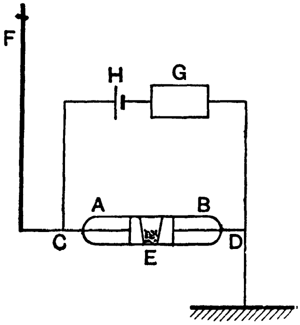


Fig. 993.—Coherer.

One great drawback to the coherer is that it does not ~~automatically~~ "decohere"; that is, the conductivity remains comparatively high after the train of waves has ceased. It recovers, however, its normal high resistance when subjected to mechanical disturbance, such as tapping, and this necessitates the employment of some device for tapping it to cause it to decohere after each train of waves.

Crystal detector.—It will be remembered that the waves, and therefore the oscillatory currents, in the receiving aerial are too rapid to produce any effect in the telephone. If, however, the negative halves of the waves, B, D and F (Fig. 994) could ~~be suppressed~~, the positive halves A, C and E, which follow each other very rapidly, would combine to produce a pull on the diaphragm of the telephone. This can be effected by the **crystal detector.**

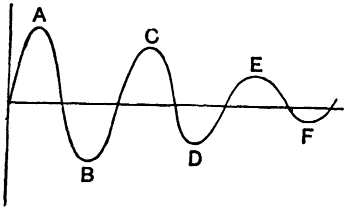


Fig. 994.—Damped wave.

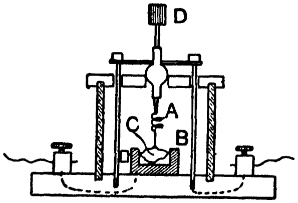


Fig. 995.—Crystal detector.

A crystal of suitable substance, carborundum, zincite, galena, hertzite or midite is placed in contact with a metal of fairly high conductivity. This contact conducts much more freely for a current in one direction than for a current in the reverse direction. The crystal detector is a highly efficient form of rectifer and has many forms. A typical form is illustrated in Fig. 995. The crystal of silicon, hertzite, midite, or some such substance, is held in a brass cup, and resting in light contact with it is a fine wire A, sometimes called a "cat's whisker." This wire should be of some non-corrodable material, such as gold or silver, and the best spot upon the crystal can be found by adjusting by means of the handle D. For efficient rectification, the point of the "cat's whisker" in contact with the crystal must be very small. The resistance of the contact when the current flows from the crystal to the wire may be thousands of ohms, while the resistance for a current flowing from the wire to the crystal is only tens of ohms. In Fig. 996 the crystal detector C is seen in use with a simple type of receiving circuit. The aerial is shown at A, and the incoming waves set up oscillations in the circuit, consisting of an adjustable inductance coil L and the condenser C_1, the telephones T being connected across the condenser. Whenever a train of waves arrives at the receiving station, a rapidly intermittent current will pass in the telephone and a short sound will be heard, one corresponding to each spark at the sending station. This will build up into a buzz or hum whose duration is equal to that for which the sending key is pressed and the sparks produced.

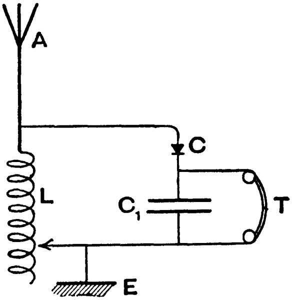


Fig. 996.—Crystal detector circuit.

It frequently happens that the rectifying power of the crystal is greatest when there is a constant e.m.f. acting across the contact. The reason for this may be seen by examining the curve connecting current across the contact and the p.d. (Fig. 997). This curve is usually concave upwards as shown. If then the constant potential difference Oa be applied by means of a battery and ab and ac be the positive and negative values of the p.d. due to the waves received, ab causes an increase ef in the current through the crystal and ac causes a decrease gh. As the former is greater than the latter the resultant is an excess of

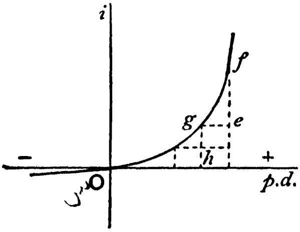


Fig. 997.—Rectification.

current in one direction, which is the condition for a sound to be heard in the telephone. In the case of many crystals at present in use the rectification is sufficiently good at the point O (Fig. 997), and the use of the auxiliary battery is unnecessary.

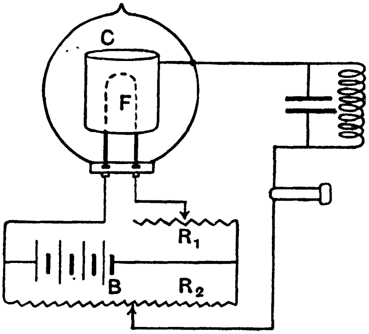


FIG. 998.—Rectifying valve.

Rectifying valve.—It is well known that hot bodies emit electrons (p. 1031), which are small charges of negative electricity. If the hot body is at a higher potential than surrounding bodies, these electrons will not pass away from it but will tend to remain upon it. On the other hand, if it is at a lower potential than surrounding bodies, the electrons, being negative charges, will be driven away from it. This stream of **negative** charges from it constitutes a **positive** electric current **towards** it. Hence the current passes through the surrounding space more readily in one direction than in the other, and the arrangement therefore forms a **rectifying valve** whose function is similar to that of the crystal dete[illegible] There are many forms of the apparatus, one being shown in Fig. 998. The hollow cylindrical conductor C surrounds the filament F of an incandescent lamp. The battery B serves to maintain the filament in a state of incandescence, and may also be used in conjunction with the resistance R_2 to maintain the necessary p.d. between F and C to obtain the best rectification.

Three-electrode valve, audion or triode.—In an incandescent lamp employed as a valve, the gas is almost entirely exhausted from it, and any current which passes on to the filament does so on account of the electrons emitted from it. If then a wire gauze or grid G (Fig. 999) surrounds the filament, the electrons emitted will charge the gauze negatively and the passage of them will nearly cease; the

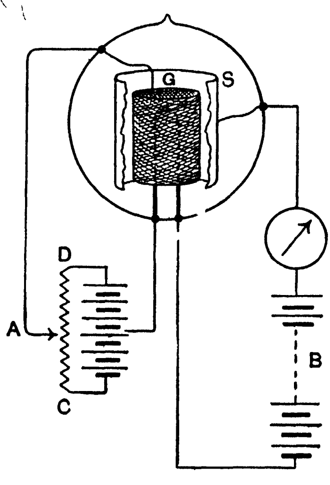


FIG. 999.

space round the filament is then practically a non-conductor for positive current towards the filament. This effect is increased if the grid be made at a negative potential with respect to the filament by bringing the movable contact A to the negative C. The battery B is then unable to send current from the sheath S to the filament.

If, however, A be raised until the grid is at a positive potential with respect to the filament, electrons will now pass freely towards the grid and reach the sheath, which means that the battery B can now produce considerable current. A small negative potential of grid over filament therefore nearly stops the current from B and a small positive potential allows a large current. This amplification is just of the kind necessary to enable feeble oscillations in the aerial to produce a large effect in the telephone. It will also be seen that the valve is capable of acting as a rectifier and so take the place of the crystal. An arrangement of a simple circuit in which the valve is used as a rectifier is shown in Fig. 1000. The variable condenser C allows the oscillatory circuit LC to be tuned, and oscillations in p.d. then occur between the grid G and the filament F. The battery B of 60 to 100 volts then produces rectified currents in the circuit consisting of the sheath or plate P and the telephones T. When used as a rectifier the very high resistance R of about a million ohms, with condenser C_1, must be employed. It is also a common practice to couple valves in series so that the amplified oscillations in the plate circuit of one are applied to the grid circuit of the next, and so are further amplified. This may be continued through a succession of

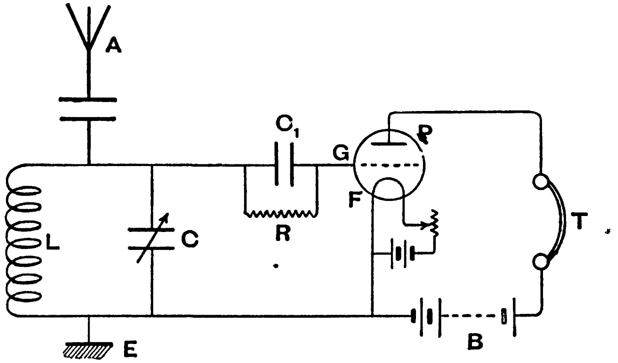


FIG. 1000.—Receiving circuit with rectifying and amplifying valve.

valves so that the feeble oscillations received at the aerial produce very intense oscillations in the telephones of the last valve. But it must be remembered that between each valve and the next there must be a transformer to reduce the high voltage oscillations in the plate circuit to low voltage oscillations to be applied to the grid of the next valve. Also the character of the transformer depends upon whether the rapid, unrectified oscillations are to be amplified, which is known as high frequency amplification, or whether the rectified oscillations of audible frequency are to be amplified, which is known as low frequency amplification.

Reaction.—A device which is largely used for multiplying the intensity of the received oscillations is to couple a coil in the telephone circuit, with the inductance in the receiving circuit, to form a transformer. Thus the oscillations in L_1 (Fig. 1001) will produce an oscillating e.m.f. in L, which, if properly timed, will increase the oscillations in the grid circuit.

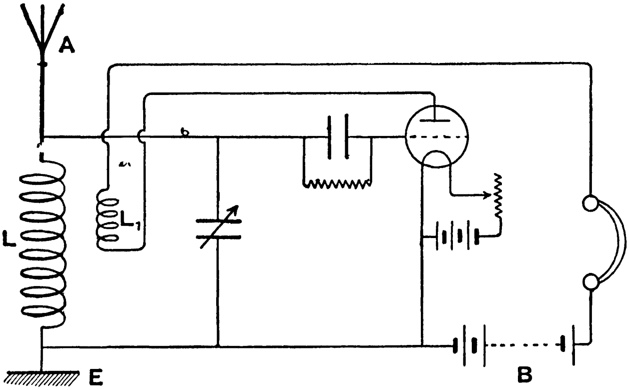


FIG. 1001.—Receiving circuit with reaction.

Wireless telephony.—In wireless telegraphy the transmitting waves are started and stopped by a key (p. 1004), and so the sounds heard may be of long or short duration, corresponding to the dashes and dots of the Morse code. But if the waves are modulated in intensity with the proper frequency, a note corresponding to that frequency is heard in the telephones. The building up of such simple notes into the sound produced by musical instruments and the human voice has been described in Chapter LIII. It only remains, then, to see how oscillations of properly modulated intensity can be produced in a sending circuit, to understand how wireless telephony is possible. The earlier attempts to vary the intensity of the oscillations by means of microphones (p. 1010) which vary the resistance of the circuit were not very successful, and were rendered unnecessary by the invention of the three-electrode valve or triode.

The production of oscillations in the current in the valve is of considerable complexity. It will be seen from Fig. 1001 that a large part of the energy of the oscillation in the plate circuit is drawn from the battery B, and that when these oscillations become sufficiently intense the transformer LL_1 will cause oscillations in the aerial and radiation of waves will result. In one condition the circuit is very sensitive, and the oscillations are on the point of ceasing. In this condition a current in one direction will increase the oscillations and in the other will reduce them. If, then, a local microphone circuit, such as M (Fig. 954), is coupled with either the grid or the plate circuit, the fluctuations of current due to the voice acting upon the microphone will cause large fluctuations of the same frequency in the oscillations in the plate circuit, and the waves emitted by the aerial will vary in intensity in a manner which copies the fluctuations produced in the microphone by the speaker. At the receiving station the sounds in the telephone will be a copy of those which actuate the microphone. The real distinction between the wireless telegraph and telephone is that in the former the waves are abruptly started and stopped by a key, and in the latter they are modulated in intensity to reproduce the sounds at the sending station.

Tuning and resonance.—It was seen on p. 1044 that every electrical circuit has a natural frequency of oscillation of current in it. If an oscillating e.m.f. acts upon a circuit, the oscillating current produced will be greatest when the frequency of the applied e.m.f. is the same as the natural frequency of oscillation for the circuit. In other words, the circuit **resounds** to the applied e.m.f. just as a vibrating body resounds to an impulse of suitable frequency (p. 713). This resonance plays a most important part in wireless telegraphy. The oscillatory circuit of the aerial must be **tuned** to the frequency of the received waves. This applies to all the oscillatory circuits used. The method of tuning is to vary the inductance or capacity or both, in each circuit, but the means of doing this and the results obtained are beyond the scope of this work. It may, however, be noted that the possibility of picking out the signals from one station and ignoring those from others rests upon this principle of tuning and resonance.

Short-wave telegraphy.—Until recently the wave-length employed for long-distance wireless telegraphy has been great, amounting in some cases to 10 kilometres. Lately, experiments have been made by Mr. Marconi with wave-lengths so small as 100 or less metres, and the results have justified this attempt. It has been found possible to transmit speech from England to Australia by these short waves. A further development is in the use of parabolic reflectors (p. 563) having the aerial in the focus. These reflectors consist of vertical wires hanging from a support in the form of a parabola, but for great efficiency and for preventing the spreading out of the beam of waves, the height and width of the parabola must be several times the wave-length of the radiation employed. Already an increase of 200 times the sensitiveness has been attained by these reflectors, and there is no reason why this sensitiveness should not be increased much further.

EXERCISES ON CHAPTER LXXXI.

1. Give a short account of the development of wireless telegraphy.

2. Describe the processes occurring during the oscillatory discharge of a condenser.

3. Give the difference between an open oscillatory circuit and a closed oscillatory circuit, for the production of electro-magnetic waves.

4. Describe some form of receiver for the detection of minute oscillatory currents.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TRIGONOMETRICAL TABLE.

Angle.	Radians.	Sine.	Tangent.	Cotangent.	Cosine.		
0°	0	0	0	∞	1	1·5708	90°
1	·0175	·0175	·0175	57·2900	·9998	1·5533	89
2	·0349	·0349	·0349	28·6363	·9994	1·5359	88
3	·0524	·0523	·0524	19·0811	·9986	1·5184	87
4	0698	·0698	·0699	14·3006	·9976	1·5010	86
5	·0873	·0872	·0875	11·4301	·9962	1·4835	85
6	·1047	·1045	·1051	9·5144	·9945	1·4661	84
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ANSWERS

PART V. MAGNETISM AND ELECTRICITY

CHAPTER LXI. p. 775.

5. 13·5 dynes.
6. 8/9 dyne, parallel to needle.
7. ±18·91 dynes.
8. 56·4 units.
10. 1·633 cm.; 1·067 dynes.

CHAPTER LXII. p. 791.

4. 0·453 c.g.s. units.
5. 5·68 sec.
7. 1/8 c.g.s. unit.
8. 25 : 7.
9. 450 c.g.s. units.
10. 9 : 16 (same direction); 41 : 16 (opposite direction).
11. 0·278 dyne.
13. (*a*) 80½°; (*b*) 36°; (*c*) −32° with axis of magnet.
14. 12500 c.g.s. units.
15. 1·236 c.g.s. unit.

CHAPTER LXIII. p. 808.

7. $\mathrm{H} = m \cos(\text{lat.})/r^3$, $\tan(\text{dip}) = 2 \tan(\text{magnetic latitude})$.
8. (*a*) 0·766; (*b*) 0·541.
9. 0·208 c.g.s. unit.
10. $\tan(\text{true dip}) = \sin \mathrm{A} \tan(\text{observed dip})$.
11. 18·4.

CHAPTER LXIV. p. 828.

2. Magnetic mcment $= 1{\cdot}508 \times 10^6$ c.g.s. units; time changes in ratio 1 : 0·98.
4. 53·39 c.g.s. units; 1005·4 c.g.s. units.
6. 3307 c.g.s. units.
7. 0·926.
8. $5{\cdot}03 \times 10^{-4}$ c.g.s. unit.
9. $\mathrm{M} = 2000$; $m = 100$; $\mathrm{I} = 100$ c.g.s. units.
10. $\mathrm{M} = 1470$ c.g.s. units; 0·00267 c.g.s. unit.
11. 5260 c.g.s. units.
12. $\mathrm{M} = 2500$ c.g.s. units; $m = 250$ c.g.s. units.
13. $1{\cdot}766 \times 10^6$ dynes.

CHAPTER LXV. p. 838.

3. 104·7 c.g.s. units.
4. 0·0487 absolute c.g.s. unit
7. $n_\mathrm{A} : n_\mathrm{B} = 10 : 6$.
8. 0·001504 to 0·1489 ampere.
9. Increase number of turns to 2292.
10. 0·943 c.g.s. unit; 0·943m dynes.
11. 0·257 absolute c.g.s. unit.
12. 0·288 amp.; 0·866 amp.

Chapter LXVI. p. 846.

3. 57·5 volts ; 143·7 watts. **4.** $7{\cdot}2 \times 10^{10}$ ergs.

5. 0·0603 H.P. ; 0·225 amp. **6.** 0·0997° C. **7.** 14·4 ohms.

9. 0·015 volt ; 25 volts. **10.** 27·8 volts ; 55·6 ohms.

11. $7{\cdot}488 \times 10^{5}$ joules ; 104 volts. **12.** 2*s.* 8·4*d.*

13. 18·4 kilowatts ; 24·7 H.P. **14.** 9·13 amp.

15. 3·58 amp. ; 3·58*d.*

Chapter LXVII. p. 857.

2. 2·73 ohms. **3.** 1·6364, 0·8182, 0·5454 amp.

4. 210 ohms. **5.** $4{\cdot}91 \times 10^{-6}$. **6.** 40·5 ohms.

7. 140 calories. **8.** 3 ohms ; 0·5 amp.

9. 0·727 amp. ; 4·36 volts. **10.** 32 : 15.

11. I $= \frac{1}{3}$ amp. ; resistance of battery = 6 ohms.

12. (*a*) 0·6 amp. ; (*b*) 0·75 amp. ; (*c*) 0·6 amp.

13. 0·976, 0·732 amp. **14.** (*a*) 0·48 amp. ; (*b*) 0·04 amp. **15.** 280 ohms.

16. Reduce external resistance to $6\frac{2}{3}$ or $3\frac{3}{4}$ ohms. **17.** 0·5 ohm ; 1·5 volt.

18. 0·0513 ohm ; 48 watts. **19.** 0·75 volt ; 0·375, 0·25 amp.

20. 18·6 H.P. **22.** 60000 ohms.

Chapter LXVIII. p. 875.

1. 3·37 cm. **2.** 343·3 ohms. **3.** 0·0240, 0·00120 ohm.

4. 0·2001 ohm ; 2·7 amp. **5.** 3·03 ohms ; 3 ohms.

8. (*a*) 0·04 ohm in parallel with G ; (*b*) 961 ohms in series with G.

9. 24·6, 8·77 milliamp. **10.** (*a*) 995·5 ohms ; (*b*) 0·004504 ohm.

11. 766·2 ohms.

Chapter LXIX. p. 896.

1. 0·5628 ohm. **2.** $E_A : E_B = 3 : 5$. **4.** 0·266 ohm.

5. 0·0000262. **6.** 3·733 : 1. **7.** 1·36 ohm.

8. G = 40 ohms. **9.** 1·15 volt.

10. 5·505 calories ; 1·966 calorie. **11.** 21·38 ohms.

12. $2{\cdot}72 \times 10^{-5}$. **13.** 1·185 ohm. **14.** 0·02 ohm. **15.** +0·4 cm.

Chapter LXX. p. 918.

2. 39·05. **6.** 20 c.c., 16° C., and 19° 25′. **7.** 0·1795 C.G.S. unit.

9. 0·0261. **11.** 20·2° C. **12.** 0·1838 gram.

15 −0·04.

CHAPTER LXXI. p. 927.

2. 60 cm. from −18 and on side opposite to +50.

3. $q^2/60$ dynes, attraction ; $q^2/900$ dynes, repulsion.

4. 0·6 dyne, from B towards D.

5. (*a*) $4(r^2 - \frac{1}{4}l^2)^2/rl^3$; (*b*) $8(r^2 + \frac{1}{4}l^2)^{\frac{3}{2}}/l^3$. **8.** F/3 dynes, repulsion.

CHAPTER LXXII. p. 944.

2. Q = 5850 C.G.S. units ; 87750 ergs.

3. $Q_A = 556 \cdot 25$; $Q_B = 333 \cdot 75$. **4.** $\sigma_1 : \sigma_2 = 1 : \sqrt[3]{2}$.

5. (*a*) $C_1 + C_2$; (*b*) $C_1C_2/(C_1 + C_2)$.

6. At first, charges $+e$ outside, $-e$ inside, potential $= +e/r_2$; (*a*) charges 0 outside, $-e$ inside, potential $= 0$; (*b*) charges $-(r_2 - r_1)e/r_2$ outside, $-r_1e/r_2$ inside, potential $= (r_1 - r_2)e/r_2^2$, where r_1 and r_2 are the radii of the inner and outer spheres respectively.

8. $Q^2/2r$ units. **9.** (*a*) 0·0444 dyne ; (*b*) +2 C.G.S. units.

10. 156 C.G.S. units ; 0·080 erg.

11. 0·2 microcoulomb ; 0·3 microfarad.

12. 14 units. **13.** From A to B.

14. 5·139 ergs ; 3·354 ergs. **15.** 134·4 ergs.

CHAPTER LXXIII. p. 956.

2. 90 C.G.S. units, 32400 ergs in air ; 36 C.G.S. units, 12960 ergs in the other medium.

6. 218·4 C.G.S. units ; 698880 ergs. **7.** 2088 C.G.S. units.

9. 150000 C.G.S. units ; 0·0333 erg. **10.** 157·1 ergs.

11. From A to B.

CHAPTER LXXV. p. 975.

2. 7·55 gauss. **3.** 20·5 C.G.S. units. **4.** 1910.

6. 4240 C.G.S. units. **9.** 0·0197 C.G.S. unit.

10. 1·25 dyne. **12.** $7 \cdot 12 \times 10^4$ C.G.S. units.

CHAPTER LXXVI. p. 987.

6. $8 \cdot 48 \times 10^{-4}$ volt. **7.** 2·13 volts.

CHAPTER LXXVII. p. 1002.

13. H.P. $= 2 \cdot 68 \times 10^{-7} n\text{N}$. **14.** 0·969 ; 0·806. **15.** 0·906 ; 0·958.

CHAPTER LXXVIII. p. 1014.

10. 3·96 pence ; 6·4 pence.

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